Questions from Homework

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⑤ $s = 160t^2 + 20t$

$v = s' = 320t + 20$

$100 = 320t + 20$

$\frac{80}{320} = \frac{320t}{320}$

$\frac{1}{4} = t$

$0.25h = t$

$15 \text{ mins} = t$

⑥ $s = t^3 - 3t^2 - 5t, \quad t \geq 0$

$v = s' = 3t^2 - 6t - 5$

$4 = 3t^2 - 6t - 5$

$0 = 3t^2 - 6t - 9$

$0 = 3(t^2 - 2t - 3)$

$0 = 3(t-3)(t+1)$

$t - 3 = 0 \mid t + 1 = 0$

$t = 3s \mid t = -1$

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⑦ $s = t^3 - 9t^2 + 18t$

$v = s' = 3t^2 - 18t + 18$

$a = s'' = 6t - 18$

a) $a = 6t - 18$

$0 = 6t - 18$

$18 = 6t$

$\boxed{3 = t}$

b) when $t = 3$

$s(3) = (3)^3 - 9(3)^2 + 18(3)$

$s(3) = 27 - 81 + 54$

$s(3) = 0m$

when $t = 3$

$v(3) = 3(3)^2 - 18(3) + 18$

$v(3) = 27 - 54 + 18$

$v(3) = -9 m/s$

④ d) $s = \frac{5t}{1+t}$ $\quad f(x)$ $\quad g(x)$

$v = \frac{5(1+t) - 5t(1)}{(1+t)^2}$

$v = \frac{5 + 5t - 5t}{(1+t)^2} = \frac{5}{(1+t)^2} = 5(1+t)^{-2}$

$a = -10(1+t)^{-3}(1)$

$a = \frac{-10}{(1+t)^3} = \frac{-10}{(1+4)^3} = \frac{-10}{125} = \boxed{\frac{-2}{25} m/s^2}$

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① $s = t^2 - 4t + 4, \quad t \geq 0$

$s' = 2t - 4$ (velocity)

a) $s'(1) = 2(1) - 4$ | $s'(3) = 2(3) - 4$

$s'(1) = -2\ m/s$ | $s'(3) = 2\ m/s$

b) Let v or $s' = 0$

$s' = 2t - 4$

$0 = 2t - 4$

$0 = 2(t - 2)$

$t - 2 = 0$

$\boxed{t = 2}$

c) Let v or $s' > 0$

$2t - 4 > 0$

$2t > 4$

$t > 2$

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③ d) $s = \sqrt{t^2 + t} = (t^2 + t)^{1/2}$

$v = \frac{1}{2}(t^2 + t)^{-1/2}(2t + 1)$

$v = \frac{2t + 1}{2(t^2 + t)^{1/2}}$

$a = \frac{2(t^2 + t)^{1/2}(2) - (2t + 1)(1)(t^2 + t)^{-1/2}(2t + 1)}{\left[2(t^2 + t)^{1/2}\right]^2}$

$a = \frac{4(t^2 + t)^{1/2} - (2t + 1)^2 (t^2 + t)^{-1/2}}{4(t^2 + t)}$

$a = \frac{(t^2 + t)^{-1/2}\left[4(t^2 + t) - (2t + 1)^2\right]}{4(t^2 + t)}$ (FOIL)

$a = \frac{4t^2 + 4t - 4t^2 - 4t - 1}{4(t^2 + t)^{3/2}}$

$a = \frac{-1}{4(t^2 + t)^{3/2}}$

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time.*

Differentiate with respect to time $(x, y, \frac{dx}{dt}, \frac{dy}{dt})$

If $xy^2 = 12$ and $\frac{dy}{dt} = 6$, Find $\frac{dx}{dt}$ when ~~$y = 2$~~

(i) Solve for x when $y = 2$

$x(2)^2 = 12$

$\frac{4x}{4} = \frac{12}{4}$

$x = 3$

(ii) Differentiate wrt

$(x)(y^2) = 12$

$\frac{dx}{dt} y^2 + x(2y\frac{dy}{dt}) = 0$

$y^2 \frac{dx}{dt} + 2xy\frac{dy}{dt} = 0$

$(2)^2 \frac{dx}{dt} + 2(3)(2)(6) = 0$

$4\frac{dx}{dt} + 72 = 0$

$4\frac{dx}{dt} = -72$

$\frac{dx}{dt} = -18$

If $x^3 + y^3 = 9$ and $\frac{dx}{dt} = 4$, Find $\frac{dy}{dt}$ when $x = 2$

(i) Find y when $x = 2$

$(2)^3 + y^3 = 9$

$8 + y^3 = 9$

$y^3 = 1$

$y = 1$

(ii) Differentiate wrt

$x^3 + y^3 = 9$

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$

$3(2)^2(4) + 3(1)^2\frac{dy}{dt} = 0$

$48 + 3\frac{dy}{dt} = 0$

$3\frac{dy}{dt} = -48$

$\frac{dy}{dt} = -16$