

## **Questions from Homework**

## Differentiation Rules

### Product Rule:

**The Product Rule** If  $f$  and  $g$  are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (\text{First}) \times (\text{Second})$$

" The derivative of the product of two functions is the the first multiplied by the derivative of second, plus the derivative of first multiplied by the second"

*Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.*

## Quotient Rule:

**The Quotient Rule** If  $f$  and  $g$  are differentiable, then

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally ...

" The denominator multiplied by the derivative of the numerator, minus the numerator multiplied by the derivative of the denominator, all over the denominator squared"

## Combining the Chain Rule With the Product and Quotient Rule:

**The Chain Rule** If  $f$  and  $g$  are both differentiable and  $F = f \circ g$  is the composite function defined by  $F(x) = f(g(x))$ , then  $F$  is differentiable and  $F'$  is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Differentiate the following function and simplify your answer:

$$y = (x^2 + 1)^3 (2 - 3x)^4$$

$$\begin{aligned} y' &= (x^2+1)^3 (4)(2-3x)^3 (-3) + 3(x^2+1)^2 (2x)(2-3x)^4 \\ &= -12(x^2+1)^3 (2-3x)^3 + 6x(x^2+1)^2 (2-3x)^4 \\ &= -6(x^2+1)^2 (2-3x)^3 \left[ 2(x^2+1) - x(2-3x) \right] \\ &= -6(x^2+1)^2 (2-3x)^3 \left[ 2x^2+2-2x+3x^2 \right] \\ &= \boxed{-6(x^2+1)^2 (2-3x)^3 (5x^2-2x+2)} \end{aligned}$$

$$g(x) = \frac{(3x+2)^2}{2x}$$

$$\begin{aligned} g'(x) &= \frac{2x(2)(3x+2)(3) - (3x+2)^2(2)}{(2x)^2} \\ &= \frac{12x(3x+2) - 2(3x+2)^2}{4x^2} \\ &= \frac{2(3x+2) \left[ 6x - (3x+2) \right]}{4x^2} \\ &= \frac{\cancel{2}(3x+2)(3x-2)}{4x^2} \\ &= \boxed{\frac{(3x+2)(3x-2)}{2x^2}} \quad \text{or} \quad \frac{9x^2-4}{2x^2} \end{aligned}$$

Differentiate the following functions and simplify your answers:

$$s = \left( \frac{2t-1}{t+2} \right)^6$$

$$\frac{ds}{dt} = 6 \left[ \frac{2t-1}{t+2} \right]^5 \left[ \frac{2t+4 - 2t+1}{(t+2)^2} \right]$$

$$\frac{ds}{dt} = 6 \left[ \frac{(2t-1)^5}{(t+2)^5} \right] \left[ \frac{5}{(t+2)^2} \right]$$

$$\frac{ds}{dt} = \frac{30(2t-1)^5}{(t+2)^7}$$

$$g(x) = (9x^{-3})(5x^3 - 1)^6$$

$$g'(x) = (9x^{-3})[6(5x^3-1)^5(15x^2)] - 27x^{-4}(5x^3-1)^6$$

$$g'(x) = 810x^{-1}(5x^3-1)^5 - 27x^{-4}(5x^3-1)^6$$

$$g'(x) = 27x^{-4}(5x^3-1)^5 \left[ 30x^3 - 5x^3 + 1 \right]$$

$$g'(x) = 27x^{-4}(5x^3-1)^5(25x^3+1)$$

$$g'(x) = \frac{27(5x^3-1)^5(25x^3+1)}{x^4}$$

**Example 1**

## Chain Rule

Let  $F(x) = f(g(x))$        $F'(x) = f'(g(x)) \cdot g'(x)$

If  $f(2) = 3$ ,  $f'(2) = 5$ ,  $g(1) = 2$  and  $g'(1) = 4$  find  $F'(1)$ .

$$\begin{aligned} F'(1) &= f'(g(1)) \cdot g'(1) \\ &= f'(2) \cdot g'(1) \\ &= 5 \cdot 4 \\ &= 20 \end{aligned}$$

$$y = 3x^2 + 7x - 1$$

$$y' = 6x + 7$$

$$\frac{dy}{dx} = 6x + 7$$

**Example 2**

If  $y = u^{10} + u^5 + 2$ , where  $u = 1 - 3x^2$ , find  $\left. \frac{dy}{dx} \right|_{x=1}$

$$\frac{dy}{du} = 10u^9 + 5u^4 \quad \frac{du}{dx} = -6x$$

when  $x = \underline{1}$   
 $u = 1 - 3(\underline{1})^2$   
 $u = 1 - 3 = \underline{\underline{-2}}$

$$\left. \frac{dy}{dx} \right|_{x=1} = \left[ \frac{dy}{du} \right] \left[ \frac{du}{dx} \right]$$

$$= [10u^9 + 5u^4] [-6x]$$

$$= [10(-2)^9 + 5(-2)^4] [-6(1)]$$

$$= [-5120 + 80] [-6]$$

$$= [-5040] [-6]$$

$$= \boxed{30240}$$



**Example 2**

If  $y = \underline{u}^{10} + \underline{u}^5 + 2$ , where  $u = \underline{1-3x^2}$ , find  $\left. \frac{dy}{dx} \right]_{x=1}$

$$y = (1-3x^2)^{10} + (1-3x^2)^5 + 2$$

$$\begin{aligned} \left. \frac{dy}{dx} \right]_{x=1} &= 10(1-3x^2)^9 (-6x) + 5(1-3x^2)^4 (-6x) \\ &= -60x(1-3x^2)^9 - 30x(1-3x^2)^4 \\ &= -30x(1-3x^2)^4 [2(1-3x^2)^5 + 1] \\ &= -30\underline{x}(1-3\underline{x^2})^4 [2(1-3\underline{x^2})^5 + 1] \\ &= -30(16)(-63) \\ &= 30240 \end{aligned}$$

# Homework

Page 103

# 3-10

③ Find  $\left. \frac{dy}{dx} \right|_{x=4}$

if  $y = u^2 - 2u^5$

$$\frac{dy}{du} = 2u - 10u^4$$

and  $u = x - \sqrt{x}$

$$\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$$

$$\frac{du}{dx} = 1 - \frac{1}{2\sqrt{x}}$$

$$\left. \frac{dy}{dx} \right|_{x=4} = \left[ \frac{dy}{du} \right] \left[ \frac{du}{dx} \right]$$

$$= [2u - 10u^4] \left[ 1 - \frac{1}{2\sqrt{x}} \right]$$

$$= [2(2) - 10(2)^4] \left[ 1 - \frac{1}{2\sqrt{4}} \right]$$

$$= (-156) \left( \frac{3}{4} \right)$$

$$= \boxed{-117}$$

when  $x=4$   $u=2$

Page 103

# 3-10

$$\textcircled{4} \left. \frac{dy}{dt} \right]_{t=1}$$

$$y = \sqrt{1+r^2}$$

$$\frac{dy}{dr} = \frac{1}{2}(1+r^2)^{-1/2} (2r)$$

$$\frac{dy}{dr} = \frac{r}{\sqrt{1+r^2}}$$

$$\frac{dr}{dt} = \frac{1(2t+1) - 2(t+1)}{(2t+1)^2}$$

$$\frac{dr}{dt} = \frac{-1}{(2t+1)^2}$$

$$r = \frac{t+1}{2t+1}$$

$$\left. \frac{dy}{dt} \right]_{t=1} = \left[ \frac{dy}{dr} \right] \left[ \frac{dr}{dt} \right]$$

$$= \left[ \frac{r}{\sqrt{1+r^2}} \right] \left[ \frac{-1}{(2t+1)^2} \right]$$

$$= \left[ \frac{2/3}{\sqrt{1+(2/3)^2}} \right] \left[ \frac{-1}{(2(1)+1)^2} \right]$$

$$= \left[ \frac{2/3}{\frac{\sqrt{13}}{3}} \right] \left[ \frac{-1}{9} \right]$$

$$= \left[ \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{\sqrt{13}} \right] \left[ \frac{-1}{9} \right]$$

$$= \boxed{\frac{-2}{9\sqrt{13}}}$$

when  $t=1$   $r = \frac{2}{3}$