Questions from homework

④ Find $\left.\frac{dy}{dt}\right]_{t=1}$ if $y=\sqrt{1+r^2}$, $y=(1+r^2)^{1/2}$ and $r=\frac{t+1}{2t+1}$ $f(x)$ $g(x)$

① when $t=1$

$r=\frac{1+1}{2(1)+1}=\frac{2}{3}$

② $\frac{dy}{dr}=\frac{1}{2}(1+r^2)^{-1/2}(2r)$

$\frac{dy}{dr}=\frac{r}{\sqrt{1+r^2}}$

③ $\frac{dr}{dt}=\frac{1(2t+1)-2(t+1)}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{2t+1-2t-2}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{-1}{(2t+1)^2}$

④ $\left.\frac{dy}{dt}\right]_{t=1}=\left[\frac{dy}{dr}\right]\left[\frac{dr}{dt}\right]$

$=\left[\frac{r}{\sqrt{1+r^2}}\right]\left[\frac{-1}{(2t+1)^2}\right]$

$=\left[\frac{2/3}{\sqrt{1+4/9}}\right]\left[-\frac{1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{\frac{13}{9}}}\right]\left[-\frac{1}{9}\right]$

$=\left[\frac{2}{3}\times\frac{3}{\sqrt{13}}\right]\left[-\frac{1}{9}\right]$

$=\frac{-2}{9\sqrt{13}}$

⑦ $y = (x^2-3)^8$ at the point $(2,1)$

$x_1 = 2$
$y_1 = 1$

(i) $y = (x^2-3)^8$

$y' = 8(x^2-3)^7(2x)$

$y' = 16x(x^2-3)^7$

(ii) $m = y'(2) = 16(2)[(2)^2-3]^7$

$m = y'(2) = 32(1) = 32$

(iii) $y - y_1 = m(x - x_1)$

$y - 1 = 32(x-2)$

$y - 1 = 32x - 64$

$y = 32x - 63$

or $0 = 32x - y - 63$

⑩ $G(x) = h(p(x))$

$h(5) = 1$

$h'(5) = 2$

$h'(1) = 3$

$p(1) = 5$

$p'(1) = 7$

find $G'(1)$

$G'(x) = h'(p(x)) \cdot p'(x)$

$G'(1) = h'(p(1)) \cdot p'(1)$

$G'(1) = h'(5) \cdot p'(1)$

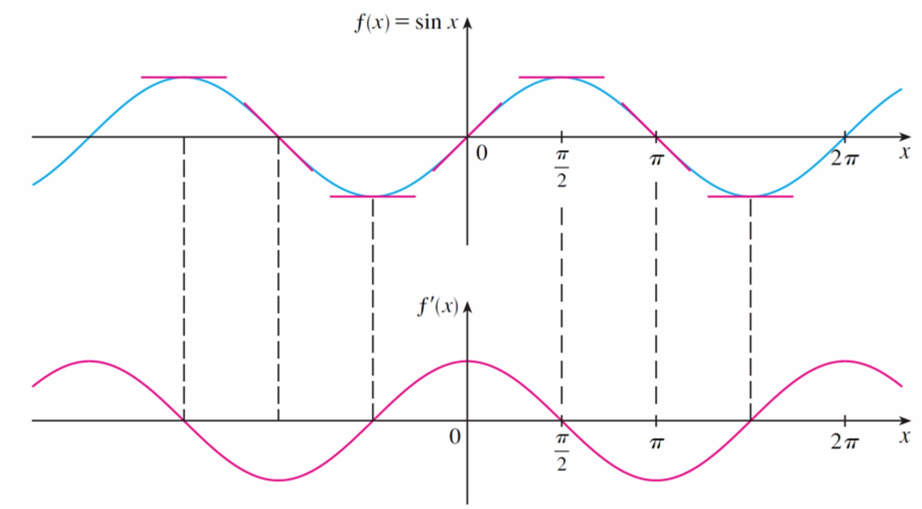
$G'(1) = (2)(7)$

$G'(1) = 14$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

    $$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$
- With some work we can also show that

  $$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $y = \sin(x + 2)$ $y = \sin(kx + d)$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$

b) $y = \sin^3 x$

c) $y = \sin^3(x^2 - 1)$

Ex #3.

Differentiate:

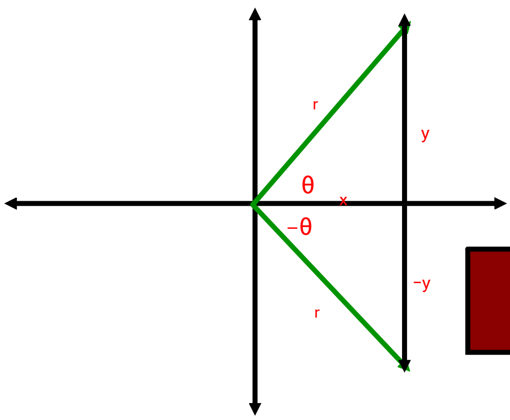
$$y = x^2 \cos x$$

Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

Negative Angles



$\sin\theta =$

$\cos(\theta) =$

$\sin(-\theta) =$

$\cos(-\theta) =$

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Attachments

Derivatives Worksheet.doc