Questions from Homework

4. Given $f(x) = 3x^2 + 2$, $g(x) = \sqrt{x + 4}$, and $h(x) = 4x - 2$, determine each combined function and state its domain.

a) $y = (f + g)(x)$ **b)** $y = (h - g)(x)$

c) $y = (g - h)(x)$ **d)** $y = (f + h)(x)$

$f(x) = 3x^2 + 2$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

$g(x) = \sqrt{x+4}$

$x + 4 \geq 0$

$x \geq -4$

D: $\{x \mid x \geq -4, x \in R\}$

or $[-4, \infty)$

$h(x) = 4x - 2$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

a) $y = f(x) + g(x)$

$y = 3x^2 + 2 + \sqrt{x+4}$

$y = 3x^2 + 2 + \sqrt{x+4}$

D: $\{x \mid x \geq -4, x \in R\}$

$[-4, \infty)$

d) $y = f(x) + h(x)$

$y = 3x^2 + 2 + (4x - 2)$

$y = 3x^2 + 2 + 4x - 2$

$y = 3x^2 + 4x$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

11. If $h(x) = (f - g)(x)$ and $f(x) = 5x + 2$, determine $g(x)$.

a) $h(x) = -x^2 + 5x + 3$

b) $h(x) = \sqrt{x - 4} + 5x + 2$

c) $h(x) = -3x + 11$

d) $h(x) = -2x^2 + 16x + 8$

Function Operations

To combine two functions, $f(x)$ and $g(x)$, multiply or divide as follows:

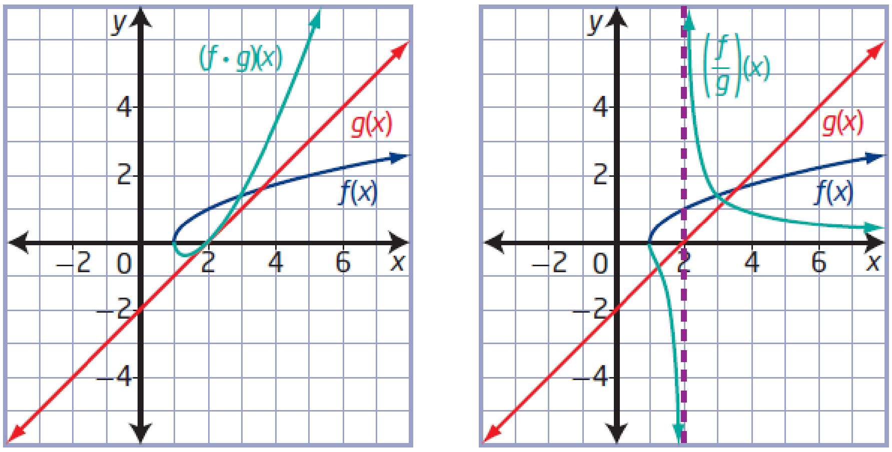
Product of Functions	*Quotient of Functions*
$h(x) = f(x)g(x)$	$h(x) = \frac{f(x)}{g(x)}$
$h(x) = (f \cdot g)(x)$	$h(x) = \left(\frac{f}{g}\right)(x)$

The domain of a product of functions is the domain common to the original functions. However, the domain of a quotient of functions must take into consideration that division by zero is undefined. The domain of a quotient, $h(x) = \frac{f(x)}{g(x)}$, is further restricted for values of x where $g(x) = 0$.

Example

Consider $f(x) = \sqrt{x - 1}$ and $g(x) = x - 2$.

The domain of $f(x)$ is $\{x \mid x \geq 1, x \in \mathrm{R}\}$, and the domain of $g(x)$ is $\{x \mid x \in \mathrm{R}\}$. So, the domain of $(f \cdot g)(x)$ is $\{x \mid x \geq 1, x \in \mathrm{R}\}$, while the domain of $\left(\frac{f}{g}\right)(x)$ is $\{x \mid x \geq 1, x \neq 2, x \in \mathrm{R}\}$



$f(x) = \sqrt{x-1}$ $(\longrightarrow)$

$x - 1 \geq 0$

$x \geq 1$

D: $\{x \mid x \geq 1, x \in \mathbb{R}\}$

or $[1, \infty)$

$g(x) = x - 2$ $(\longleftrightarrow)$

D: $\{x \mid x \in \mathbb{R}\}$

or $(-\infty, \infty)$

Key Ideas

- The combined function $h(x) = (f \cdot g)(x)$ represents the product of two functions, $f(x)$ and $g(x)$.
- The combined function $h(x) = \left(\frac{f}{g}\right)(x)$ represents the quotient of two functions, $f(x)$ and $g(x)$, where $g(x) \neq 0$.
- The domain of a product or quotient of functions is the domain common to both $f(x)$ and $g(x)$. The domain of the quotient $\left(\frac{f}{g}\right)(x)$ is further restricted by excluding values where $g(x) = 0$.
- The range of a combined function can be determined using its graph.

Example 1

Determine the Product of Functions

Given $f(x) = (x + 2)^2 - 5$ and $g(x) = 3x - 4$, determine $h(x) = (f \cdot g)(x)$. State the domain and range of $h(x)$.

Solution

$f(x) = (x+2)(x+2) - 5$

$f(x) = x^2 + 2x + 2x + 4 - 5 = x^2 + 4x - 1$

To determine $h(x) = (f \cdot g)(x)$, multiply the two functions.

$h(x) = (f \cdot g)(x)$

$h(x) = f(x)g(x)$

$h(x) = ((x + 2)^2 - 5)(3x - 4)$

$h(x) = (x^2 + 4x - 1)(3x - 4)$

$h(x) = 3x^3 - 4x^2 + 12x^2 - 16x - 3x + 4$

$h(x) = 3x^3 + 8x^2 - 19x + 4$

How can you tell from the original functions that the product is a cubic function?

$f(x) = x^2 + 4x - 1$	$g(x) = 3x - 4$	$h(x) = 3x^3 + 8x^2 - 19x + 4$
D: $\{x \mid x \in R\}$	D: $\{x \mid x \in R\}$	D: $\{x \mid x \in R\}$
or $(-\infty, \infty)$	or $(-\infty, \infty)$	or $(-\infty, \infty)$
		R: $\{y \mid y \in R\}$
		or $(-\infty, \infty)$

Example 2

Determine the Quotient of Functions

Consider the functions $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$.

a) Determine the equation of the function $h(x) = \left(\frac{g}{f}\right)(x)$.

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) State the domain and range of $h(x)$.

$f(x) = x^2+x-6$ | $g(x) = 2x+6$ | $h(x) = \frac{2}{x-2}$

D: $\{x \mid x \in \mathbb{R}\}$ | D: $\{x \mid x \in \mathbb{R}\}$ | D: $\{x \mid x \neq -3, 2, x \in \mathbb{R}\}$

Solution

a) To determine $h(x) = \left(\frac{g}{f}\right)(x)$, divide the two functions.

$$h(x) = \left(\frac{g}{f}\right)(x)$$

$$h(x) = \frac{g(x)}{f(x)}$$

Common

$$h(x) = \frac{2x + 6}{x^2 + x - 6}$$

Simple Trinomial

$-2 \times 3 = -6$

$-2 + 3 = 1$

$$h(x) = \frac{2(x + 3)}{(x + 3)(x - 2)} \quad \text{Factor.}$$

$$h(x) = \frac{2\cancel{(x + 3)}^{1}}{\cancel{(x + 3)}_{1}(x - 2)}$$

$$h(x) = \frac{2}{x - 2},\ x \neq -3,\ 2 \quad \text{Identify any non-permissible values.}$$

hole @ $x = -3$

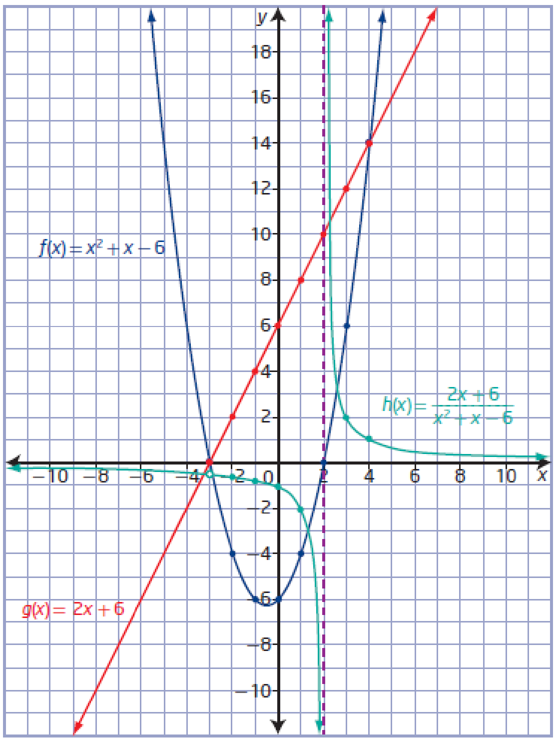
$$h(x) = \frac{2}{x-2}$$

$$h(-3) = \frac{2}{-3-2} = \frac{2}{-5} = -\frac{2}{5}$$

$\boxed{\left(-3, -\tfrac{2}{5}\right)}$

b) **Method 1: Use Paper and Pencil**

x	$f(x) = x^2 + x - 6$	$g(x) = 2x + 6$	$h(x) = \frac{2}{x-2}, x \neq -3, 2$	
-3	0	0	does not exist	hole @ $(-3, -2/5)$
-2	-4	2	$-\frac{1}{2}$	
-1	-6	4	$-\frac{2}{3}$	
0	-6	6	-1	
1	-4	8	-2	
2	0	10	undefined	VA @ $x=2$
3	6	12	2	
4	14	14	1	



How are the y-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

c) The function $f(x) = x^2 + x - 6$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = 2x + 6$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = \left(\frac{g}{f}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $f(x) = 0$.

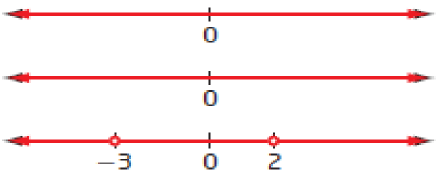
Since the function $h(x)$ does not exist at $\left(-3, -\frac{2}{5}\right)$ and is undefined at $x = 2$, the domain is $\{x \mid x \neq -3, x \neq 2, x \in \mathrm{R}\}$. This is shown in the graph by the point of discontinuity at $\left(-3, -\frac{2}{5}\right)$ and the vertical asymptote that appears at $x = 2$.

How do you know there is a point of discontinuity and an asymptote?

Domain of $f(x) = x^2 + x - 6$

Domain of $g(x) = 2x + 6$

Domain of $h(x) = \left(\frac{g}{f}\right)(x)$



The range of $h(x)$ is $\left\{y \mid y \neq 0, -\frac{2}{5}, y \in \mathrm{R}\right\}$.

horizontal asymptote

hole @ $x = -3$

$h(x) = \frac{2}{x-2}$

$h(-3) = \frac{2}{-3-2} = \frac{2}{-5} = -\frac{2}{5}$ $\quad (-3, -2/5)$

Homework

finish #1-9 on page 496-497

① d) $f(x)=\sqrt{x-1}$ and $g(x)=\sqrt{6-x}$

$h(x)=f(x)g(x)$

$h(x)=(\sqrt{x-1})(\sqrt{6-x})$

$h(x)=\sqrt{(x-1)(6-x)}$

$h(x)=\sqrt{6x-x^2-6+x}$

$h(x)=\sqrt{-x^2+7x-6}$

Ex: $\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$

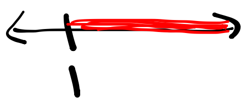
$\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$

$f(x)=\sqrt{x-1}$

$x-1\geq 0$

$x\geq 1$

D: $\{x\,|\,x\geq 1, x\in\mathbb{R}\}$



$g(x)=\sqrt{6-x}$

$6-x\geq 0$

$-x\geq -6$

$x\leq 6$

D: $\{x\,|\,x\leq 6, x\in\mathbb{R}\}$

$h(x)=\sqrt{-x^2+7x-6}$

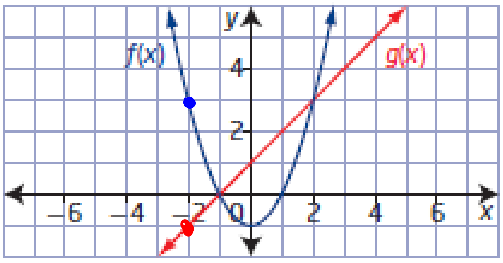
D: $\{x\,|\,1\leq x\leq 6, x\in\mathbb{R}\}$

$k(x)=\frac{f(x)}{g(x)}=\frac{\sqrt{x-1}}{\sqrt{6-x}}$

D: $\{x\,|\,1\leq x\leq 6, x\neq 6, x\in\mathbb{R}\}$

Homework

2. Use the graphs of $f(x)$ and $g(x)$ to evaluate the following.



a) $(f \cdot g)(-2)$ b) $(f \cdot g)(1)$

c) $\left(\frac{f}{g}\right)(0)$ d) $\left(\frac{f}{g}\right)(1)$

a) $f(-2) \cdot g(-2)$

$= 3 \cdot -1$

$= -3$

10.2 Products and Quotients of Functions, pages 496 to 498

1. a) $h(x) = x^2 - 49$, $k(x) = \dfrac{x+7}{x-7}$, $x \neq 7$

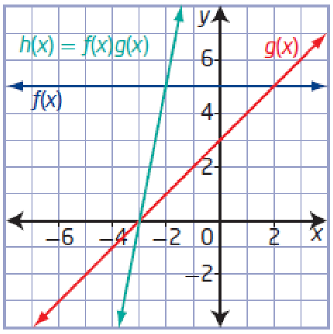
b) $h(x) = 6x^2 + 5x - 4$, $k(x) = \dfrac{2x-1}{3x+4}$, $x \neq -\frac{4}{3}$

c) $h(x) = (x+2)\sqrt{x+5}$, $k(x) = \dfrac{\sqrt{x+5}}{x+2}$, $x \geq -5$, $x \neq -2$

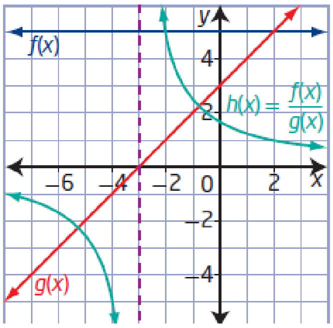
d) $h(x) = \sqrt{-x^2 + 7x - 6}$, $k(x) = \dfrac{\sqrt{x-1}}{\sqrt{6-x}}$, $1 \leq x < 6$

2. a) -3 **b)** 0 **c)** -1 **d)** 0

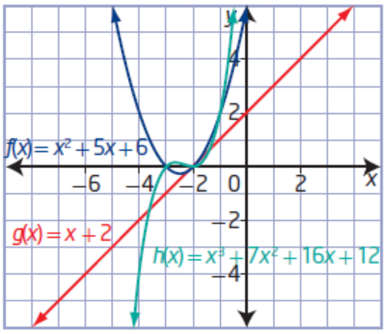
3. a)



b)

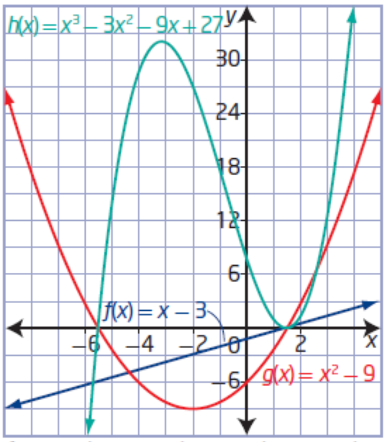


4. a) $h(x) = x^3 + 7x^2 + 16x + 12$



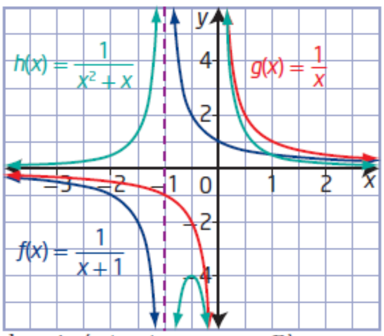
domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$

b) $h(x) = x^3 - 3x^2 - 9x + 27$



domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$

c) $h(x) = \dfrac{1}{x^2 + x}$



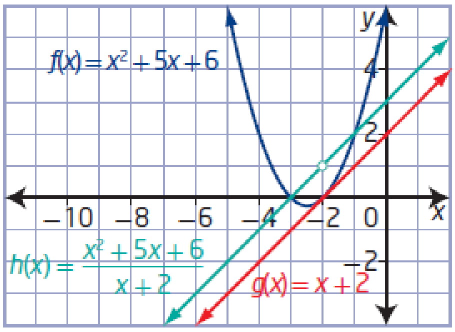
domain $\{x \mid x \neq 0, -1, x \in \mathbb{R}\}$, range $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathbb{R}\}$

$f(x) = \dfrac{1}{x+1}$ D: $\{x \mid x \neq -1, x \in R\}$

$g(x) = \dfrac{1}{x}$ D: $\{x \mid x \neq 0, x \in R\}$

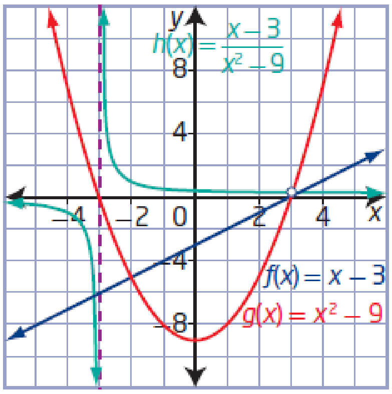
$h(x) = \dfrac{1}{x^2 + x}$ D: $\{x \mid x \neq -1, 0, x \in R\}$

5. a) $h(x) = x + 3,\ x \neq -2$



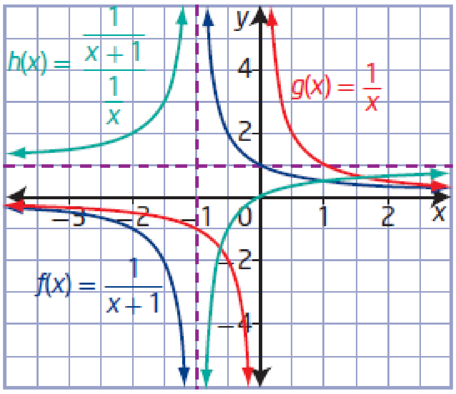
domain $\{x \mid x \neq -2,\ x \in \mathrm{R}\}$,
range $\{y \mid y \neq 1,\ y \in \mathrm{R}\}$

b) $h(x) = \dfrac{1}{x + 3},\ x \neq \pm 3$



domain $\{x \mid x \neq \pm 3,\ x \in \mathrm{R}\}$,
range $\left\{y \mid y \neq 0,\ \frac{1}{6},\ y \in \mathrm{R}\right\}$

c) $h(x) = \dfrac{x}{x + 1},\ x \neq -1, 0$



domain $\{x \mid x \neq -1, 0,\ x \in \mathrm{R}\}$,
range $\{y \mid y \neq 0, 1,\ y \in \mathrm{R}\}$

6. a) $y = x^3 + 3x^2 - 10x - 24$

b) $y = \dfrac{x^2 - x - 6}{x + 4},\ x \neq -4$ **c)** $y = \dfrac{2x - 1}{x + 4},\ x \neq -4$

d) $y = \dfrac{x^2 - x - 6}{x^2 + 8x + 16},\ x \neq -4$

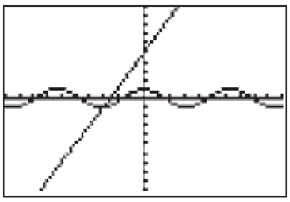
7. a) $g(x) = 3$ **b)** $g(x) = -x$

c) $g(x) = \sqrt{x}$ **d)** $g(x) = 5x - 6$

8. a) $g(x) = x + 7$ **b)** $g(x) = \sqrt{x + 6}$

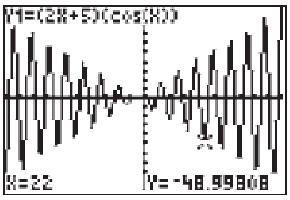
c) $g(x) = 2$ **d)** $g(x) = 3x^2 + 26x - 9$

9. a)



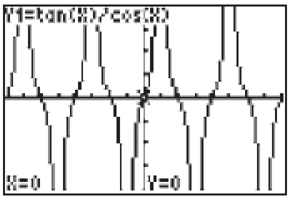
$f(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$,
range
$\{y \mid -1 \leq y \leq 1,\ y \in \mathrm{R}\}$

b)



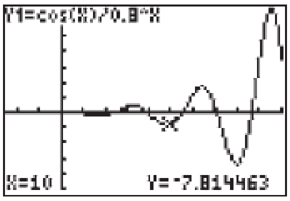
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

10. a)



domain $\{x \mid x \neq (2n - 1)\frac{\pi}{2}, n \in \mathrm{I}, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

b)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

11. a) $y = \dfrac{f(x)}{g(x)}$ **b)** $y = f(x)f(x)$

c) The graphs of $y = \dfrac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.