## Questions from Homework

Ex 10.4

4 c) $36, 18, 9, \ldots, \frac{9}{128}$

$a = 36$

$r = \frac{18}{36} = \frac{1}{2}$

$t_n = \frac{9}{128}$

$t_n = ar^{n-1}$

$\frac{9}{128} = (36)\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$

$* \left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$

$* \frac{\log(1/512)}{\log(1/2)}$

$9 = n-1$

$10 = n$

2 d) $2^{50}, 2^{48}, 2^{46}, \ldots$

$a = 2^{50}$

$r = \frac{2^{48}}{2^{50}} = 2^{48-50} = 2^{-2}$

$t_{15} = (2^{50})(2^{-2})^{14}$

$t_{15} = (2^{50})(2^{-28})$

$t_{15} = 2^{22}$

6 $t_3 = \frac{1}{9}$ | $t_7 = 9$

$t_3 = ar^{3-1}$ | $t_7 = ar^{7-1}$

$t_3 = ar^2$ | $t_7 = ar^6$

$\boxed{ar^2 = \frac{1}{9}}$ | $\boxed{ar^6 = 9}$

$\frac{ar^6 = 9}{ar^2 = 1/9}$

$r^4 = 81$

$\boxed{r = \pm 3}$

$ar^2 = \frac{1}{9}$

$a(3)^2 = \frac{1}{9}$

$9a = \frac{1}{9}$

$\boxed{a = \frac{1}{81}}$

$\frac{1}{81}, \pm\frac{1}{27}, \frac{1}{9}, \pm\frac{1}{3}$

$\boxed{t_4 = \pm\frac{1}{3}}$

4 b) $16, -8, 4, \ldots, \frac{1}{4}$

$n = ?$

$a = 16$

$r = \frac{-8}{16} = \frac{-1}{2}$

$t_n = \frac{1}{4}$

$t_n = ar^{n-1}$

$\frac{1}{4} = (16)\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(-\frac{1}{2}\right)^{n-1}$

Get common base.

$\frac{\log(1/64)}{\log(1/2)} = 6$

$* \left(-\frac{1}{2}\right)^6 = \left(-\frac{1}{2}\right)^{n-1}$

$6 = n-1$

$\boxed{7 = n}$

6 $t_{10} = 2560$ | $t_5 = 80$

$t_{10} = ar^{10-1}$ | $t_5 = ar^{5-1}$

$t_{10} = ar^9$ | $t_5 = ar^4$

$2560 = ar^9$ | $80 = ar^4$

$\boxed{ar^9 = 2560}$ | $\boxed{ar^4 = 80}$

$\frac{ar^9 = 2560}{ar^4 = 80}$

$r^5 = 32$

$\boxed{r = 2}$

$ar^4 = 80$

$a(2)^4 = 80$

$16a = 80$

$\boxed{a = 5}$

$t_{12} = (5)(2)^{12-1}$

$t_{12} = (5)(2)^{11}$

$t_{12} = (5)(2048) = \boxed{10240}$

Questions from Homework

① a) $3, 6, 12, \ldots$ geometric $r = \frac{6}{3} = \frac{12}{6} = 2$

c) $-5, -7, -9, \ldots$ arithmetic $d = -7-(-5) = -2$

② c) $-\frac{1}{6}, \frac{1}{3}, -\frac{2}{3}, \ldots$

Given:

$a = -\frac{1}{6}$

$r = \frac{1}{3} \div -\frac{1}{6} = \frac{1}{3} \times -\frac{6}{1} = -\frac{6}{3} = -2$

$t_n = ar^{n-1}$

$t_7 = (-\frac{1}{6})(-2)^{7-1}$

$t_7 = (-\frac{1}{6})(-2)^6$

$t_7 = (-\frac{1}{6})(64)$

$t_7 = -\frac{64}{6} = -\frac{32}{3}$

③ e) $\frac{p^2}{q}, \frac{p^3}{2q}, \frac{p^4}{4q}$

$a = \frac{p^2}{q}$

$r = \frac{p^3}{2q} \div \frac{p^2}{q} = \frac{p^3}{2q} \cdot \frac{q}{p^2} = \frac{p^{\cancel{3}}\cancel{q}}{2\cancel{p^2}\cancel{q}} = \frac{p}{2}$

$t_n = ar^{n-1}$

$t_{10} = (\frac{p^2}{q})(\frac{p}{2})^{10-1}$

$t_{10} = (\frac{p^2}{q})(\frac{p}{2})^9$

$t_{10} = (\frac{p^2}{q})(\frac{p^9}{512}) = \frac{p^{11}}{512q}$

⑤ b) $t_5 = 8$

$t_n = ar^{n-1}$

$t_5 = ar^{5-1}$

$t_5 = ar^4$

$8 = ar^4$

$t_{10} = \frac{1}{4}$

$t_n = ar^{n-1}$

$t_{10} = ar^{10-1}$

$t_{10} = ar^9$

$\frac{1}{4} = ar^9$

$\frac{1}{4} \times \frac{1}{8}$

Elimination

$\frac{\frac{1}{4} = ar^9}{8 = ar^4}$

$\frac{1}{32} = r^5$

$\boxed{\frac{1}{2} = r}$

$8 = ar^4$

$8 = a(\frac{1}{2})^4$

$16 \cdot 8 = \frac{1}{\cancel{16}} a \cdot 16$

$\boxed{128 = a}$

$t_3 = (128)(\frac{1}{2})^{3-1}$

$= 128(\frac{1}{2})^2$

$= 128(\frac{1}{4})$

$= 32$

④ b) $16, -8, 4, \ldots, \frac{1}{4}$ (Solving for n)

$a = 16$

$r = -\frac{1}{2}$

$t_n = \frac{1}{4}$

$t_n = ar^{n-1}$

$\frac{1}{4} \times \frac{1}{16}$

$\frac{\frac{1}{4} = (16)(-\frac{1}{2})^{n-1}}{16}$

$\frac{1}{64} = (-\frac{1}{2})^{n-1}$

$\cancel{(-\frac{1}{2})}^6 = \cancel{(-\frac{1}{2})}^{n-1}$

$\frac{\log(\frac{1}{64})}{\log(\frac{1}{2})} = 6$

$6 = n-1$

$7 = n$

# Arithmetic Series

Series: The sum of the terms of a sequence. The sum is usually finite: 1+2+3+4+5. However it could be infinite: 2+4+8+16+... You can find the sum of many finite series and certain types of infinite series by using formulas.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Ex: 2+5+8+11+14

$t_5 = 14$ $S_5 = 40$

Sum of the first five terms

$t_4 = 11$ $S_4 = 26$

:

Find the sum of the first 100 terms of the arithmetic series 1+4+7+10+...

a = 1
d = $t_2$-$t_1$ = 3
n = 100

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{100} = \frac{100}{2}[2(1) + (100-1)3]$$

$$S_{100} = 50(2 + 297)$$

$$S_{100} = 50(299)$$

$$S_{100} = 14950$$

# Find the sum of the following series

$$\frac{1}{2} + 1 + \frac{3}{2} + 2, \ldots + 20$$

$d = 1 - \frac{1}{2}$
$= \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$

Hint: How many terms are there?

$a = \frac{1}{2}$
$d = \frac{1}{2}$
$t_n = 20$
$n =$

$$t_n = a + (n-1)d$$
$$20 = \frac{1}{2} + (n-1)\left(\frac{1}{2}\right)$$
$$20 = \cancel{\frac{1}{2}} + \frac{n}{2} - \cancel{\frac{1}{2}}$$
$$2 \cdot 20 = \frac{n}{\cancel{2}} \cdot \cancel{2}$$
$$40 = n$$

$$S_{40} = \frac{40}{2}\left(\frac{1}{2} + \frac{20}{1}\right)$$
$$S_{40} = 20\left(\frac{1}{2} + \frac{40}{2}\right)$$
$$S_{40} = \left(\frac{\cancel{20}^{10}}{1}\right)\left(\frac{41}{\cancel{2}_1}\right) = 410$$
$$S_{40} = \frac{820}{2}$$
$$S_{40} = 410$$

$$20 = \frac{1}{2} + (n-1)\left(\frac{1}{2}\right)$$

$$\frac{\cancel{2}}{1} \cdot \frac{39}{\cancel{2}} = (n-1)\cancel{\left(\frac{1}{2}\right)} \cancel{\div \frac{1}{2}}$$

$$39 = n-1$$

$$40 = n$$

(Solve for n)

How many terms are in the series:
3+8+13+...+248 if its sum is 6275?

5 5

$a = 3$
$d = 5$
$S_n = 6275$
$t_n = 248$

$$S_n = \frac{n}{2}(a + t_n)$$

$$6275 = \frac{n}{2}(3 + 248)$$

$$2 \cdot \quad 6275 = 2 \cdot \frac{n}{\cancel{2}}(251)$$

$$\frac{12550}{251} = \frac{251n}{251}$$

$$50 = n$$

# Find the indicated sums of the following series:

$S_{15}$ of 2+6+10....

$n=15$

$a=2$

$d=4$

$S_{15}=\frac{15}{2}[2(2)+(15-1)(4)]$

$=\frac{15}{2}[4+14(4)]$

$=\frac{15}{2}[4+56]$

$=\frac{15}{2}(60)$

$\boxed{=450}$

$S_{20}$ of -15-10-5+...

$-10-(-15)=5$

$-5-(-10)=5$

$n=20$

$a=-15$

$d=5$

$S_{20}=\frac{20}{2}[2(-15)+(20-1)(5)]$

$=10[-30+19(5)]$

$=10[-30+95]$

$=10(65)$

$\boxed{=650}$

# Homework

#1-8

7) $t_2 = -1$

$t_2 = a + (2-1)d$

$t_2 = a + d$

$-1 = a + d$

$a + d = -1$

$t_{12} = 19$

$t_1 = a + (12-1)d$

$t_1 = a + 11d$

$19 = a + 11d$

$a + 11d = 19$

$a + 11d = 19$

$(-)\ a + d = -1$

$10d = 20$

$\boxed{d = 2}$

$a + d = -1$

$a + (2) = -1$

$\boxed{a = -3}$