Questions from Homework

① n) $y = \sin\left(\frac{1}{x}\right) = \sin x^{-1}$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2} = \frac{-1}{x^2}$

$y' = \cos(x^{-1}) \cdot (-x^{-2})$

$y' = -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

q) $y = x\cos\frac{1}{x} = x\cos(x^{-1})$

Product Rule

$f'g + fg'$

$y' = 1\cos\left(\frac{1}{x}\right) + x\left(-\sin\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)$

$y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x}\sin\left(\frac{1}{x}\right)$

l) $y = \frac{\sin x}{1+\cos x}$

Quotient Rule

$\frac{f'g - fg'}{(g)^2}$

$y' = \frac{(\cos x)(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$y' = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

Pythagorean Identity

$\sin^2 x + \cos^2 x = 1$

$y' = \frac{(1+\cos x)}{(1+\cos x)^2} = \boxed{\frac{1}{1+\cos x}}$

Questions from Homework

m) $y = (1+\cos^2 x)^6 = [1+(\cos x)^2]^6$

$y' = 6[1+(\cos x)^2]^5 (2\cos x \cdot -\sin x)$

$y' = 6(1+\cos^2 x)^5 (-2\sin x \cos x)$

Double Angle Identity
$\sin 2x = 2\sin x \cos x$

$y' = -6(1+\cos^2 x)^5 (\sin 2x)$

i) $y = 3\sin^4 (2-x)^{-1} = 3[\sin(2-x)^{-1}]^4$

$y' = 12[\sin(2-x)^{-1}]^3 \cos(2-x)^{-1} \cdot -1(2-x)^{-2}(-1)$

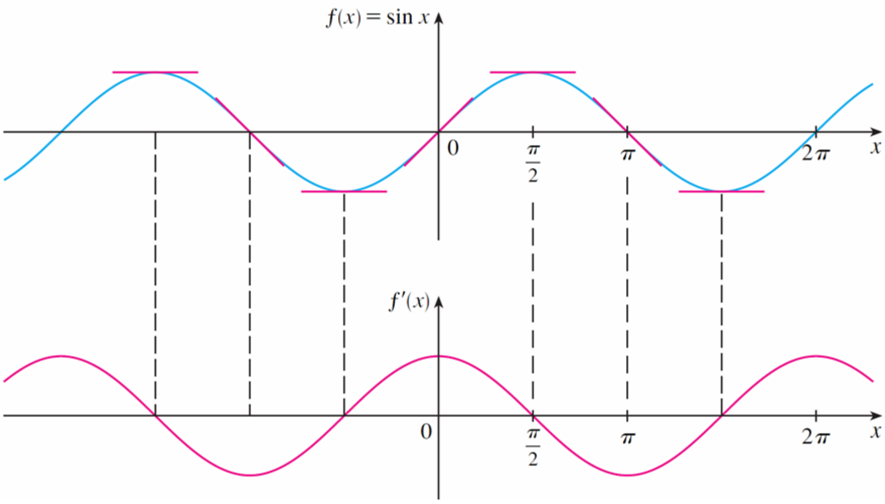
$y' = 12\sin^3(2-x)^{-1} \cos(2-x)^{-1} (2-x)^{-2}$

$y' = \frac{12\sin^3(2-x)^{-1}\cos(2-x)^{-1}}{(2-x)^2}$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h \to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h \to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

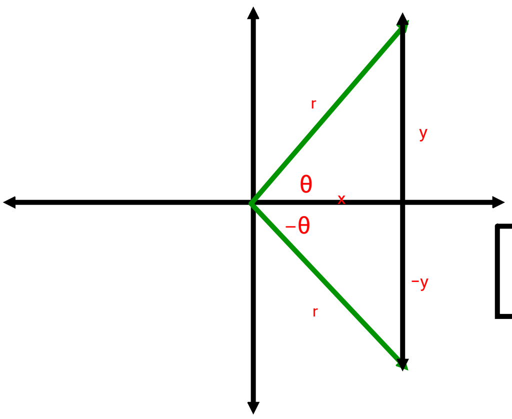
$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Negative Angles



$\sin\theta = \frac{y}{r}$

$\cos(\theta) = \frac{x}{r}$

$\sin(-\theta) =$

$\cos(-\theta) =$

$\sin(-\theta) =$

$\cos(-\theta) =$

Ex: 7.2

① a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

Let's Practice...

Differentiate the following:

$$f(x) = \frac{1}{1+\tan x} = (1+\tan x)^{-1}$$

$$f'(x) = -1(1+\tan x)^{-2}(\sec^2 x)$$

$$f'(x) = \frac{-\sec^2 x}{(1+\tan x)^2}$$

Ex #2.

Differentiate:

$$f(x) = 2\csc^3(3x^2) = 2[\csc(3x^2)]^3$$

$$f'(x) = 6[\csc(3x^2)]^2(-\csc(3x^2)\cot(3x^2)\cdot 6x)$$

$$f'(x) = 6\csc^2(3x^2)(-6x\csc(3x^2)\cot(3x^2))$$

$$f'(x) = -36x\csc^3(3x^2)\cot(3x^2)$$

Homework

Worksheet on derivatives of trigonometric functions

Page 314 #3 a,c,e Ex (7.2)

Page 319 #1 Ex (7.3)

③ a) Find the equation of the tangent to the curve $y = 2\sin x$ @ $(\frac{\pi}{6}, 1)$ $x_1 = \frac{\pi}{6}$, $y_1 = 1$

(i) Find y'

$y' = 2\cos x$

(ii) Find $y'(\frac{\pi}{6})$

$m = y'(\frac{\pi}{6}) = 2\cos(\frac{\pi}{6})$

$= 2\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$

(iii) $y - y_1 = m(x - x_1)$

$y - 1 = \sqrt{3}(x - \frac{\pi}{6})$

$y - 1 = x\sqrt{3} - \frac{\pi\sqrt{3}}{6}$

$6y - 6 = 6x\sqrt{3} - \pi\sqrt{3}$

$\boxed{0 = 6x\sqrt{3} - 6y - \pi\sqrt{3} + 6}$

Attachments

Derivatives Worksheet.doc