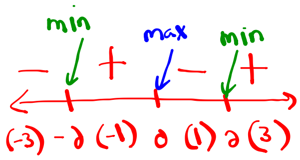
Questions from homework

2 e) $h(x) = x^4 - 8x^2 + 6$

$h'(x) = 4x^3 - 16x$

$h'(x) = 4x(x^2 - 4)$

$h'(x) = 4x(x+2)(x-2)$



CV: $4x = 0$ | $x+2=0$ | $x-2=0$

$x = 0$ | $x = -2$ | $x = 2$

$x = -2, 0, 2$

Increasing on $(-2,0) + (2,\infty)$

Decreasing on $(-\infty,-2) + (0,2)$

$h(x) = x^4 - 8x^2 + 6$

$h(-2) = (-2)^4 - 8(-2)^2 + 6$

$h(-2) = 16 - 32 + 6 = -10$

local min @ $(-2, 10)$

$h(x) = x^4 - 8x^2 + 6$

$h(0) = (0)^4 - 8(0)^2 + 6$

$h(0) = 0 - 0 + 6 = 6$

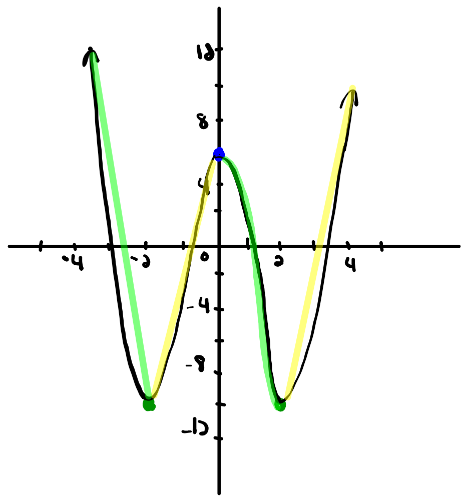
local max @ $(0, 6)$

$h(x) = x^4 - 8x^2 + 6$

$h(2) = (2)^4 - 8(2)^2 + 6$

$h(2) = 16 - 32 + 6 = -10$

local min @ $(2, -10)$



Questions from homework

② f) $h(x) = 3x^5 - 5x^3$

$h'(x) = 15x^4 - 15x^2$

$h'(x) = 15x^2(x^2 - 1)$

$h'(x) = 15x^2(x+1)(x-1)$

max neither min

$+ \quad - \quad - \quad +$

(-2) −1 (-0.5) 0 (0.5) 1 (2)

CV: $15x^2 = 0$, $x^2 = 0$, $x = 0$ | $x+1=0$, $x=-1$ | $x-1=0$, $x=1$

Increasing on $(-\infty, -1)$ + $(1, \infty)$

Decreasing on $(-1, 0)$ + $(0, 1)$

$h(x) = 3x^5 - 5x^3$

$h(-1) = 3(-1)^5 - 5(-1)^3$

$h(-1) = 3(-1) - 5(-1)$

$h(-1) = -3 + 5$

$h(-1) = 2$

$(-1, 2)$ max

$h(x) = 3x^5 - 5x^3$

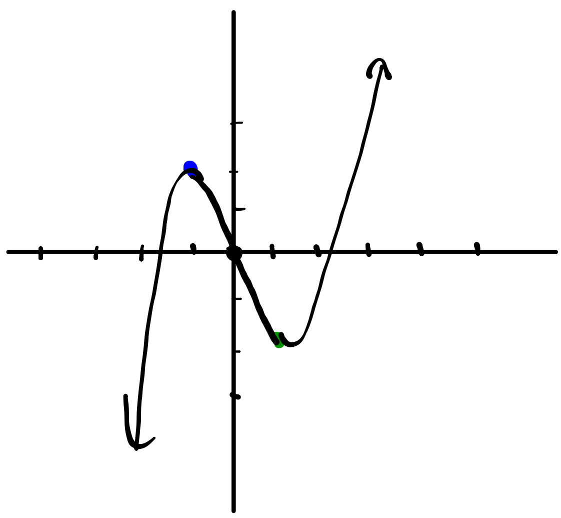
$h(1) = 3(1)^5 - 5(1)^3$

$h(1) = 3(1) - 5(1)$

$h(1) = 3 - 5$

$h(1) = -2$

$(1, -2)$ min



Questions from homework

③ a) $f(x) = 2x^{2/3}(3 - 4x^{1/3}) = 6x^{2/3} - 8x$

$f'(x) = 4x^{-1/3} - 8$

$= \frac{4}{x^{1/3}} - \frac{8}{1} \rightarrow \frac{4}{x^{1/3}} - \frac{8x^{1/3}}{x^{1/3}}$

$= \frac{4 - 8x^{1/3}}{x^{1/3}} \rightarrow \frac{4 - 8\sqrt[3]{x}}{\sqrt[3]{x}}$

CV: $4 - 8x^{1/3} = 0$ | $x^{1/3} = 0$

$4 = 8x^{1/3}$ | $x = 0$

$\frac{1}{2} = x^{1/3}$

$\frac{1}{8} = x$

$0.125 = x$

min max

$-$ $+$ $-$

(-1) 0 (0.1) 0.125 (1)

Decreasing on $(-\infty, 0)$ and $(\frac{1}{8}, \infty)$

Increasing on $(0, \frac{1}{8})$

min $(x = 0)$

$h(x) = 6x^{2/3} - 8x$

$h(0) = 6(0)^{2/3} - 8(0)$

$h(0) = 0$

$(0, 0)$ min

max $(x = \frac{1}{8})$

$h(x) = 6x^{2/3} - 8x$

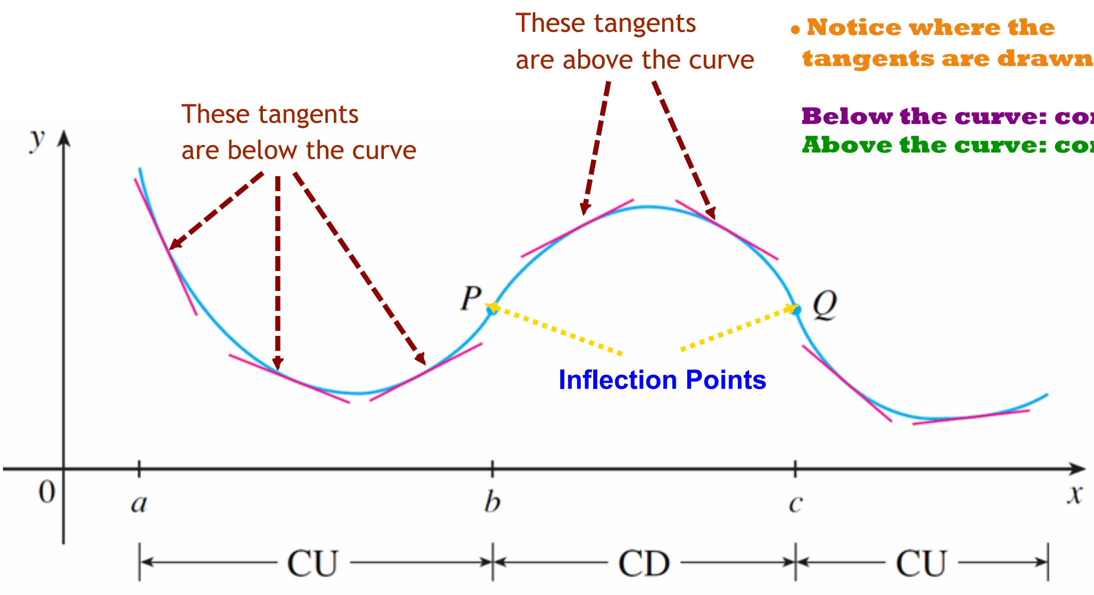
$h(\frac{1}{8}) = 6(\frac{1}{8})^{2/3} - 8(\frac{1}{8})$

$h(\frac{1}{8}) = \frac{3}{2} - 1$

$h(\frac{1}{8}) = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

$(\frac{1}{8}, \frac{1}{2})$ max

Concavity



- Notice where the tangents are drawn...

Below the curve: concave up
Above the curve: concave down

- In general, the graph of f is called ***concave upward*** on an interval I if it lies above all its tangents. $f''(x)>0$
- It is called ***concave downward*** on I if it lies below all of these tangents. $f''(x)<0$
- A point where a curve changes its direction of concavity is called an ***inflection point***.

If $f'(x)>0$ then $f(x)$ is increasing,
so if $f''(x)>0$ then $f'(x)$ is increasing.

Concavity Test

(a) If $f''(x)>0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x)<0$ for all x in I, then the graph of f is concave downward on I.

Thus there is a point of inflection at any point where the second derivative changes sign.

Determine where the curve $y = x^3 - 3x^2 + 4x - 5$ is concave upward and concave downward

Find the points of inflection

$y = x^3 - 3x^2 + 4x - 5$

$y' = 3x^2 - 6x + 4$

$y'' = 6x - 6$

$y'' = 6(x-1)$

I.P. @ $x=1$

$-$ | $+$

(0) 1 (2)

CV: $6 \neq 0$ | $x - 1 = 0$, $x = 1$

CV: $x = 1$

Concave Up on $(1, \infty)$ $x > 1$

Concave Down on $(-\infty, 1)$ $x < 1$

$y = x^3 - 3x^2 + 4x - 5$

$y = (1)^3 - 3(1)^2 + 4(1) - 5$

$y = 1 - 3 + 4 - 5$

$y = -3$

$(1, -3)$ Inflection Point

Determine where the curve $y = \dfrac{x}{x^2+1}$ is concave upward and concave downward

Find the points of inflection

$$y = \frac{x}{x^2+1}$$

$$y' = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{-x^2+1}{(x^2+1)^2}}$$

$$y'' = \frac{(x^2+1)^2(-2x) - (x^2+1)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$y'' = \frac{-2x(x^2+1)^2 + 4x(x^2-1)(x^2+1)}{(x^2+1)^4}$$

$-x^2-1 \quad +2x^2-2$

$$y'' = \frac{2x\cancel{(x^2+1)}\left[-(x^2+1)+2(x^2-1)\right]}{(x^2+1)^{\cancel{4}\,3}}$$

$$y'' = \frac{2x(x^2-3)}{(x^2+1)^3}$$ ← Always positive

CV: $2x = 0$, $x = 0$ | $x^2-3=0$, $x^2 = 3$, $x = \pm\sqrt{3}$

IP IP IP

$- \quad + \quad - \quad +$

$(-2)\ -\sqrt{3}\ (-1)\ 0\ (1)\ \sqrt{3}\ (2)$

$\boxed{\text{CU on } (-\sqrt{3},0) \text{ and } (\sqrt{3},\infty) \quad \text{CD on } (-\infty,-\sqrt{3}) \text{ and } (0,\sqrt{3})}$

Inflection Points: $y = \dfrac{x}{x^2+1}$

$f(-\sqrt{3}) = -\dfrac{\sqrt{3}}{4}$ $\quad \left(-\sqrt{3}, -\dfrac{\sqrt{3}}{4}\right)$

$f(0) = \dfrac{0}{1} = 0$ $\quad (0,0)$

$f(\sqrt{3}) = \dfrac{\sqrt{3}}{4}$ $\quad \left(\sqrt{3}, \dfrac{\sqrt{3}}{4}\right)$

homework

Second Derivative Test for Local Extrema

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

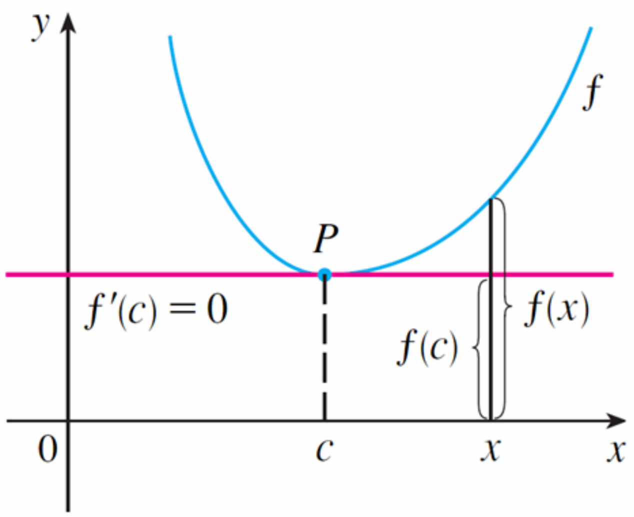


FIGURE 6
$f''(c) > 0$, f is concave upward

Example:

Examine the function $f(x) = x^4 - 4x^3$ with respect to...

- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values

Solution

Example:

Using the function: $f(x) = \dfrac{x^2}{x-7}$

Determine each of the following...

- Intercepts
- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values