Slant Asymptotes:

② d) $y = \frac{(x-1)^3}{x^2} = \frac{(x-1)(x-1)(x-1)}{x^2} = \frac{x^3-3x^2+3x-1}{x^2}$

(i) x int ($y=0$)

$x^2 \cdot 0 = \frac{(x-1)^3}{x^2} \cdot x^2$

$0 = (x-1)^3$

$0 = x-1$

$1 = x$

$(1,0)$

(ii) y-int ($x=0$)

$y = \frac{(0-1)^3}{0^2} = \frac{-1}{0}$

undefined

No y-int

(iii) VA: $x^2 = 0$

$x = 0$

$\lim_{x \to 0^-} \frac{(-)}{(+)} = -\infty$

$(x=-0,1)$

$\lim_{x \to 0^+} \frac{(-)}{(+)} = -\infty$

$(x=0,1)$

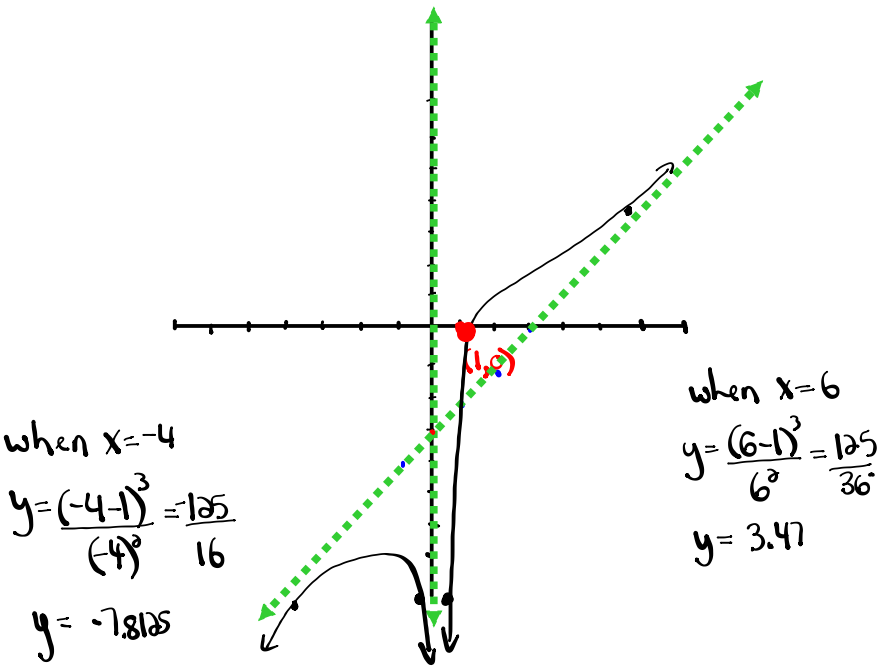
(iv) SA:

$$\begin{array}{r|l} x^2 & x^3-3x^2+3x-1 \\ & -(x^3) \\ \hline & -3x^2+3x-1 \\ & -(-3x^2) \\ \hline & 3x-1 \end{array} \quad x-3 \quad R$$

$y = x-3$

$m = \frac{1}{1}$ rise / run

$b = -3$ y-int



when $x=-4$

$y = \frac{(-4-1)^3}{(-4)^2} = \frac{-125}{16}$

$y = -7.8125$

when $x=6$

$y = \frac{(6-1)^3}{6^2} = \frac{125}{36}$

$y = 3.47$

Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \leq 0$$

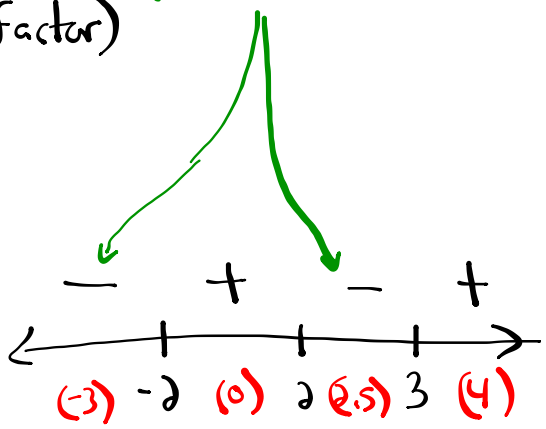
where does this function have y values that are less than or equal to zero

$y = x^3 - 3x^2 - 4x + 12$ (factor)

$y = x^2(x-3) - 4(x-3)$

$y = (x-3)(x^2-4)$

$y = (x-3)(x-2)(x+2)$



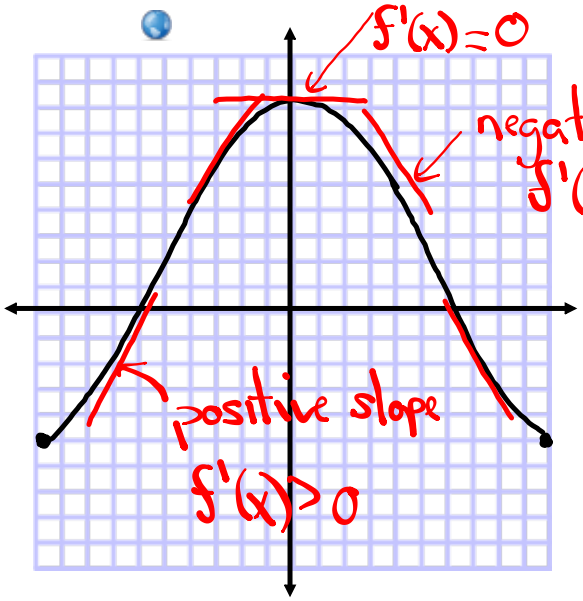
$x \in (-\infty, -2] \cup [2, 3]$

Find x int ($y = 0$)

$0 = (x-3)(x-2)(x+2)$

$x-3=0$	$x-2=0$	$x+2=0$
$x=3$	$x=2$	$x=-2$

Increasing and Decreasing Functions



slope of tangent line = f'(x)

Test for Increasing and Decreasing Functions

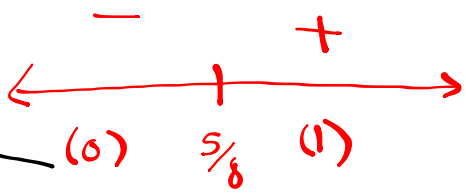
1. If $f'(x) > 0$ for all x in an interval I, then f is ~~increasing~~ on I. f'(x) is positive
2. If $f'(x) < 0$ for all x in an interval I, then f is ~~decreasing~~ on I f'(x) is negative

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

$f'(x) = 1 - 5x + 4x^2$

$f'(x) = -5 + 8x$



CV: $f'(x) = 0$ or undefined

$0 = -5 + 8x$

$5 = 8x$

$5/8 = x$

CV: $x = \frac{5}{8} = 0.625$

Thus f will be increasing on the interval $\left(\frac{5}{8}, \infty\right)$

Similarly,

Thus f will be decreasing on the interval $\left(-\infty, \frac{5}{8}\right)$

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

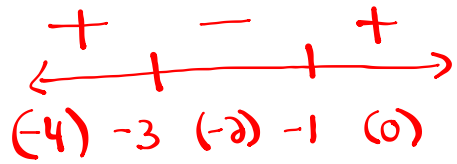
Solution

$y = x^3 + 6x^2 + 9x + 2$

$y' = 3x^2 + 12x + 9$

$y' = 3(x^2 + 4x + 3)$

$y' = 3(x+3)(x+1)$



CV: $y' = 0$ or undefined

$0 = 3(x+3)(x+1)$

$3 \neq 0$ | $x+3=0$, $x=-3$ | $x+1=0$, $x=-1$

CV: $x = -3, -1$

Increasing on $(-\infty, -3) \cup (-1, \infty)$

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

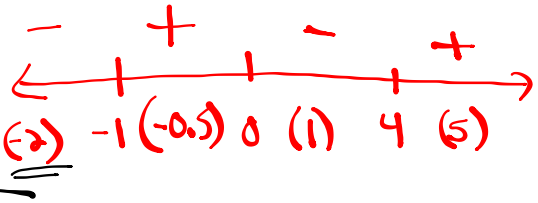
First we compute the derivative and factor it:

$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$



CV: $f'(x) = 0$ or undefined

$0 = 4x(x-4)(x+1)$

$4x = 0$	$x - 4 = 0$	$x + 1 = 0$
$x = 0$	$x = 4$	$x = -1$

CV: $x = -1, 0, 4$

Increasing on $(-1,0) + (4,\infty)$

Decreasing on $(-\infty,-1) + (0,4)$

Homework