

① a) $v_0 = 7.5 \text{ m/s [N]}$ $t = 3.0 \text{ s}$
 $v_f = 10.0 \text{ m/s [E } 40^\circ \text{ N]}$

Step 1: Horizontal Vertical

$v_{0E} = 0 \text{ m/s}$ $v_{0N} = 7.5 \text{ m/s}$

$v_{fE} = 10 \cos 40^\circ$ $v_{fN} = 10 \sin 40^\circ$
 $= 7.66 \text{ m/s}$ $= 6.43 \text{ m/s}$

Step 2: $\vec{a}_E = \frac{v_{fE} - v_{0E}}{t}$ $\vec{a}_N = \frac{v_{fN} - v_{0N}}{t}$

$= \frac{7.66 - 0}{3.0}$ $= \frac{6.43 - 7.5}{3.0}$

$= 2.55 \text{ m/s}^2$ $= -0.357 \text{ m/s}^2$

Step 3: $\vec{a} = \sqrt{(\vec{a}_E)^2 + (\vec{a}_N)^2}$ $\theta = \tan^{-1}\left(\frac{0.357}{2.55}\right)$

$\vec{a} = \sqrt{(2.55)^2 + (-0.357)^2}$ $\theta = 8^\circ$

$\vec{a} = 2.57 \text{ m/s}^2$

$\vec{a} = 2.57 \text{ m/s}^2 \text{ [E } 8^\circ \text{ S]}$

① b) v_f after 6.0 s

$v_f = v_0 + at$

Step 1: $v_{fE} = v_{0E} + \vec{a}_E t$

$= 0 \text{ m/s} + (2.55 \text{ m/s}^2)(6.0 \text{ s})$
 $= 15.3 \text{ m/s}$

$v_{fN} = v_{0N} + \vec{a}_N t$

$= 7.5 \text{ m/s} + (-0.357)(6)$
 $= 5.358 \text{ m/s}$

Step 2: $\vec{v}_f = \sqrt{(v_{fE})^2 + (v_{fN})^2}$

$= \sqrt{(15.3)^2 + (5.358)^2}$
 $= 16.2 \text{ m/s}$

$\theta = \tan^{-1}\left(\frac{5.358}{15.3}\right)$

$= 19^\circ$

$v_f = 16.2 \text{ m/s [E } 19^\circ \text{ N]}$

$$\textcircled{2} \quad \vec{V}_0 = 6.5 \text{ m/s [E } 20^\circ \text{ S]} \\ \vec{a} = 2.1 \text{ m/s}^2 \text{ [E } 60^\circ \text{ N]} \\ t = 18 \text{ s}$$

$$\text{Step 1: a) } \vec{V}_f = ? \quad \vec{V}_f = \vec{V}_0 + \vec{a}t$$

Horizontal:

$$\vec{V}_{fE} = 6.5 \cos 20^\circ \\ = 6.11 \text{ m/s}$$

$$\vec{a}_E = 2.1 \cos 60^\circ \\ = 1.05 \text{ m/s}^2$$

Vertical:

$$\vec{V}_{fN} = -6.5 \sin 20^\circ \\ = -2.22 \text{ m/s}$$

$$\vec{a}_N = 2.1 \sin 60^\circ \\ = 1.82 \text{ m/s}^2$$

$$\text{Step 2: } \vec{V}_{fE} = \vec{V}_{0E} + \vec{a}_E t \quad \vec{V}_{fN} = \vec{V}_{0N} + \vec{a}_N t \\ = 6.11 + (1.05)(18) \quad = -2.22 + (1.82)(18) \\ = 25.01 \text{ m/s} \quad = 30.54 \text{ m/s}$$

$$\text{Step 3: } \vec{V}_f = \sqrt{(25.01)^2 + (30.54)^2} \quad \theta = \tan^{-1} \left(\frac{30.54}{25.01} \right) \\ \vec{V}_f = 39.5 \text{ m/s} \quad = 51^\circ$$

$$\boxed{\vec{V}_f = 39.5 \text{ m/s [E } 51^\circ \text{ N]}}$$

$$\textcircled{2} \text{ b) } \vec{d} = ? \quad \vec{d} = \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\text{Step 1: } \vec{d}_E = \vec{V}_{0E} t + \frac{1}{2} \vec{a}_E t^2 \\ = (6.11)(18) + \frac{1}{2}(1.05)(18)^2 \\ \text{Found in part a} \\ = 109.98 + 170.1 \\ = 280.1 \text{ m}$$

$$\vec{d}_N = \vec{V}_{0N} t + \frac{1}{2} \vec{a}_N t^2 \\ = (-2.22)(18) + \frac{1}{2}(1.82)(18)^2 \\ \text{Found in part a} \\ = -39.96 + 294.84 \\ = 254.9 \text{ m}$$

$$\vec{d} = \sqrt{(280.1)^2 + (254.9)^2} \quad \theta = \tan^{-1} \left(\frac{254.9}{280.1} \right) \\ = 378.7 \text{ m} \quad = 42^\circ$$

$$\boxed{\vec{d} = 378.7 \text{ m [E } 42^\circ \text{ N]}}$$

Applications of Vectors

(3) $\vec{v}_a = 9.2 \text{ m/s [E } 25^\circ \text{ N]}$
 $\vec{v}_f = 11 \text{ m/s [E } 14^\circ \text{ S]}$
 $t = 7.9 \text{ s}$

Step 1: a) $\vec{a} = ?$

$\begin{aligned} \text{Horizontal} \\ \vec{v}_{0E} &= 9.2 \cos 25^\circ \\ &= 8.34 \text{ m/s} \end{aligned}$	$\begin{aligned} \text{Vertical} \\ \vec{v}_{0N} &= 9.2 \sin 25^\circ \\ &= 3.89 \text{ m/s} \end{aligned}$
$\begin{aligned} \vec{v}_{fE} &= 11 \cos 14^\circ \\ &= 10.67 \text{ m/s} \end{aligned}$	$\begin{aligned} \vec{v}_{fN} &= -11 \sin 14^\circ \\ &= -2.66 \text{ m/s} \end{aligned}$

Step 2: $\vec{a}_E = \frac{\vec{v}_{fE} - \vec{v}_{0E}}{t}$ $\vec{a}_N = \frac{\vec{v}_{fN} - \vec{v}_{0N}}{t}$

$\begin{aligned} &= \frac{10.67 - 8.34}{7.9} \\ &= \underline{0.29 \text{ m/s}^2} \end{aligned}$	$\begin{aligned} &= \frac{-2.66 - 3.89}{7.9} \\ &= \underline{-0.83 \text{ m/s}^2} \end{aligned}$
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Step 3: $\vec{a} = \sqrt{(\vec{a}_E)^2 + (\vec{a}_N)^2}$ $\phi = \tan^{-1}\left(\frac{0.83}{0.29}\right)$

$$\begin{aligned} &= \sqrt{(0.29)^2 + (-0.83)^2} && = 71^\circ \\ &= 0.88 \text{ m/s}^2 \end{aligned}$$

$\vec{a} = 0.88 \text{ m/s}^2 \text{ [E } 71^\circ \text{ S]}$

(3) b) $\vec{d} = ?$

$\vec{d}_E = \vec{v}_{0E} t + \frac{1}{2} \vec{a}_E t^2$

Use values from part (a)

$$\begin{aligned} &= (8.34)(7.9) + \frac{1}{2}(0.29)(7.9)^2 \\ &= 65.89 + 9.05 \\ &= \underline{74.9 \text{ m}} \end{aligned}$$

$\vec{d}_N = \vec{v}_{0N} t + \frac{1}{2} \vec{a}_N t^2$

$$\begin{aligned} &= (3.89)(7.9) + \frac{1}{2}(-0.83)(7.9)^2 \\ &= 30.73 + (-25.9) \\ &= \underline{4.83 \text{ m}} \end{aligned}$$

$\vec{d} = \sqrt{(\vec{d}_E)^2 + (\vec{d}_N)^2}$ $\phi = \tan^{-1}\left(\frac{4.83}{74.9}\right)$

$$\begin{aligned} &= \sqrt{(74.9)^2 + (4.83)^2} && = 3.7^\circ \\ &= 75.1 \text{ m} \end{aligned}$$

$\vec{d} = 75.1 \text{ m [E } 3.7^\circ \text{ N]}$

c) mass = 55 kg $\vec{F}_{\text{avg}} = ?$

$$\begin{aligned} \vec{F} &= m\vec{a} \\ &= (55 \text{ kg})(0.88 \text{ m/s}^2 \text{ [E } 71^\circ \text{ S]}) \\ &= \underline{48.4 \text{ N [E } 71^\circ \text{ S]}} \end{aligned}$$

(4) a)

$\vec{C} = \vec{A} + \vec{B}$
 $\vec{B} = \vec{C} - \vec{A}$
 (d)

Step 1:

Horizontal	Vertical
$d_{AE} = 75 \cos 67^\circ$ $= 29.3 \text{ km}$	$d_{AN} = 75 \sin 67^\circ$ $= 69.0 \text{ km}$
$d_{CE} = 93 \cos 26^\circ$ $= 83.6 \text{ km}$	$d_{CN} = -93 \sin 26^\circ$ $= -40.8 \text{ km}$

Step 2:

$\vec{d}_{BE} = \vec{d}_{CE} - \vec{d}_{AE}$ $= 83.6 - 29.3$ $= 54.3 \text{ m}$	$\vec{d}_{BN} = \vec{d}_{CN} - \vec{d}_{AN}$ $= -40.8 - 69.0$ $= -109.8$
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Step 3:

$d = \sqrt{(54.3)^2 + (-109.8)^2}$ $= 122.4 \text{ km}$	$\phi = \tan^{-1}\left(\frac{109.8}{54.3}\right)$ $= 64^\circ$
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$\vec{d} = 122.4 \text{ km [E } 64^\circ \text{ S]}$

b) $\vec{V}_{mn} = ?$ $\vec{V} = \frac{\Delta \vec{d}}{t} = \frac{122.4 \text{ km}}{0.5 \text{ h}} = 244.8 \text{ km/h [E } 64^\circ \text{ S]}$

(5)

Horizontal	Vertical
$\vec{V}_x = 5.0 \text{ m/s}$	$\vec{V}_{oy} = 0$
$\vec{d}_{ox} = 0 \text{ m}$	$\vec{V}_{fy} = ?$
$\vec{d}_{fx} = ?$	$\vec{d}_{oy} = 78.4 \text{ m}$
	$\vec{d}_{fy} = 0 \text{ m}$
	$\vec{a} = -9.81 \text{ m/s}^2$

a) $\vec{d}_{fy} = \vec{d}_{oy} + \vec{V}_{oy}t + \frac{1}{2}at^2$
 $0 = 78.4 + (0)(t) + \frac{1}{2}(-9.81)t^2$
 $0 = 78.4 + (-4.9)t^2$
 $-78.4 = -4.9t^2$
 $\frac{-78.4}{-4.9} = \frac{-4.9t^2}{-4.9}$
 $16 = t^2$
 $4 \text{ sec} = t$

b) $\vec{V}_x = \frac{\vec{d}_{fx} - \vec{d}_{ox}}{t}$ $5.0 \text{ m/s} = \frac{\vec{d}_{fx} - 0}{4 \text{ s}}$ $20.0 \text{ m} = \vec{d}_{fx}$	c) $\vec{V}_{fy} = \vec{V}_{oy} + \vec{a}t$ $\vec{V}_{fy} = 0 + (-9.81)(4)$ $\vec{V}_{fy} = -39.24 \text{ m/s}$
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$\vec{V}_f = \sqrt{(\vec{V}_x)^2 + (\vec{V}_{fy})^2}$ $\phi = \tan^{-1}\left(\frac{\vec{V}_{fy}}{\vec{V}_x}\right)$
 $= \sqrt{(5.0)^2 + (-39.24)^2}$ $\phi = \tan^{-1}\left(\frac{39.24}{5.0}\right)$
 $= 39.6 \text{ m/s}$ $\phi = 82.7^\circ$

⑥

Horizontal	Vertical
$\vec{V}_x = 24 \text{ m/s}$	$\vec{V}_{oy} = 0 \text{ m/s}$
$\vec{d}_{ox} = 0 \text{ m}$	$\vec{d}_{oy} = 15 \text{ m}$
$\vec{d}_{fx} = ?$	$\vec{d}_{fy} = 0 \text{ m}$

2) i) $\vec{d}_{fy} = \vec{d}_{oy} + \vec{V}_{oy}t + \frac{1}{2}\vec{a}t^2$
 $0 = 15 + (0)t + \frac{1}{2}(-9.81)t^2$
 $0 = 15 + (-4.9)t^2$
 $-15 = -4.9t^2$
 $\frac{-15}{-4.9} = \frac{-4.9t^2}{-4.9}$
 $3.061 = t^2$
 $\underline{1.75 \text{ s} = t}$

ii) $\vec{V}_x = \frac{\vec{d}_{fx} - \vec{d}_{ox}}{t}$
 $24 = \frac{\vec{d}_{fx} - 0}{1.75 \text{ s}}$
 $\underline{42.0 \text{ m} = \vec{d}_{fx}}$

b) $\vec{V}_{fy} = \vec{V}_{oy} + \vec{a}t$
 $= 0 + (-9.81)(1.75)$
 $= \underline{-17.17 \text{ m/s}}$

c) $\vec{V}_f = \sqrt{(\vec{V}_x)^2 + (\vec{V}_{fy})^2}$
 $= \sqrt{(24)^2 + (-17.17)^2}$
 $= 29.5 \text{ m/s}$
 $\phi = \tan^{-1}\left(\frac{\vec{V}_{fy}}{\vec{V}_x}\right)$
 $= \tan^{-1}\left(\frac{17.17}{24}\right)$
 $= 35.6^\circ$
 $29.5 \text{ m/s}, 35.6^\circ$
 to the horizontal
 (E 35.6°S)

⑦ $\vec{V} = 21 \text{ m/s} \quad \theta = 35^\circ$

Horizontal	Vertical
$\vec{V}_{ox} = 21 \cos 35^\circ$ $= 17.2 \text{ m/s}$	$\vec{V}_{oy} = 21 \sin 35^\circ$ $= 12.0 \text{ m/s}$
$\vec{d}_{ox} = 0 \text{ m}$	$\vec{d}_{oy} = 60.0 \text{ m}$
$\vec{d}_{fx} = ?$	$\vec{d}_{fy} = 0 \text{ m}$

$\vec{d}_{fy} = \vec{d}_{fo} + \vec{V}_{oy}t + \frac{1}{2}\vec{a}t^2$
 $-60 = 0 + (12.0)t - \frac{1}{2}(-9.81)t^2$
 $-60 = 12.0t - 4.9t^2$
 $4.9t^2 - 12.0t - 60 = 0$

$t = \frac{12 \pm \sqrt{(12)^2 - 4(4.9)(-60)}}{2(4.9)}$
 $= \frac{12 \pm \sqrt{1320}}{9.81}$
 $= \frac{12 \pm 36.33}{9.81}$
 $\underline{t = 4.93 \text{ s}} \quad \text{or } t = \cancel{2.48 \text{ s}}$

$\vec{V}_x = \frac{\vec{d}_{fx} - \vec{d}_{ox}}{t}$
 $17.2 = \frac{\vec{d}_{fx} - 0}{4.93}$
 $\underline{\vec{d}_{fx} = 84.8 \text{ m}}$

Horizontal	Vertical
$\vec{v}_{ox} = 15 \cos 42^\circ$ $= 11.15 \text{ m/s}$	$\vec{v}_{oy} = 15 \sin 42^\circ$ $= 10.04 \text{ m/s}$
$\vec{d}_{ox} = ?$	$\vec{d}_{oy} = 5.3 \text{ m}$
$\vec{d}_{fx} = ?$	$\vec{d}_{fy} = 0 \text{ m}$
$\vec{d}_{fy} = \vec{d}_{oy} + \vec{v}_{oy}t + \frac{1}{2}at^2$	
$-5.3 = 0 + (10.04)t + \frac{1}{2}(-9.81)(t^2)$	
$-5.3 = 10.04t - 4.9t^2$	
$4.9t^2 - 10.04t - 5.3 = 0$	
$t = \frac{10.04 \pm \sqrt{(10.04)^2 - 4(4.9)(-5.3)}}{2(4.9)}$	
$= \frac{10.04 \pm \sqrt{204.6816}}{9.81}$	
$t = 2.48 \text{ s} \quad \text{or} \quad t = -0.43 \text{ s}$	
$\vec{v}_{fy} = \vec{v}_{oy} + at$ $= (10.04) + (-9.81)(2.48)$ $= -14.3 \text{ m/s}$	$\vec{v}_f = \sqrt{(\vec{v}_{fx})^2 + (\vec{v}_{fy})^2}$ $\vec{v}_f = \sqrt{(11.15)^2 + (-14.3)^2}$ $\vec{v}_f = 18.1 \text{ m/s}$
$\theta = \tan^{-1}\left(\frac{\vec{v}_{fy}}{\vec{v}_{fx}}\right)$ $\theta = \tan^{-1}\left(\frac{-14.3}{11.15}\right)$ $\theta = 52^\circ$	$\vec{v}_f = 18.1 \text{ m/s}$ at 52° below the horizontal.

Horizontal	Vertical
$\vec{d}_{ox} = 0 \text{ m}$	$\vec{d}_{oy} = 0 \text{ m}$ } lands on
$\vec{d}_{fx} = 75 \text{ cm} = 0.75 \text{ m}$	$\vec{d}_{fy} = 0 \text{ m}$ } the ground
$\vec{v}_x = \vec{v} \cos 55^\circ$	$\vec{v}_{oy} = \vec{v} \sin 55^\circ$
① $\vec{v}_x = \frac{\vec{d}_{fx} - \vec{d}_{ox}}{t}$	Rearrange for t: $t = \frac{0.75}{\vec{v} \cos 55^\circ}$
$\vec{v}_x = \frac{0.75 - 0}{t}$	
$\therefore \vec{v} \cos 55^\circ = \frac{0.75}{t}$	
② $\vec{d}_{fy} = \vec{d}_{oy} + \vec{v}_{oy}t + \frac{1}{2}at^2$	
$0 = 0 + (\vec{v} \sin 55^\circ)t + \frac{1}{2}(-9.81)t^2$	
$0 = (\vec{v} \sin 55^\circ)\left(\frac{0.75}{\vec{v} \cos 55^\circ}\right) + \frac{1}{2}(-9.81)\left(\frac{0.75}{\vec{v} \cos 55^\circ}\right)^2$	
$0 = 0.75 \tan 55^\circ - 4.9 \left[\frac{0.75^2}{\vec{v}^2 \cos^2 55^\circ} \right]$	
$0 = 1.0711 - 4.9 \left[\frac{0.5625}{\vec{v}^2 (0.3276)} \right]$	
$-1.0711 = -4.9 \left(\frac{1.7097}{\vec{v}^2} \right)$	
$\frac{-1.0711 \vec{v}^2}{-1.0711} = \frac{-8.37753}{-1.0711}$	
$\vec{v}^2 = 7.8214$	
$\vec{v} = 2.796$	
$\vec{v} = 2.8 \text{ m/s}$	

$\vec{V}_2' = ?$ $m_1 = 0.155 \text{ kg}$ $m_2 = 0.125 \text{ kg}$
 $\vec{V}_1 = 2.5 \text{ m/s}$ $\vec{V}_1' = 1.4 \text{ m/s}$ $\vec{V}_2 = 0 \text{ m/s}$
 $\vec{V}_2' = ?$

$\vec{V}_1 = 2.5 \text{ m/s}$ at 38° $\vec{V}_1' = 1.4 \text{ m/s}$

x-direction

$$m_1 \vec{V}_{1x} + m_2 \vec{V}_{2x} = m_1 \vec{V}'_{1x} + m_2 \vec{V}'_{2x}$$

$$\vec{V}'_{2x} = \frac{m_1 \vec{V}_{1x} + m_2 \vec{V}_{2x} - m_1 \vec{V}'_{1x}}{m_2}$$

$$\vec{V}'_{2x} = \frac{(0.155)(2.5) + (0) - (0.155)(1.4 \cos 38^\circ)}{0.125}$$

$$\vec{V}'_{2x} = -1.73 \text{ m/s}$$

y-direction

$$m_1 \vec{V}_{1y} + m_2 \vec{V}_{2y} = m_1 \vec{V}'_{1y} + m_2 \vec{V}'_{2y}$$

$$\vec{V}'_{2y} = \frac{m_1 \vec{V}_{1y} + m_2 \vec{V}_{2y} - m_1 \vec{V}'_{1y}}{m_2}$$

$$\vec{V}'_{2y} = \frac{(0) + (0) - (0.155)(-1.4 \sin 38^\circ)}{0.125}$$

$$\vec{V}'_{2y} = 1.07 \text{ m/s}$$

$\vec{V}_2' = \sqrt{(\vec{V}'_{2x})^2 + (\vec{V}'_{2y})^2}$ $\phi = \tan^{-1} \left(\frac{\vec{V}'_{2y}}{\vec{V}'_{2x}} \right)$
 $\vec{V}_2' = \sqrt{(1.73)^2 + (1.07)^2}$ $\phi = \tan^{-1} \left(\frac{1.07}{1.73} \right)$
 $\vec{V}_2' = 2.03 \text{ m/s}$ $\phi = 31.7^\circ$

$\vec{V}_2' = 2.0 \text{ m/s}, 32^\circ \text{ up from the positive } x\text{-axis}$

(11) $m_{\text{total}} = 3.5 \text{ kg}$ $\vec{V}_3' = ?$

$m_1 = 1.3 \text{ kg}$ $m_2 = 1.2 \text{ kg}$ $m_3 = 3.5 - 1.3 - 1.2 = 1.0 \text{ kg}$
 $\vec{V}_1 = 1.8 \text{ m/s}$ $\vec{V}_2 = 2.5 \text{ m/s}$ $\vec{V}_3 = ?$
 $\alpha = 52^\circ \text{ ccw from } +x\text{-axis}$ $\alpha = 61^\circ \text{ cw from } -x\text{-axis}$

x-direction

$$m_1 \vec{V}'_{1x} + m_2 \vec{V}'_{2x} + m_3 \vec{V}'_{3x} = 0$$

$$\vec{V}'_{3x} = \frac{-m_1 \vec{V}'_{1x} - m_2 \vec{V}'_{2x}}{m_3}$$

$$\vec{V}'_{3x} = \frac{-(1.3)(1.8 \cos 52^\circ) - (1.2)(-2.5 \cos 61^\circ)}{1.0}$$

$$\vec{V}'_{3x} = 0.0138$$

y-direction

$$m_1 \vec{V}'_{1y} + m_2 \vec{V}'_{2y} + m_3 \vec{V}'_{3y} = 0$$

$$\vec{V}'_{3y} = \frac{-m_1 \vec{V}'_{1y} - m_2 \vec{V}'_{2y}}{m_3}$$

$$\vec{V}'_{3y} = \frac{-(1.3)(1.8 \sin 52^\circ) - (1.2)(2.5 \sin 61^\circ)}{1.0}$$

$$\vec{V}'_{3y} = -4.47$$

$\vec{V}_3 = \sqrt{(\vec{V}'_{3x})^2 + (\vec{V}'_{3y})^2}$ $\phi = \tan^{-1} \left(\frac{\vec{V}'_{3y}}{\vec{V}'_{3x}} \right)$
 $\vec{V}_3 = \sqrt{(0.0138)^2 + (4.47)^2}$ $\phi = \tan^{-1} \left(\frac{4.47}{0.0138} \right)$
 $\vec{V}_3 = 4.47$ $\phi = 89.8^\circ$

$\vec{V}_3 = 4.5 \text{ m/s}, 89.8^\circ \text{ down from the } +x\text{-axis}$

(12) $m = 75.0 \text{ kg}$ $F_a = 535 \text{ N}$
 $\vec{F}_{\text{net } x} = 15.0 \text{ N}$ $\theta = 28.0^\circ$ to the horizontal

$\vec{F}_{ax} = 535 \cos 28^\circ$
 $= 472 \text{ N}$

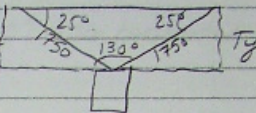
$\vec{F}_{ay} = -535 \sin 28^\circ$
 $= -251 \text{ N}$

$\vec{F}_g = mg$
 $= (75.0)(9.81)$
 $= -736 \text{ N}$

x-direction:
 $\vec{F}_{\text{net } x} = \sum \text{ forces in x-direction}$
 $\vec{F}_{\text{net } x} = \vec{F}_{ax} + \vec{F}_f$
 $15.0 = 472 + \vec{F}_f$
 $\vec{F}_f = -457 \text{ N}$
 $|\vec{F}_f| = 457 \text{ N}$

y-direction:
 $\vec{F}_{\text{net } y} = \sum \text{ forces in y-direction}$
 $\vec{F}_{\text{net } y} = \vec{F}_{ay} + \vec{F}_f + \vec{F}_g$
 $\vec{F}_f = \vec{F}_{\text{net } y} - \vec{F}_{ay} - \vec{F}_g$
 $\vec{F}_f = 0 - (-251) - (-736)$
 $\vec{F}_f = 987 \text{ N}$

$\mu = \frac{|\vec{F}_f|}{\vec{F}_n} = \frac{457 \text{ N}}{987 \text{ N}} = \boxed{0.46}$

(13) Largest mass = ? 

$\sin \theta = \frac{T_y}{T}$

$T_y = T \sin \theta$
 $T_y = 1750 \sin 25^\circ$
 $\boxed{T_y = 740 \text{ N}}$ (max weight can be supported)

$F_{\text{net } y} = \sum \text{ forces in y-direction}$
 $0 = F_g + 2T_y$
 $0 = F_g + 2(740)$
 $-1480 = F_g$

$F_g = mg$
 $-1480 = m(-9.81)$
 $\frac{-1480}{-9.81} = \frac{-9.81}{-9.81}$
 $\boxed{151 \text{ kg} = m}$

14) $m = 33 \text{ kg}$ $\theta = 35^\circ$ $\mu = 0.13$

a) $\vec{a} = 0.75 \text{ m/s}^2$ $\vec{F}_a = P$

$F_{net\ x} = \sum \text{forces in } x\text{-dir.}$
 $F_{net\ x} = F_a + F_{gx} + F_f$

i) $\vec{F}_{net\ x} = m \vec{a}_x$
 $= (33)(0.75)$
 $= \underline{24.8 \text{ N}}$

ii) $F_f = \mu F_N$ $F_N = F_{gy}$
 $= (0.13)(265 \text{ N})$ $= mg \cos \theta$
 $= \underline{34.5 \text{ N}}$ $= (33)(9.81) \cos 35^\circ$
 $= 265 \text{ N}$

iii) $F_{gx} = F_g \sin \theta$
 $= mg \sin \theta$
 $= (33)(9.81) \sin 35^\circ$
 $= \underline{186 \text{ N}}$

$F_{net\ x} = F_a + F_{gx} + F_f$
 $24.8 = F_a + 186 \text{ N} - 34.5$
 $\underline{24.8 = F_a + 151.5}$

15) $m = 25 \text{ kg}$ $\theta = 33^\circ$ $\mu = 0.38$

$F_{gy} = -mg \cos 33^\circ$
 $= -(25)(9.81) \cos 33^\circ$
 $= -205.68$

$F_y = ma_y$ $*a_y = 0$
 $F_{gy} + F_N = ma_y$
 $-205.68 + F_N = 0$
 $F_N = \underline{205.68 \text{ N}}$

$F_x = ma_x$ $F_{gx} = mg \sin 33^\circ$
 $F_{gx} + F_f = ma_x$ $= (25)(9.81) \sin 33^\circ$
 $133.57 - 78.158 = 25 \cdot \vec{a}_x$ $= 133.57$
 $55.412 = 25 \cdot \vec{a}_x$ $F_f = -\mu F_N$
 $2.2 \text{ m/s}^2 = \vec{a}$ $= -(0.38)(205.68)$
 $= \underline{78.158 \text{ N}}$

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$\phi = 20^\circ$ $\mu = 0.19$
 $m_2 = 25 \text{ kg}$ $\vec{a} = ?$
 $m_1 = 16 \text{ kg}$

$-F_{g2}$ $+ |F_N| = |F_{gy}|$

$\sum \text{Forces} = \sum_i m_i \vec{a}$
 $-F_{g2} + F_{gy} + F_f = (m_1 + m_2) \cdot \vec{a}$
 $-m_2 g + m_1 g \sin \phi + \mu F_N = (m_1 + m_2) \cdot \vec{a}$
 $-m_2 g + m_1 g \sin \phi + \mu m_1 g \cos \phi = (m_1 + m_2) \vec{a}$
 $-(25)(9.81) + (16)(9.81) \sin 20^\circ + (0.19)(16)(9.81) \cos 20^\circ = (16+25) \vec{a}$
 $-245.25 + 53.68 + 28.02 = 41 \vec{a}$
 $\frac{-163.55}{41} = \frac{41 \vec{a}}{41}$
 $\boxed{-4.0 \text{ m/s}^2 = \vec{a}}$

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A uniform 150 kg beam, 10.0 m long, supports a 275 kg box that is 2.5 m from the right support column. Calculate the magnitude of the forces on the beam exerted by each of the vertical support columns.

$F_1 = ?$ $T_1 = 0$
 $F_4 = ?$ $T_4 = 0$

5.0 m 2.5 m 2.5 m

F_2 F_3
 T_2 T_3

i) $\sum \tau_{\text{net}} = T_1 + T_2 + T_3 + T_4$
 $0 = 0 - r_2 F_2 - r_3 F_3 + r_4 F_4$
 $0 = -(5)(150)(9.81) - (7.5)(275)(9.81) + (10)F_4$
 $0 = -7358 - 20233 + 10 F_4$
 $\frac{27591}{10} = \frac{10 F_4}{10}$
 $\boxed{2759 \text{ N} = F_4}$

ii) Move pivot point to F_4 , and solve for F_1 :

$\sum \tau_{\text{net}} = T_1 + T_2 + T_3 + T_4$
 $0 = -10 F_1 + (5)(150)(9.81) + (2.5)(275)(9.81) + 0$
 $0 = -10 F_1 + 7358 + 6744$
 $\frac{-14102}{10} = \frac{-10 F_1}{10}$
 $\boxed{1410 \text{ N} = F_1}$

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$\tau_{\text{net}} = \tau_1 + \tau_2 + \tau_3 + \tau_4$
 $0 = 0 - (7.5)(60)(9.81)\cos 30^\circ - (10)(m)(9.81)\cos 30^\circ + (15)(1800)\sin 60^\circ$
 $0 = -3823 - 85m + 23383$
 $-19560 = -85m$
 $230 \text{ kg} = m$

max T in cable = 1800 N
 beam = 15 m $\beta = 60^\circ$
 $m_{\text{beam}} = 60 \text{ kg}$ $\alpha = 30^\circ$
 $m_{\text{hanging mass}} = ?$, 10 m from hinge

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$\sum \text{torques} = \tau_2 + \tau_3 + \tau_4$
 $\tau_2 = -r_2 F_2 \cos \beta$
 $= -(1.5)(75)(9.81)\cos 55^\circ$
 $= -633 \text{ Nm}$
 $\tau_3 = -r_3 F_3 \cos \beta$
 $= -(2.5)(25)(9.81)\cos 55^\circ$
 $= -351.7 \text{ Nm}$
 $\tau_4 = +r_4 F_4 \sin \beta$
 $= (5)F_4 \sin 55^\circ$
 $= 4.1 F_4$

$\tau_{\text{net}} = \tau_2 + \tau_3 + \tau_4$
 $0 = -633 - 351.7 + 4.1 F_4$
 $984.7 = 4.1 F_4$
 $240 \text{ N} = F_4$

$F_{\text{net}x} = F_f + F_4$
 $0 = F_f + (-240)$
 $240 \text{ N} = F_f$

$F_{\text{net}y} = F_N + F_2 + F_3$
 $0 = F_N - (75)(9.81) - (25)(9.81)$
 $0 = F_N - 735.75 - 245.25$
 $0 = F_N - 981$
 $981 \text{ N} = F_N$

$F_f = \mu F_N$
 $\mu = \frac{F_f}{F_N}$
 $\mu = \frac{240 \text{ N}}{981 \text{ N}}$