Ch. 1 Practice Test

(10) 5, 36, 67, 98, 129, 160 (Arithmetic)

Given:

$t_1 = 5$, $t_6 = 160$

$t_n = a + (n-1)d$

$t_1 = a + (1-1)d$

$t_1 = a + 0d$

$t_1 = a$

$\boxed{5 = a}$

$t_n = a + (n-1)d$

$t_6 = a + (6-1)d$

$t_6 = a + 5d$

$\boxed{160 = a + 5d}$

$160 = a + 5d$

$160 = 5 + 5d$

$\frac{155}{5} = \frac{5d}{5}$

$\boxed{31 = d}$

b) general term

$t_n = a + (n-1)d$

$t_n = 5 + (n-1)(31)$

$t_n = 5 + 31n - 31$

$\boxed{t_n = 31n - 26}$

Series + Sequence

① $\underline{80000}$, ____, ____, ____, $\underline{117128}$

$a = 80000$

$t_n = ar^{n-1}$

$t_5 = 117128$

$\frac{117128}{80000} = \frac{\cancel{80000}r^{5-1}}{\cancel{80000}}$

$n = 5$

$r = ?$

$(1.4641)^{1/4} = (r^4)^{1/4}$

ARoI = ?

$\boxed{1.1 = r}$

ARoI = $100(1.1-1) = 10\%$

Series & Sequence

5) a) $\sum_{n=1}^{5} n^2+1$

$= [(1)^2+1] + [(2)^2+1] + [(3)^2+1] + [(4)^2+1] + [(5)^2+1]$

$= 2 + 5 + 10 + 17 + 26$

$= 60$

b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

$a = 3$

$r = \frac{1}{2}$

$S_n = \frac{a}{1-r}$

$S_n = \frac{3}{1-\frac{1}{2}}$

$S_n = \frac{3}{\frac{2}{2}-\frac{1}{2}}$

$S_n = \frac{3}{\frac{1}{2}} = 3 \times 2 = 6$

Series & Sequence

5 c) $1+5+9+ \dots +77$

$\underbrace{1 \quad 5}_{4} \quad \underbrace{5 \quad 9}_{4}$

$a = 1$

$t_n = 77$

$d = 4$

① Find n:

$t_n = a + (n-1)d$

$77 = 1 + (n-1)4$

$76 = 4(n-1)$

$19 = n-1$

$\boxed{20 = n}$

② Find S_{20}

$S_{20} = \frac{20}{2}[1+77]$

$= 10(78)$

$\boxed{= 780}$

5 b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \dots$

$\underbrace{3 \quad \frac{3}{2}}_{1/2} \quad \underbrace{\frac{3}{2} \quad \frac{3}{4}}_{1/2}$

$a = 3$

$r = \frac{1}{2}$

$S_n = \frac{3}{1-\frac{1}{2}} = \frac{3}{\frac{1}{2}} = 3 \cdot 2 = 6$

4 c) $\lim_{n \to \infty} (-1)^{n+1} n^2$

$= 1, -4, 9, -16, 25, -36, 49, -64, 81, -100$

= Diverging (has no limit)

Series & Sequence

⑥ Find "a", "r", and S_5

$t_n = ar^{n-1}$

$\frac{1}{9} \div 9 = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

$t_3 = 9$

$t_3 = ar^{3-1}$

$t_3 = ar^2$

$9 = ar^2$

$ar^2 = 9$

$t_7 = \frac{1}{9}$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$\frac{1}{9} = ar^6$

$ar^6 = \frac{1}{9}$

Elimination

$$\frac{ar^6 = \frac{1}{9}}{ar^2 = 9}$$

$r^4 = \frac{1}{81}$

$\boxed{r = \pm\frac{1}{3}}$

$ar^2 = 9$

$a\left(\pm\frac{1}{3}\right)^2 = 9$

$a\left(\frac{1}{9}\right) = 9$

$\cancel{9} \cdot \frac{a}{\cancel{9}} = 9 \cdot 9$

$\boxed{a = 81}$

when $r = \frac{1}{3}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_5 = \frac{81\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\frac{1}{3} - 1}$

$S_5 = \frac{81\left(\frac{1}{243} - \frac{243}{243}\right)}{\frac{1}{3} - \frac{3}{3}}$

$S_5 = 81\left(\frac{-242}{243}\right) \div -\frac{2}{3}$

$S_5 = \cancel{81}^{121}\left(\frac{-\cancel{242}}{\cancel{243}}\right)\left(\frac{-\cancel{3}}{\cancel{2}}\right)$

$S_5 = 121$

when $r = -\frac{1}{3}$

$S_5 = \frac{81\left(\left(-\frac{1}{3}\right)^5 - 1\right)}{-\frac{1}{3} - 1}$

$S_5 = \frac{81\left(\frac{-1}{243} - \frac{243}{243}\right)}{-\frac{1}{3} - \frac{3}{3}}$

$S_5 = \cancel{81}^{61}\left(\frac{-\cancel{244}}{\cancel{243}}\right)\left(-\frac{\cancel{3}}{\cancel{4}}\right)$

$S_5 = 61$

Series & Sequence

$t_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

① $t_{12} = 15$

$t_{12} = a + (12-1)d$

$t_{12} = a + 11d$

$15 = a + 11d$

$\boxed{a + 11d = 15}$

$S_{15} = 105$

$S_{15} = \frac{15}{2}[2a + (15-1)d]$

$S_{15} = 7.5[2a + 14d]$

$S_{15} = 15a + 105d$

$105 = 15a + 105d$

$15a + 105d = 105$

$\boxed{a + 7d = 7}$

Elimination

$a + 11d = 15$

$(-)\ a + 7d = 7$

$\frac{4d}{4} = \frac{8}{4}$

$\boxed{d = 2}$

↳ $a + 7d = 7$

$a + 7(2) = 7$

$a + 14 = 7$

$\boxed{a = -7}$

$t_1 = -7$

$t_2 = -5$

$t_3 = -3$

Series & Sequence

$\_\_\_, \frac{1}{5}, \_\_\_, \_\_\_, 25, \_\_\_, \_\_\_$

$t_2 = \frac{1}{5}$, $t_5 = 25$

$t_2 = ar^{2-1}$ $t_5 = ar^{5-1}$

$t_2 = ar^1$ $t_5 = ar^4$

$\frac{1}{5} = ar$ $25 = ar^4$

$\boxed{ar = \frac{1}{5} \quad ar^4 = 25}$

$$\frac{ar^4 = 25}{ar = 1/5}$$

$r^3 = 125$

$r = 5$

$ar = 1/5$

$a(5) = \frac{1}{5}$

$5a = \frac{1}{5}$

$\boxed{a = \frac{1}{25}}$

Permutations + Combinations

binomial theore

- used to expand $(x + y)^n$, $n \in N$
- each term has the f $_nC_k(x)^{n-k}(y)^k$, where $k + 1$ is the term number

You can use the **binomial theorem** to expand any power of a binomial expression.

$$(x + y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + \cdots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

In this chapter, all binomial expansions will be written in descending order of the exponent of the first term in the binomial.

The following are some important observations about the expansion of $(x + y)^n$, where x and y represent the terms of the binomial and $n \in N$:

- the expansion contains $n + 1$ terms
- the number of objects, k, selected in the combination $_nC_k$ can be taken to match the number of factors of the second variable selected; that is, it is the same as the exponent on the second variable
- the general term, t_{k+1}, has the form

$${}_nC_k(x)^{n-k}(y)^k$$

the same

- the sum of the exponents in any term of the expansion is n

Permutations + Combinations

① 3 digit # from 0, 1, 2, 8, and 9
with no repititions

$\underline{4} \times \underline{4} \times \underline{3} = 48$ C

can't use 0 as digit one

⑮ $\left(y - \frac{2}{y^2}\right)^5$

$n = 5$
$x = y$
$y = \frac{-2}{y^2}$

$_5C_0(y)^5\left(\frac{-2}{y^2}\right)^0 + {_5C_1}(y)^4\left(\frac{-2}{y^2}\right)^1 + {_5C_2}(y)^3\left(\frac{-2}{y^2}\right)^2 + {_5C_3}(y)^2\left(\frac{-2}{y^2}\right)^3 + {_5C_4}(y)^1\left(\frac{-2}{y^2}\right)^4 + {_5C_5}(y)^0\left(\frac{-2}{y^2}\right)^5$

$1(y^5)(1) + 5(y^4)\left(\frac{-2}{y^2}\right) + 10(y^3)\left(\frac{4}{y^4}\right) + 10(y^2)\left(\frac{-8}{y^6}\right) + 5(y)\left(\frac{16}{y^8}\right) + 1(1)\left(\frac{-32}{y^{10}}\right)$

$y^5 - \frac{10y^4}{y^2} + \frac{40y^3}{y^4} - \frac{80y^2}{y^8} + \frac{80y}{y^8} - \frac{32}{y^{10}}$

$\boxed{y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}}$

Permutations + Combinations

5 $(2x^2+3y)^7 \rightarrow 3^{rd}$ term

$${}_7C_2(2x^2)^5(3y)^2$$

$$21(32x^{10})(9y^2)$$

$$\boxed{6048x^{10}y^2}$$

Permutations + Combinations

② $\left(x^2 - \frac{x}{2}\right)^5$

$${}_5C_0(x^2)^5\left(-\frac{x}{2}\right)^0 + {}_5C_1(x^2)^4\left(-\frac{x}{2}\right)^1 + {}_5C_2(x^2)^3\left(-\frac{x}{2}\right)^2 + {}_5C_3(x^2)^2\left(-\frac{x}{2}\right)^3 + {}_5C_4(x^2)^1\left(-\frac{x}{2}\right)^4 + {}_5C_5(x^2)^0\left(-\frac{x}{2}\right)^5$$

$$1(x^{10})(1) + 5(x^8)\left(\frac{-x}{2}\right) + 10(x^6)\left(\frac{x^2}{4}\right) + 10(x^4)\left(\frac{-x^3}{8}\right) + 5(x^2)\left(\frac{x^4}{16}\right) + 1(1)\left(\frac{-x^5}{32}\right)$$

$$x^{10} - \frac{5x^9}{2} + \frac{5}{2}x^8 - \frac{5}{4}x^7 + \frac{5}{16}x^6 - \frac{x^5}{32}$$

Permutations + Combinations

Expand: $\left(x-\frac{1}{2}\right)^4$

$${}_4C_0(x)^4\left(-\tfrac{1}{2}\right)^0+{}_4C_1(x)^3\left(-\tfrac{1}{2}\right)^1+{}_4C_2(x)^2\left(-\tfrac{1}{2}\right)^2+{}_4C_3(x)^1\left(-\tfrac{1}{2}\right)^3+{}_4C_4(x)^0\left(-\tfrac{1}{2}\right)^4$$

$$(1)(x^4)(1)+(4)(x^3)\left(-\frac{1}{2}\right)+(6)(x^2)\left(\frac{1}{4}\right)+(4)(x)\left(-\frac{1}{8}\right)+(1)(1)\left(\frac{1}{16}\right)$$

$$x^4-\frac{4x^3}{2}+\frac{6x^2}{4}-\frac{4x}{8}+\frac{1}{16}$$

$$\boxed{x^4-2x^3+\frac{3}{2}x^2-\frac{1}{2}x+\frac{1}{16}}$$

Permutations + Combinations

4 c) Case 1 (All Black)

$_{26}C_5 = \boxed{65\,780}$

Case 2 (4 Black + 1 Red)

$_{26}C_4 \times {}_{26}C_1 = 14\,950 \times 26 = \boxed{388\,700}$

Case 3 (3 black + 2 red)

$_{26}C_3 \times {}_{26}C_2 = 2600 \times 325 = \boxed{845000}$

Total $= 845\,000 + 388\,700 + 65\,780 = 1\,299\,480$

Permutations + Combinations

⑤ 14 Letters
- 5 are burnt
- 9 are good

3 good + 2 bad

${}_9C_3 \times {}_5C_2$

84×10

840

Limits

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Limits

① c) $\lim_{a\to b} \dfrac{(a+2b)^2-9b^2}{a-b}$

$\lim_{a\to b} \dfrac{(a+2b+3b)(a+2b-3b)}{(a-b)}$

$\lim_{a\to b} \dfrac{(a+5b)\cancel{(a-b)}}{\cancel{(a-b)}} = 6b$

① f) $\lim_{x\to 0} \dfrac{\frac{1}{x+5}-\frac{1}{5}}{(x^2+5x)(5(x+5))}$ CD: $5(x+5)$

$\lim_{x\to 0} \dfrac{5-1(x+5)}{x(x+5)(5(x+5))}$

$\lim_{x\to 0} \dfrac{5-x-5}{5x(x+5)^2}$

$\lim_{x\to 0} \dfrac{-\cancel{x}}{5\cancel{x}(x+5)^2} = \dfrac{-1}{5(25)} = \dfrac{-1}{125}$

Limits

② Let $f(x) = \begin{cases} -x^2+2, & \text{if } x<1 \\ 1, & \text{if } x=1 \\ -2x+3, & \text{if } 1<x\leq 3 \\ -3, & \text{if } x>3 \end{cases}$

$-x^2+2$

x	y
○ 1	1
0	2
-1	1
-2	-2

1

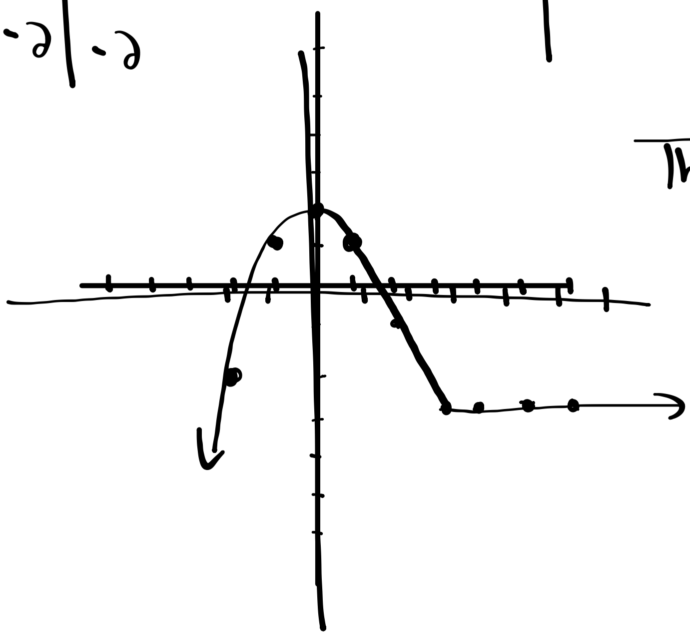
x	y
• 1	1

$-2x+3$

x	y
○ 1	1
2	-1
• 3	-3

-3

x	y
○ 3	-3
4	-3
5	-3
6	-3



The function is continuous

Limits

$$\lim_{x\to 0} \frac{\cancel{3(x+3)}\frac{1}{\cancel{x+3}} - \frac{1}{\cancel{3}}\cancel{3}(x+3)}{x \cdot 3(x+3)}$$

CD: $3(x+3)$

$$\lim_{x\to 0} \frac{3-(x+3)}{3x(x+3)}$$

$$\lim_{x\to 0} \frac{3-x-3}{3x(x+3)}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{3\cancel{x}(x+3)} = \frac{-1}{3(3)} = -\frac{1}{9}$$

Differentiation

Product: $(f\cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$

$$y = \underbrace{(3x^2-5)}_{f(x)}\underbrace{(4x^3+3x)}_{g(x)}$$

$$y' = 6x(4x^3+3x) + (3x^2-5)(12x^2+3)$$

$$y' = \underline{24x^4} + \underline{18x^2} + \underline{36x^4} + \underline{9x^2} - \underline{60x^2} - 15$$

$$\boxed{y' = 60x^4 - 33x^2 - 15}$$

Quotient: $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$y = \frac{x^2+2 \leftarrow f(x)}{3x+5 \leftarrow g(x)}$$

$$y' = \frac{2x(3x+5) - 3(x^2+2)}{(3x+5)^2}$$

$$y' = \frac{6x^2+10x-3x^2-6}{(3x+5)^2} = \boxed{\frac{3x^2+10x-6}{(3x+5)^2}}$$

Chain: $(f\circ g)'(x) = f'(g(x))\cdot g'(x)$

$$y = \sqrt{4x^3-6x} = (4x^3-6x)^{1/2}$$

$$y' = \frac{1}{2}(4x^3-6x)^{-1/2}(12x^2-6)$$

$$y' = \frac{12x^2-6}{2(4x^3-6x)^{1/2}} = \frac{\cancel{6}^{\,3}(2x^2-1)}{\cancel{2}\sqrt{4x^3-6x}} = \boxed{\frac{3(2x^2-1)}{\sqrt{4x^3-6x}}}$$

$$\frac{6x^2-3}{\sqrt{4x^3-6x}}$$

Combo:

$$y = (3x^2+5)^3\sqrt{4x-2} = \underbrace{(3x^2+5)^3}_{f(x)}\underbrace{(4x-2)^{1/2}}_{g(x)}$$

$$y' = \underbrace{3(3x^2+5)^2(6x)}_{f'(x)}\underbrace{(4x-2)^{1/2}}_{g(x)} + \underbrace{(3x^2+5)^3}_{f(x)}\underbrace{\left(\frac{1}{2}\right)(4x-2)^{-1/2}(4)}_{g'(x)}$$

$$y' = 18x(3x^2+5)^2(4x-2)^{1/2} + 2(3x^2+5)^3(4x-2)^{-1/2}$$

$$\cdot\ 36x^2 - 18x + 3x^2 + 5$$

$$y' = 2(3x^2+5)^2(4x-2)^{-1/2}\left[9x(4x-2) + (3x^2+5)\right]$$

$$y' = 2(3x^2+5)^2(4x-2)^{-1/2}(39x^2-18x+5)$$

$$y' = \frac{2(3x^2+5)^2(39x^2-18x+5)}{\sqrt{4x-2}}$$

$$\frac{(3x^2+5)^2}{(3x^2+5)^2} = (3x^2+5)^0 = 1 \quad \Bigg| \quad \frac{(4x-2)^{1/2}}{(4x-2)^{-1/2}} = (4x-2)^{1/2-(-1/2)} = (4x-2)^1$$

Differentiation

$-\frac{1}{2} \times 3 = -\frac{3}{2}$

$-\frac{1}{2} - 1$

$-\frac{1}{2} - \frac{2}{2} = -\frac{3}{2}$

② b) $f(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$

$f'(x) = -\frac{3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}}$

④ b) $y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{3+x^2}$ $\begin{matrix} f \\ g \end{matrix}$

$\frac{f'g - fg'}{(g)^2}$

$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - x^{1/2}(2x)}{(3+x^2)^2}$

$x^{1/2} \cdot x^1$

$x^{1/2+1}$

$x^{1/2+2/2}$

$x^{3/2}$

$y' = \frac{\frac{1}{2x^{1/2}}(3+x^2) - 2x^{3/2}}{(3+x^2)^2}$

$y' = \frac{\cancel{2x^{1/2}}\frac{(3+x^2)}{\cancel{2x^{1/2}}} - 2x^{3/2} \cdot 2x^{1/2}}{2x^{1/2}(3+x^2)^2}$

Complex Fraction:

CD: $2x^{1/2}$

$2x^{3/2} \cdot 2x^{1/2}$

$4x^{3/2+1/2} = 4x^{4/2}$

$y' = \frac{3+x^2-4x^2}{2\sqrt{x}(3+x^2)^2}$

$y' = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$

⑥ b) $y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$

$y' = -8(x-1)^{-3/2}(1) = -8(x-1)^{-3/2} = \frac{-8}{(x-1)^{3/2}}$

$= \frac{-8}{\sqrt{(x-1)^3}}$

Differentiation

$$f(x) = \boxed{3x^2 + 2x - 7}$$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 7$$

$$= 3(x^2 + 2xh + h^2) + 2x + 2h - 7$$

$$= \boxed{3x^2 + 6xh + 3h^2 + 2x + 2h - 7}$$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{+ 2x} + 2h \cancel{- 7} - (\cancel{3x^2} \cancel{+ 2x} \cancel{- 7})}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}} = \boxed{6x + 2}$$

Differentiation

$$f(x) = \frac{(3x^2+5)^3}{\sqrt{2x-7}} \qquad \frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{\overbrace{3(3x^2+5)^2(6x)}^{f'}\overbrace{(2x-7)^{1/2}}^{g} - \overbrace{(3x^2+5)^3}^{f}\overbrace{\left(\tfrac{1}{2}\right)(2x-7)^{-1/2}(2)}^{g'}}{\left[\sqrt{2x-7}\right]^2}$$

$$f'(x) = \frac{18x(3x^2+5)^2(2x-7)^{1/2} - (3x^2+5)^3(2x-7)^{-1/2}}{2x-7}$$ Factor

$36x^2 - 126x - 3x^2 - 5$

$$f'(x) = \frac{(3x^2+5)^2(2x-7)^{-1/2}\left[18x(2x-7) - (3x^2+5)\right]}{(2x-7)}$$

$$f'(x) = \frac{(3x^2+5)^2(33x^2-126x-5)}{(2x-7)^{3/2}}$$

Differentiation

3) $y = \sqrt[7]{2x^2 + \sqrt{x^7 - 8x\sqrt{3-x}}} = \left[2x^2 + \left(x^7 - 8x(3-x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{7}}$

$y' = \frac{1}{7}\left[2x^2 + \left(x^7 - 8x(3-x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{-\frac{6}{7}}\left[4x + \frac{1}{2}\left(x^7 - 8x(3-x)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(7x^6 - \left(8(3-x)^{\frac{1}{2}} + 8x\left(\frac{1}{2}\right)(3-x)^{-\frac{1}{2}}(-1)\right)\right)\right]$

Derivatives Exam Review:

④ b) $y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{(3+x^2)}$

$$y' = \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{2\cdot\frac{3}{2}x^{-1/2} + 2\cdot\frac{1}{2}x^{3/2} - 2\cdot 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{x^{-1/2}(3 + x^2 - 4x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3-3x^2}{2x^{1/2}(3+x^2)^2} = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$