

$$\textcircled{1} \text{ dn } y = \frac{(x+1)^3}{(x+2)^5(x+3)^7}$$

$$\ln y = \ln \frac{(x+1)^3}{(x+2)^5(x+3)^7}$$

$$\ln y = \ln(x+1)^3 - \ln(x+2)^5 - \ln(x+3)^7$$

$$\ln y = 3\ln(x+1) - 5\ln(x+2) - 7\ln(x+3)$$

$$\frac{y'}{y} = 3\left(\frac{1}{x+1}\right) - 5\left(\frac{1}{x+2}\right) - 7\left(\frac{1}{x+3}\right)$$

$$\cancel{y} \cdot \frac{y'}{y} = \left[ \frac{3}{x+1} - \frac{5}{x+2} - \frac{7}{x+3} \right] y$$

$$y' = \left[ \frac{3}{x+1} - \frac{5}{x+2} - \frac{7}{x+3} \right] \left[ \frac{(x+1)^3}{(x+2)^5(x+3)^7} \right]$$

$$\textcircled{a} \text{ b) } y = x^{\sqrt{x}}$$

$$\ln y = \ln x^{\sqrt{x}}$$

$$\ln y = (\sqrt{x})(\ln x) \quad \text{product rule}$$

$$\frac{y'}{y} = \frac{1}{2} x^{-1/2} (\ln x) + x^{1/2} \left( \frac{1}{x} \right)$$

$$\frac{y'}{y} = \frac{1}{2} x^{-1/2} \ln x + x^{-1/2}$$

$$\cancel{y} \cdot \frac{y'}{\cancel{y}} = x^{-1/2} \left( \frac{1}{2} \ln x + 1 \right) \cdot y$$

$$y' = x^{-1/2} \left( \frac{1}{2} \ln x + 1 \right) x^{\sqrt{x}}$$

$$y' = x^{\sqrt{x} - 1/2} \left( \frac{1}{2} \ln x + 1 \right)$$

$$\textcircled{3} \quad y = x^x \quad @ \quad (2, 4)$$

Find  $y'$

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = (x) \ln x \quad \text{product}$$

$$\frac{y'}{y} = 1 \ln x + x \left( \frac{1}{x} \right)$$

$$y \cdot \frac{y'}{y} = (\ln x + 1) y$$

$$y' = (\ln x + 1) x^x$$

@ sub in  $x=2$

$$y' = (\ln 2 + 1) 2^2$$

$$y' = 4(\ln 2 + 1)$$

$$y' = 4\ln 2 + 4$$

$$m = 4\ln 2 + 4$$

$$\textcircled{3} \quad y - y_1 = m(x - x_1)$$

$$y - 4 = (4\ln 2 + 4)(x - 2)$$

$$y - 4 = 4x\ln 2 - 8\ln 2 + 4x - 8$$

$$y = 4x\ln 2 + 4x - 8\ln 2 - 8 + 4$$

$$y = 4x(\ln 2 + 1) - 8\ln 2 - 4$$

$$y = 4x(\ln 2 + 1) - 4(2\ln 2 + 1)$$

$$0 = 4x(\ln 2 + 1) - y - 4(2\ln 2 + 1)$$

## Warm Up

Differentiate each of the following:

1.  $f(x) = 6^{x^3} + \ln(\tan^{-1} 2x^4)$

$$f'(x) = 6^{x^3} (\ln 6) 3x^2 + \left( \frac{1}{\tan^{-1} 2x^4} \right) \left( \frac{1}{1 + (2x^4)^2} \right) (8x^3)$$

2.  $y = (8x - 1)^{\sqrt{x}}$

$$\ln y = \ln (8x - 1)^{\sqrt{x}}$$

$$\ln y = (\sqrt{x}) (\ln(8x - 1))$$

$$\cancel{y} \cdot \frac{y'}{\cancel{y}} = \left[ \frac{1}{2\sqrt{x}} \ln(8x - 1) + \sqrt{x} \left( \frac{8}{8x - 1} \right) \right] \cdot y$$

$$y' = \left[ \frac{\ln(8x - 1)}{2\sqrt{x}} + \frac{8\sqrt{x}}{8x - 1} \right] (8x - 1)^{\sqrt{x}}$$

## Derivative Rules

### Exponential Functions

$$d(b^u) = b^u \cdot (\ln b) \cdot du, \text{ where } b \in R$$

$$d(e^u) = e^u \cdot du, \text{ base is Euler's number}$$

### Logarithmic Functions

$$d(\log_b u) = \frac{1}{u \ln b} \cdot du, \text{ where } b \in R$$

$$d(\ln u) = \frac{1}{u} \cdot du, \text{ base is Euler's number}$$

### Inverse Trigonometric Functions

$$\frac{d(\sin^{-1} u)}{du} = \frac{1}{\sqrt{1-u^2}} du$$

$$\frac{d(\csc^{-1} u)}{du} = \frac{-1}{u\sqrt{u^2-1}} du$$

$$\frac{d(\cos^{-1} u)}{du} = \frac{-1}{\sqrt{1-u^2}} du$$

$$\frac{d(\sec^{-1} u)}{du} = \frac{1}{u\sqrt{u^2-1}} du$$

$$\frac{d(\tan^{-1} u)}{du} = \frac{1}{1+u^2} du$$

$$\frac{d(\cot^{-1} u)}{du} = \frac{-1}{u^2+1} du$$

Quiz Monday: Derivatives of Transcendental Functions

- Inverse Trigonometric Functions
- Exponential Functions
- Logarithmic Functions

## **Practice Test**



## Solutions

① a)  $f(x) = \log_5(x^3) + e^{\sin 5x}$   
 $f'(x) = \frac{3x^2}{x^3 \ln 5} + e^{\sin 5x} (\cos 5x)(5)$

b)  $y = \cos^{-1}(\sqrt{x}) - \ln(\ln x^3)$   
 $y' = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} (x^{-1/2}) - \frac{1}{\ln x^3} \left( \frac{1}{x^3} \right) (3x^2)$

c)  $h(t) = \frac{5^{\tan t}}{\ln(3t^4 + 5)}$   
 $h'(t) = \frac{\ln(3t^4 + 5) [5^{\tan t} (\ln 5) (\sec^2 t)] - 5^{\tan t} \left[ \frac{1}{3t^4 + 5} \cdot 12t^3 \right]}{[\ln(3t^4 + 5)]^2}$

d)  $y = (5 - 2x^2)^x$   
 $\ln y = \ln(5 - 2x^2)^x$   
 $\ln y = x \ln(5 - 2x^2)$   
 $\frac{1}{y} \cdot y' = x \left( \frac{-4x}{5 - 2x^2} \right) + \ln(5 - 2x^2)$   
 $y' = \left[ x \left( \frac{-4x}{5 - 2x^2} \right) + \ln(5 - 2x^2) \right] (5 - 2x^2)^x$

① e)  $y = \tan^{-1}(\ln^3(x^5-1))$   
 $y = \tan^{-1}(\ln(x^5-1))^3$   
 $y' = \frac{1}{1+(\ln^3(x^5-1))^2} [3(\ln(x^5-1))^2 \left(\frac{1}{x^5-1}\right) (5x^4)]$

f)  $g(x) = 4^{5x} e^{\sin^{-1}x}$   
 $g'(x) = 4^{5x} (e^{\sin^{-1}x})' \left(\frac{1}{1-x}\right) \left(\frac{1}{\sqrt{1-x^2}}\right) + 4^{5x} (\ln 4)(5) (e^{\sin^{-1}x})$

②  $y = \frac{(x^2-2x)^3 (8x^5)}{\sqrt{(5-x^2)^5} (e^{x^2+2})}$   
 $\ln y = \ln \left[ \frac{(x^2-2x)^3 (8x^5)}{\sqrt{(5-x^2)^5} (e^{x^2+2})} \right]$   
 $\ln y = 3 \ln(x^2-2x) + \ln 8x^5 - \frac{5}{2} \ln(5-x^2) - (x^2+2) \ln e$   
 $\ln y = 3 \ln(x^2-2x) + \ln 8x^5 - \frac{5}{2} \ln(5-x^2) - (x^2+2)$   
 $\frac{1}{y} \cdot y' = \left[ 3 \left( \frac{2x-2}{x^2-2x} \right) + \frac{40x^4}{8x^5} - \frac{5}{2} \left( \frac{-2x}{5-x^2} \right) - 5x^4 \right] y$   
 $y' = \left[ \frac{6x-6}{x^2-2x} + \frac{5}{x} + \frac{5x}{(5-x^2)} - 5x^4 \right] \left[ \frac{(x^2-2x)^3 (8x^5)}{\sqrt{(5-x^2)^5} (e^{x^2+2})} \right]$



③  $e^{3x-y^5} = 5^{xy^3}$

$$e^{3x-y^5} (3 - 5y^4 y') = 5^{xy^3} (\ln 5) (xy^2 y' + y^3)$$

$$3e^{3x-y^5} - 5y^4 y' e^{3x-y^5} = 5^{xy^3} \ln 5 (xy^2 y' + y^3)$$

$$3e^{3x-y^5} - 5y^4 y' \ln 5 y^3 = y' (5^{xy^3} \ln 5 (3xy^2 + 5y^4 e^{3x-y^5}))$$

$$\frac{3e^{3x-y^5} - 5y^4 \ln 5 y^3}{5^{xy^3} \ln 5 (3xy^2 + 5y^4 e^{3x-y^5})} = y'$$

④  $z = \sin^{-1}(y-3) - y^3 \quad y = 3e^x + x^2$

$$z = \sin^{-1}(3e^x + x^2 - 3) - (3e^x + x^2)^3$$

$$z' = \frac{1}{\sqrt{1-(3e^x+x^2-3)^2}} (3e^x + 2x) - 3(3e^x+x^2)^2 (3e^x+2x)$$

$$z' = \frac{3e^x+2x}{\sqrt{1-(3e^x+x^2-3)^2}} - 3(3e^x+x^2)^2 (3e^x+2x)$$

$$z'(0) = \frac{3+0}{1} - 3(3+0)^2 (3+0)$$

$$z'(0) = 3 - 81$$

$$z'(0) = -78$$

⑤  $F(x) = x^2 e^{2x}$

$$F'(x) = x^2(e^{2x})'(2) + 2x e^{2x}$$
$$= 2x^2 e^{2x} + 2x e^{2x}$$
$$= 2x e^{2x}(x+1)$$

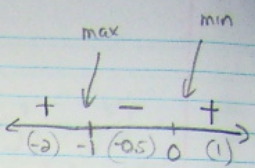
CV'  $x = 0, -1$

$$F(-1) = (-1)^2 e^{2(-1)}$$
$$= 1 e^{-2}$$
$$= 1 \cdot \left(\frac{1}{e^2}\right)$$
$$= \frac{1}{e^2}$$

$(-1, \frac{1}{e^2})$  Local max

$$F(0) = (0)^2 e^{2(0)}$$
$$= 0$$

$(0, 0)$  Local min



4. Find  $\frac{dz}{dx}$  at  $x=0$  given that  $z = \sin^{-1}(y-3) - y^3$  and  $y = 3e^x + x^2$

$$z = \sin^{-1}(y-3) - y^3 \quad \left| \quad y = 3e^x + x^2 \quad \left| \quad \begin{array}{l} \text{When } x=0 \\ y = 3e^{(0)} + (0)^2 \\ y = 3(1) + 0 \\ \underline{\underline{y = 3}} \end{array} \right. \right.$$

$$\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}} - 3y^2 \quad \left| \quad \frac{dy}{dx} = 3e^x + 2x$$

Since  $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$$\frac{dz}{dx} = \left[ \frac{1}{\sqrt{1-(y-3)^2}} - 3y^2 \right] [3e^x + 2x]$$

$$\frac{dz}{dx} = \left[ \frac{1}{\sqrt{1-(3-3)^2}} - 3(3)^2 \right] [3e^0 + 2(0)]$$

$$\frac{dz}{dx} = (-26)(3) = \boxed{-78}$$

May 06

1 a)  $f(x) = \frac{1}{x^2} \ln 5 + e^{\sin 5x} (\cos 5x)$       b)  $y' = \frac{-1}{\sqrt{1-(z/3)^2}} - \frac{1}{\ln x^2} \left(\frac{1}{x^2}\right) (2x)$

c)  $h'(t) = \frac{5^{\tan t} \ln 5 (\sec^2 t) \ln(3t^2+5) - 5^{\tan t} \left(\frac{1}{3t^2+5}\right)}{[\ln(3t^2+5)]^2}$

d)  $\ln y = x \ln(5-2x^2)$   
 $\frac{1}{y} y' = [\ln(5-2x^2) + x \left(\frac{1}{5-2x^2}\right) (-4x)] y$   
 $y' = \left(\frac{\ln(5-2x^2) - 4x^2}{5-2x^2}\right) (5-2x^2)^x$

e)  $y' = \frac{1}{1 + [\ln^2(x^2-1)]^2} \left[ 3[\ln(x^2-1)]^2 \left(\frac{1}{x^2-1}\right) (5x^2) \right]$

f)  $g'(x) = 4^{5x} \ln 4 (5) e^{\sin^2 \sqrt{x}} + 4^{5x} e^{\sin^2 \sqrt{x}} \left(\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-1/2}\right)$

2.  $\ln y = 3 \ln(x^2-2x) + \ln \pi x^2 - \frac{5}{2} \ln(5-x^2) - (x^2+2)$   
 $\frac{1}{y} y' = \left[ 3 \left(\frac{2x-2}{x^2-2x}\right) + \frac{40x^2}{\pi x^2} - \frac{5}{2} \left(\frac{-2x}{5-x^2}\right) - 5x^2 \right] y$   
 $y' = \left(\frac{3(2x-2)}{x^2-2x} + \frac{40}{\pi} + \frac{5x}{5-x^2} - 5x^2\right) \left(\frac{(x^2-2x)^3 (\pi x^2)}{(5-x^2)^{3/2} (e^{x^2+2})}\right)$

3.  $e^{3x-y^5} (3-5y^4 \frac{dy}{dx}) = 5^{xy^3} \ln 5 (y^3 x (3y^2 \frac{dy}{dx}))$   
 $3e^{3x-y^5} - 5y^4 e^{3x-y^5} \frac{dy}{dx} = 5^{xy^3} \ln 5 y^3 + 3xy^2 5^{xy^3} \ln 5 \frac{dy}{dx}$   
 $\frac{3e^{3x-y^5} - 5^{xy^3} \ln 5 y^3}{3xy^2 5^{xy^3} \ln 5 + 5y^4 e^{3x-y^5}} = \frac{dy}{dx}$

4.  $\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}} (1) - 3y^{-2}$       with  $x=0$   
 $\frac{dy}{dx} = 3e^x + 2x$   
 $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = \frac{dz}{dy} \cdot dx$   
 at  $x=0 \dots y=3+0$   
 $\frac{dz}{dy} = \frac{1}{\sqrt{1-0^2}} - 27$        $\frac{dy}{dx} = 3e^0 + 2(0)$   
 $= -26$        $= 3$   
 $\frac{dz}{dx} = -78$

5.  $f'(x) = 2x e^{2x} + x^2 e^{2x} (2)$   
 $f'(x) = 2x e^{2x} (1+x)$   
 critical values:  $x=-1, x=0$

	$2x$	$e^{2x}$	$1+x$	$f'/f$	
$(-\infty, -1)$	-	+	-	+	Inc
$(-1, 0)$	-	+	+	-	Dec
$(0, \infty)$	+	+	+	+	Inc

Local MAX:  $(-1, \frac{1}{e^2})$   
 Local MIN:  $(0, 0)$

$$\textcircled{1} \text{ c) } h(t) = \frac{5^{\tan t}}{\ln(3t^4 + 5)}$$

$$h'(t) = \frac{\underbrace{5^{\tan t}}_{f(x)} \underbrace{(\ln 5)(\sec^2 t)}_{f'(x)} \underbrace{(\ln(3t^4 + 5))}_{g(x)} - \underbrace{5^{\tan t}}_{f(x)} \underbrace{\left(\frac{12t^3}{3t^4 + 5}\right)}_{g'(x)}}{[\ln(3t^4 + 5)]^2}$$

$g(x)$

## Attachments

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Review of Transcendentals.doc

logs & arcfuctions test 2006.doc