Questions from Homework

② c) $t_n = \frac{1}{3n}$

$t_1 = \frac{1}{3}$ $t_2 = \frac{1}{6}$ $t_3 = \frac{1}{9}$ $t_4 = \frac{1}{12}$

$0.\overline{3}$, $0.1\overline{6}$, $0.\overline{1}$, $0.08\overline{3}$

$d = t_2 - t_1$
$= \frac{1}{6} - \frac{1}{3}$
$= \frac{3}{18} - \frac{6}{18}$
$= -\frac{3}{18} = \cancel{-\frac{1}{6}}$

$d = t_3 - t_2$
$= \frac{1}{9} - \frac{1}{6}$
$= \frac{2}{18} - \frac{3}{18}$
$= \cancel{-\frac{1}{18}}$

③ d) $5\sqrt{2}, 4\sqrt{2}, 3\sqrt{2}, \ldots 2\sqrt{2}, 1\sqrt{2}$

Given:

$a = 5\sqrt{2}$

$d = 4\sqrt{2} - 5\sqrt{2}$
$= -1\sqrt{2}$
$= -\sqrt{2}$

general term $(n = n)$

$t_n = a + (n-1)d$

$t_n = 5\sqrt{2} + (n-1)(-\sqrt{2})$

$t_n = 5\sqrt{2} - n\sqrt{2} + \sqrt{2}$

$t_n = 6\sqrt{2} - n\sqrt{2}$

$t_n = \sqrt{2}(6-n)$

⑨ f) $5a-3b, 4a-2b, 3a-b, \ldots, -5a+7b$

Given

$n = ?$

$a = 5a-3b$

$d = -a+b$

$4a-2b-(5a-3b)$

$4a-2b-5a+3b$

$-a+b$

$t_n = -5a+7b$

$t_n = a+(n-1)d$

$-5a+7b = 5a-3b + (n-1)(-a+b)$

$-10a+10b = (n-1)(-a+b)$

$10\cancel{(-a+b)} = (n-1)\cancel{(-a+b)}$

$10 = n-1$

$\boxed{11 = n}$

Geometric Sequences

Ex: 2, 4, 8, 16, 32

2 2 2 2

Sequences of numbers that follow a pattern of multiplying a fixed number from one term to the next are called geometric sequences.

- To find the next term, multiply the previous term by a common ratio.
- In the sequence 2, 4, 8, 16, 32 we are multiplying by 2.
- This common ratio is called **"r" (r = t_2/t_1).**
- The first term is still called **"a"** or **"t_1".**
- The second term is called **"t_2".**
- The last term or an indicated term is called **"t_n".** general term
- The position of a term or the number of terms is called **"n".**

Geometric Sequences

Remember $r = t_2/t_1$

Find "r" and the next term!

1, 2, 4, 8,... 16

$r = \frac{t_2}{t_1} = \frac{2}{1} = 2$

16, -8, 4, -2, 1,... $-\frac{1}{2}$

$r = \frac{-8}{16} = \frac{4}{-8} = \frac{-2}{4} = -\frac{1}{2}$

0.01, 0.06, 0.36, 2.16, ..., 12.96

6 6 6

$r = 6$

Geometric Sequences

To find any given term in a geometric sequence we use the following formula:

$$\boxed{t_n = ar^{n-1}}$$

Examples

$r = \frac{t_4}{t_3} = \frac{-1}{4} \div \frac{1}{2}$

$= \frac{-1}{4} \times \frac{2}{1} = \frac{-2}{4} = \frac{-1}{2}$

Find the indicated term

1. $2, -1, \frac{1}{2}, \frac{-1}{4} \ldots$

$a = 2$

$r = \frac{-1}{2}$

$n = 9$

$t_9 = (2)\left(\frac{-1}{2}\right)^{9-1}$

$t_9 = (2)\left(\frac{-1}{2}\right)^8$

$t_9 = (2)\left(\frac{1}{256}\right)$

$t_9 = \frac{2}{256} = \frac{1}{128}$

look for

$\boxed{y^x}$ or $\boxed{x^y}$

or $\boxed{\wedge}$

exponent

We can also determine the number of terms in the sequence.

$$\boxed{t_n = ar^{n-1}}$$

How many terms are in the following sequences?
(Solve for "n")

9, 27, 81,... 2187 $a=9$ $r=3$ $t_n=2187$

$\frac{2187}{9} = \frac{(9)(3)^{n-1}}{9}$

$243 = 3^{n-1}$ * $\frac{\log 243}{\log 3} = 5$

$\cancel{3}^{\boxed{5}} = \cancel{3}^{n-1}$

$5 = n-1$

$\boxed{6 = n}$

Solve for "n"

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \frac{1}{1024}$

$t_n = ar^{n-1}$

$a = \frac{1}{2}$

$r = \frac{1}{4} \div \frac{1}{2}$

$= \frac{1}{4} \times \frac{2}{1}$

$= \frac{2}{4} = \frac{1}{2}$

$t_n = \frac{1}{1024}$

$\frac{\frac{1}{1024}}{\frac{1}{2}} = \frac{(\cancel{\frac{1}{2}})(\frac{1}{2})^{n-1}}{\cancel{\frac{1}{2}}}$

$\frac{1}{1024} \cdot 2 = \left(\frac{1}{2}\right)^{n-1}$

$\frac{2}{1024} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$ * $\frac{\log(1/512)}{\log(1/2)} = 9$

$\cancel{\left(\frac{1}{2}\right)}^9 = \cancel{\left(\frac{1}{2}\right)}^{n-1}$

$9 = n-1$

$\boxed{10 = n}$

geometric $t_n = ar^{n-1}$

Find "a", "r", and "t_n" for the following sequences!

3, 12, 48, 192, 768

$t_2 = 12$, $t_5 = 768$

$t_n = ar^{n-1}$
$t_2 = ar^{2-1}$
$t_2 = ar^1$
$12 = ar$

$t_n = ar^{n-1}$
$t_5 = ar^{5-1}$
$t_5 = ar^4$
$768 = ar^4$

Elimination

$\div \dfrac{768 = ar^4}{12 = ar}$ $\boxed{\sqrt[3]{\ }}$

$64 = r^3$ 3 $\boxed{\sqrt[x]{\ }}$

$\boxed{4 = r}$

$12 = ar$
$\dfrac{12}{4} = \dfrac{a(4)}{4}$
$\boxed{3 = a}$

general term (n=n)

$t_n = ar^{n-1}$
$t_n = (3)(4)^{n-1}$

$(r^3)^{1/3} = (64)^{1/3}$
$r = 4$

$(r^2)^{1/2} = (64)^{1/2}$
$r = 8$

what if:

$t_n = ar^{n-1}$
$t_n = (8)(2)^{n-1}$
$t_n = (2^3)(2)^{n-1}$
$t_n = 2^{3+n-1} = 2^{n+2}$

Homework

#1- #6