Questions from homework

⑥ a) $G(x) = (x^4 - x + 1)^2 (x^2 - 2)^3$

$$G'(x) = 2(x^4 - x + 1)(4x^3 - 1)(x^2 - 2)^3 + 3(x^2 - 2)^2(2x)(x^4 - x + 1)^2$$

$$G'(x) = 2(4x^3 - 1)(x^4 - x + 1)(x^2 - 2)^3 + 6x(x^4 - x + 1)^2(x^2 - 2)^2$$

$4x^5 - 8x^3 - x^2 + 2 + 3x^5 - 3x^2 + 3x$

$$G'(x) = 2(x^4 - x + 1)(x^2 - 2)^2\left[(4x^3 - 1)(x^2 - 2) + 3x(x^4 - x + 1)\right]$$

$$G'(x) = 2(x^4 - x + 1)(x^2 - 2)^2(7x^5 - 8x^3 - 4x^2 + 3x + 2)$$

$(t + t^2)'$

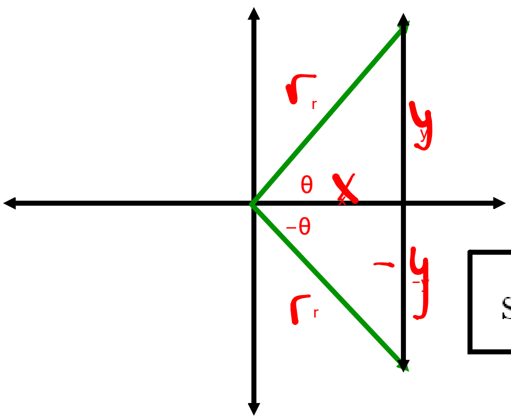
m) $y = \left(t + \sqrt[3]{t + t^2}\right)^{20} = \left(t + (t + t^2)^{1/3}\right)^{20}$

$$y' = 20\left(t + \sqrt[3]{t + t^2}\right)^{19}\left(1 + \frac{1}{3}(t + t^2)^{-2/3}(1 + 2t)\right)$$

$$y' = 20\left(t + \sqrt[3]{t + t^2}\right)^{19}\left(1 + \frac{1 + 2t}{3(t + t^2)^{2/3}}\right)$$

Negative Angles

$\sin(-4x) = -\sin(4x)$ $\cos(-4x) = \cos 4x$



$\sin\theta = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$

$\sin(-\theta) = -\frac{y}{r}$ $\cos(-\theta) = \frac{x}{r}$

$\boxed{\sin(-\theta) = -\sin\theta}$ $\boxed{\cos(-\theta) = \cos\theta}$

Ex: 7.2

① a) $y = \cos(-4x)$

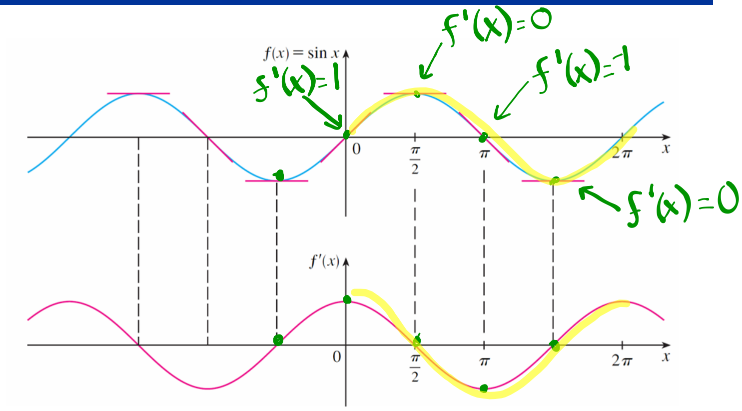
Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

Derivatives of Trigonometric Functions

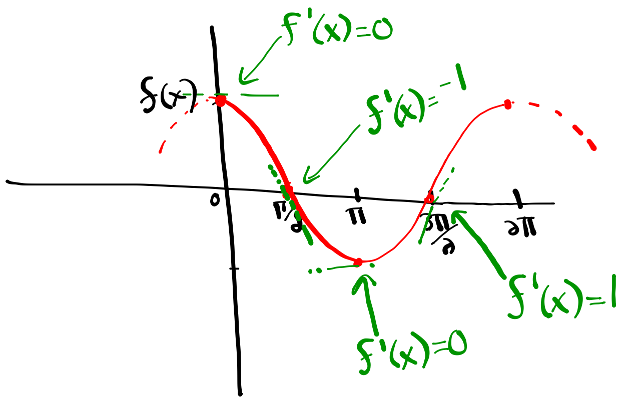
The Sine Function

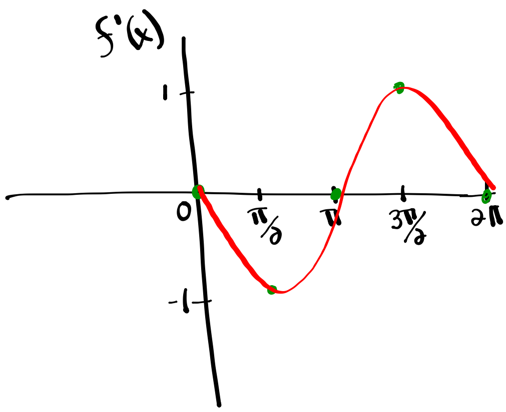
- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



if $f(x) = \sin x$

$f'(x) = \cos x$





Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

    $$\lim_{h\to 0} \sin x = \sin x \text{ and } \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

  $$\lim_{h\to 0} \frac{\sin h}{h} = 1 \text{ and } \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

$f(x) = \sin x$

$f'(x) = \cos x$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du \qquad \frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du \qquad \frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du \qquad \frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Ex: $f(x) = \tan(5x^2)$ $\quad u = 5x^2$

$du = 10x$

$f'(x) = \sec^2(5x^2) \cdot 10x$

$f'(x) = 10x\sec^2(5x^2)$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $u = 3x$ $du = 3$

$y' = \cos(3x) \cdot 3$

$y' = 3\cos(3x)$

$y = \sin(x+2)$ $u = x+2$ $du = 1$

$y' = \cos(x+2) \cdot 1$

$y' = \cos(x+2)$

$y = \sin(kx+d)$ $u = kx+d$ $du = k$

$y' = \cos(kx+d) \cdot k$

$y' = k\cos(kx+d)$

Ex #2.

Differentiate:

$y = (\sin x)^3$ $u = x$ $du = 1$

$y = [\sin(x^2 - 1)]^3$

$u = x^3$ $du = 3x^2$

$u = x^2 - 1$ $du = 2x$

a) $y = \sin(x^3)$

b) $y = \sin^3 x$

c) $y = \sin^3(x^2 - 1)$

$y' = \cos(x^3) \cdot 3x^2$

$y' = 3x^2 \cos(x^3)$

$y' = 3(\sin x)^2 (\cos x) \cdot 1$

$y' = 3\sin^2 x \cos x$

$y' = 3[\sin(x^2-1)]^2 \cos(x^2-1) 2x$

$y' = 6x \sin^2(x^2-1) \cos(x^2-1)$

Ex #3.

Differentiate: Product Rule $f'g + fg'$

$$y = (x^2)(\cos x)$$

$$y' = 2x\cos x + x^2(-\sin x)(1)$$

$$y' = 2x\cos x - x^2\sin x$$

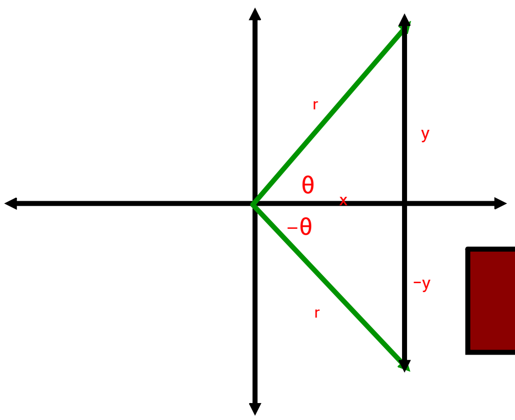
$$y' = x(2\cos x - x\sin x)$$

Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

Negative Angles



$\sin\theta =$ $\cos(\theta) =$

$\sin(-\theta) =$ $\cos(-\theta) =$

Attachments

Derivatives Worksheet.doc