$\boxed{\text{Laws of Logarithms}}$

$$\log_b M + \log_b N = \log_b (MN)$$

$$\log_b M - \log_b N = \log_b \left(\frac{M}{N} \right)$$

$$\log_b (N^p) = p \log_b (N)$$

Warm Up

Review of laws of logarithms...

Given that $\log_x M = -3$, $\log_x N = 5$ **and** $\log_x P = 4$, **evaluate the following logarithmic expression:**

$$\log_x \left[\frac{(M^3 N)^2 \sqrt{P}}{MP} \right]$$

$$\log_x \left[\frac{M^6 N^2 P^{1/2}}{MP} \right]$$

$$\log_x \left[M^5 N^2 P^{-1/2} \right]$$

$$\log_x \left[\frac{M^5 N^2}{P^{1/2}} \right]$$

$$\log_x M^5 + \log_x N^2 - \log_x P^{1/2}$$

$$5\log_x M + 2\log_x N - \frac{1}{2}\log_x P$$

$$5(-3) + 2(5) - \frac{1}{2}(4)$$

$$-15 + 10 - 2$$

$$-7$$

Solve the following equation: $$\frac{3^{x-1}}{5 * 2^{3x}} = 6^{1-2x}$$

take log of both sides

$$\log\left(\frac{3^{x-1}}{5 \cdot 2^{3x}}\right) = \log 6^{1-2x}$$

$$\log 3^{x-1} - \log 5 - \log 2^{3x} = \log 6^{1-2x}$$

$$(x-1)\log 3 - \log 5 - 3x\log 2 = (1-2x)\log 6$$

$$x\log 3 - \log 3 - \log 5 - 3x\log 2 = \log 6 - 2x\log 6$$

$$x\log 3 - 3x\log 2 + 2x\log 6 = \log 6 + \log 3 + \log 5$$

$$x(\log 3 - 3\log 2 + 2\log 6) = \log 6 + \log 3 + \log 5$$

$$x = \frac{\log 6 + \log 3 + \log 5}{\log 3 - 3\log 2 + 2\log 6}$$

$$x = \frac{\log 6 + \log 3 + \log 5}{\log 3 - \log 2^3 + \log 6^2}$$

$$x = \frac{\log(6 \cdot 3 \cdot 5)}{\log\left(\frac{3 \cdot 36}{8}\right)}$$

$$x = \frac{\log 90}{\log 13.5} \simeq 1.73$$

Questions from Homework

Rule: $d(\ln u) = \frac{1}{u}\,du = \frac{du}{u}$

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$u = \left(\frac{x}{2x+3}\right)^{1/2} = \frac{(x)^{1/2}}{(2x+3)^{1/2}}$

① k) $y = \ln\sqrt{\frac{x}{2x+3}} = \ln\left(\frac{x}{2x+3}\right)^{1/2}$

$y' = \frac{(2x+3)^{1/2}}{x^{1/2}}\left[\frac{1}{2}\left(\frac{x}{2x+3}\right)^{-1/2} \cdot \frac{1(2x+3)-2x}{(2x+3)^2}\right]$

$y' = \frac{(2x+3)^{1/2}}{x^{1/2}}\left[\frac{1}{2}\,\frac{(2x+3)^{1/2}}{x^{1/2}} \cdot \frac{3}{(2x+3)^2}\right]$

$y' = \frac{3\cancel{(2x+3)}}{2x\,(2x+3)^{\cancel{2}}} = \frac{3}{2x(2x+3)} = \frac{3}{4x^2+6x}$

m) $y = \ln(\sec x + \tan x)$

$u = \sec x + \tan x$

$du = \sec x \tan x \cdot 1 + \sec^2 x \cdot$

$= \sec x \tan x + \sec^2 x$

$y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$y' = \frac{\sec x\,\cancel{(\tan x + \sec x)}}{\cancel{(\sec x + \tan x)}} = \sec x$

Questions from Homework

① j) $y = \ln\left(\frac{x+1}{x-1}\right)$ $u = \frac{x+1}{x-1}$

$1 \div \frac{x+1}{x-1}$

$1 \times \frac{x-1}{x+1} = \frac{x-1}{x+1}$

$$y' = \left(\frac{x-1}{x+1}\right)\left(\frac{1(x-1) - 1(x+1)}{(x-1)^2}\right)$$

$$y' = \left(\frac{x-1}{x+1}\right)\left(\frac{x-1-x-1}{(x-1)^2}\right)$$

$$y' = \left(\frac{\cancel{x-1}}{x+1}\right)\left(\frac{-2}{(x-1)^{\cancel{2}}}\right)$$

$$y' = \frac{-2}{(x+1)(x-1)} = \frac{-2}{x^2-1}$$

n) $y = \tan[\ln(1-3x)]$ $u = \ln(1-3x)$

$$y' = \sec^2[\ln(1-3x)] \cdot \frac{1}{1-3x} \cdot -3$$

$$y' = \frac{-3\sec^2[\ln(1-3x)]}{1-3x}$$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

$\log_3 x = \frac{\ln x}{\ln 3}$

$= \frac{\log M}{\log N} = \frac{\log_e M}{\log_e N} = \frac{\ln M}{\ln N}$

Rule: $d(\log_b u) = \frac{1}{u \ln b} du = \frac{du}{u \ln b}$

Differentiate:

$y = \log_6 x^3$

$b=6$
$u=x^3$
$du=3x^2$

$y' = \frac{1}{x^3 \ln 6} \cdot 3x^2$

$y' = \frac{3\cancel{x^2}}{x^{\cancel{3}} \ln 6} = \frac{3}{x \ln 6}$

$y = \log(5x^4)$

$b=10$
$u=5x^4$
$du=20x^3$

$y' = \frac{20x^3}{5x^4 \ln 10} = \frac{4}{x \ln 10}$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

$b = 3$
$u = 9x$
$du = 9$

Rule:

$$d(b^u) = b^u(\ln b)du \text{ , where } b \in R$$

$y = 3^{9x}$

$y' = 3^{9x}(\ln 3)9$

Try this one... $y = \pi^{x^5}$

$b = \pi$
$u = x^5$
$du = 5x^4$

$y' = \pi^{x^5}(\ln \pi)5x^4$

Practice Problems:

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#1 #2 a #3 #4

#5 #6 #7 #8

③ d) $g(x) = \dfrac{1 + \log_3 x}{x}$

$$g'(x) = \frac{\left(\frac{1}{\cancel{x}\ln 3} \cdot 1\right)\cancel{x} - 1(1 + \log_3 x)}{x^2}$$

$$g'(x) = \frac{\frac{1}{\ln 3}^{\ln 3} - 1^{\ln 3} - \log_3 x^{\ln 3}}{x^2 \ln 3}$$

CD: ln3

$$g'(x) = \frac{1 - \ln 3 - \ln 3(\log_3 x)}{x^2 \ln 3}$$

$\frac{\ln x}{\ln 3}$

$$g'(x) = \frac{1 - \ln 3 - \cancel{\ln 3}\left(\frac{\ln x}{\cancel{\ln 3}}\right)}{x^2 \ln 3} = \boxed{\frac{1 - \ln 3 - \ln x}{x^2 \ln 3}}$$

④ a) $y = x^3 + 3^x$

$b = 3$
$u = x$
$du = 1$

$y' = 3x^2 + 3^x(\ln 3)(1)$

$y' = 3x^2 + 3^x \ln 3$

$\frac{x^1}{x^{1/2}} = x^{1/2}$

④ c) $y = (x)(5^{\sqrt{x}})$

$b = 5$
$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}$

$y' = 1(5^{\sqrt{x}}) + x(5^{\sqrt{x}})\ln 5\left(\frac{1}{2}x^{-1/2}\right)$

$y' = \frac{5^{\sqrt{x}}}{1} + \frac{\sqrt{x}\,5^{\sqrt{x}}\ln 5}{2}$

$y' = \frac{2(5^{\sqrt{x}})}{2} + \frac{\sqrt{x}\,5^{\sqrt{x}}\ln 5}{2}$

$y' = \frac{2(5^{\sqrt{x}}) + \sqrt{x}\,5^{\sqrt{x}}\ln 5}{2}$

$y' = \frac{5^{\sqrt{x}}(2 + \sqrt{x}\ln 5)}{2}$

⑤ (1) find $f'(x)$

(2) Sub in x-value to find m

(3) $y-y_1=m(x-x_1)$

⑤ c) $y=10^x$ @ $(1,10)$

(i) $y=10^x$

$y'=10^x \ln 10 \cdot 1$

$y'=10^x \ln 10$

(ii) $y'=10^1 \ln 10$

$y'=10\ln 10$

$m=10\ln 10$

(iii) $y-y_1=m(x-x_1)$

$y-10=10\ln 10(x-1)$

$y-10=10x\ln 10-10\ln 10$

$0=10x\ln 10-y-10\ln 10+10$

$0=10x\ln 10-y-10(\ln 10-1)$

(7) $y = x\ln x$

$y' = 1\ln x + x\left(\frac{1}{x}\cdot 1\right)$

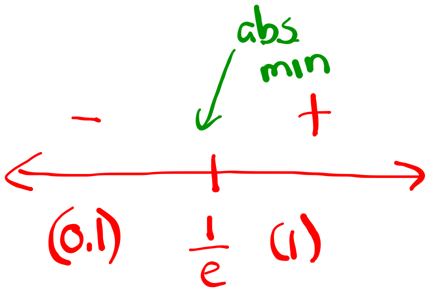
$y' = \ln x + 1$

CV: $0 = \ln x + 1$

$-1 = \ln x$ (log)

$e^{-1} = x$ (exp)

$\boxed{\frac{1}{e} = x}$



Find min

$y = x\ln x$

$y = \left(\frac{1}{e}\right)\ln\left(\frac{1}{e}\right)$

$y = \frac{1}{e}\boxed{\ln(e^{-1})}$

$y = \frac{1}{e}(-1) = \frac{-1}{e}$

$\boxed{\left(\frac{1}{e}, \frac{-1}{e}\right)}$