Questions from Homework

Applications of Derivatives

Now that we know how to calculate derivatives, we use them in this unit to compute, *velocity, acceleration, and other rates of change.*

Suppose that an object moves along a straight line. (Think of a ball being thrown vertically upward or a car being driven along a road or a stone being dropped from a cliff.) The position function is $s = f(t)$, where s is the **displacement (directed distance)** of the object from the origin at time t. Recall that the **(instantaneous) velocity** of the object at time t is defined as the limit of average velocities over shorter and shorter time intervals.

In short, the ***velocity*** is the derivative of the position function and in Leibniz notation we write.

$$v = \frac{ds}{dt}$$

ex: $s = 4t^2 + 7t$ (displacement)

$s' = 8t + 7$ (velocity)

$s'' = 8$ (acceleration)

Velocity

If a stone is dropped from a cliff that is 122.5 m high, then its height in metres after *t* seconds is $h = 122.5 - 4.9t^2$

$h' = -9.8t$ (velocity)

a) Find its velocity after 1 *s* and 2 *s*.
b) When will the stone hit the ground? $(h=0)$
c) With what velocity will it hit the ground?

a) $h'(1) = -9.8(1)$
$= -9.8 m/s$

$h'(2) = -9.8(2)$
$= -19.6 m/s$

b) $h = 122.5 - 4.9t^2$

$0 = 122.5 - 4.9t^2 \longrightarrow$

$\frac{4.9t^2}{4.9} = \frac{122.5}{4.9}$

$t^2 = 25$

$t = \pm 5$

$t = 5$ sec.

by factoring

$0 = 4.9(25 - t^2)$

$0 = 4.9(5-t)(5+t)$

$5 - t = 0$ | $5 + t = 0$

$\boxed{5 = t}$ | $t = -5$

c) $h'(5) = -9.8(5)$
$= -49 m/s$

The position of a particle moving on a line is given by the equation $s = 2t^3 - 21t^2 + 60t,$

where *t* is measured in seconds and *s* in meters

$s' = 6t^2 - 42t + 60$ (velocity)

a) Find its velocity after 3 *s* and 6 *s*.

b) When is the particle at rest? $(v = 0)$

a) $s'(3) = 6(3)^2 - 42(3) + 60$
$= 54 - 126 + 60$
$= -12\ m/s$

$s'(6) = 6(6)^2 - 42(6) + 60$
$= 216 - 252 + 60$
$= 24\ m/s$

b) $0 = 6t^2 - 42t + 60$ ← Factor
$0 = 6(t^2 - 7t + 10)$

$-2 \times -5 = 10$
$-2 + -5 = -7$

$0 = 6(t-5)(t-2)$

$t - 5 = 0$ | $t - 2 = 0$
$t = 5$ | $t = 2$

The particle is at rest at 2 sec and 5 sec.

Applcations of Derivatives

Now that we know how to calculate derivatives, we use them in this unit to compute, *velocity, acceleration, and other rates of change.*

If an object moves along a straight line, its **acceleration** is the rate of change of velocity with respect to time. Therefore, the acceleration *a(t)* at time *t* is the derivative of the velocity function.

$$a(t) = v'(t) = \frac{dv}{dt}$$

Since the ***velocity*** is the derivative of the position function $s = f(t)$, it follows that the acceleration is the *second derivative* of the position function:

h(t) = height
h'(t) = velocity
h''(t) = acceleration

So

$$v(t) = s'(t) = \frac{ds}{dt}$$

$$a(t) = v'(t) = s''(t)$$

In Leibniz Notation,

$$a = \frac{d^2 s}{dt^2}$$

Acceleration

The position function of a particle is $s = t^3 + 2t^2 + 2t$
given by where s **is measured in meters and** t **in seconds**

a) Find the velocity and acceleration as a funciton of time.
b) Find the acceleration at 3 s

a) $s = t^3 + 2t^2 + 2t$

$s' = 3t^2 + 4t + 2$ (velocity)

$s'' = 6t + 4$ (acceleration)

b) $s''(3) = 6(3) + 4$

$= 18 + 4$

$= 22\ m/s^2$

If a ball is thrown upward with an initial velocity of 24.5 m/s, then its distance above the ground in meters after *t* seconds is

$$s = 24.5 - 4.9t^2$$

a) Find the acceleration of the ball.

$v = -9.8t$

$a = -9.8$

a) $a = -9.8 m/s^2$ (constant acceleration)

Notice that the acceleration in *Example 2* is a constant, and is called the *acceleration due to gravity* . The fact that it is negative means that **the ball slows down as it rises and speeds up as it falls.**

- In general, a negative acceleration indicates that the velocity is decreasing

- Likewise, a positive acceleration means that the velocity is increasing

Homework

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