Chapter Review For Final Exam:

Ch.1 → (Inverse Functions)

Ch.2 → (Radical Functions)

Ch.7 → (Exponential Functions) $y = 2^x$

Ch.8 → (Logarithmic Functions) $y = \log_2 x$

Ch.4 → (Trig + Unit Circle)

Ch.5 → (Trig Functions)

Ch.6 → (Trig Identities)

Ch.2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

Ch.7

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

Ch.8

$y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Ch.5

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

Chapter 2 Radical Functions

$$\sqrt{x+8} - 6 = x$$

Solve for x:

$$\left(\sqrt{x+8}\right)^2 = (x+6)^2$$

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2+12x+36$$

$$0 = x^2+11x+28$$

$\underline{7} \times \underline{4} = 28$

$\underline{7} + \underline{4} = 11$

$$0 = (x+7)(x+4)$$

$x+7=0$	$x+4=0$
$x=-7$	$x=-4$

Test $x=-4$ is a solution

$\sqrt{x+8} = x+6$

$\sqrt{-4+8}$	$-4+6$
2	2 ✓

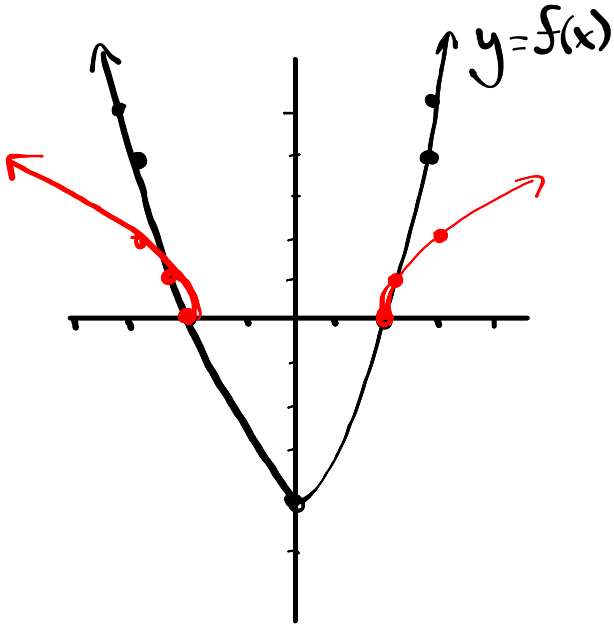
Test $x=-7$ is extraneous

$\sqrt{x+8} = x+6$

$\sqrt{-7+8}$	$-7+6$
1	-1 ✗

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. State the domain and range of each.



$y = f(x)$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -4, y \in \mathbb{R}\}$ or $[-4, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in \mathbb{R}\}$

$(-\infty, -2]$ and $[2, \infty)$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2}+7=71$

(b) $9^{2x+1}=81(27^{x})$

a) $2^{2x+2}+7=71$

$2^{2x+2}=64$

$\frac{\log 64}{\log 2}=6$

$\cancel{2}^{2x+2}=\cancel{2}^{6}$

$2x+2=6$

$\frac{2x}{2}=\frac{4}{2}$

$x=2$

b) $9^{2x+1}=81(27^{x})$

$\frac{\log 9}{\log 3}=2$ $\quad \frac{\log 81}{\log 3}=4$ $\quad \frac{\log 27}{\log 3}=3$

$(3^{2})^{2x+1}=3^{4}(3^{3})^{x}$

$3^{4x+2}=3^{4}\cdot 3^{3x}$

$\cancel{3}^{4x+2}=\cancel{3}^{3x+4}$

$4x+2=3x+4$

$x=2$

Ex: $y=3(2)^{2x+8}-1$

$y=3(2)^{2(x+4)}-1$

a=3 → vertical stretch by a factor of 3
no vertical reflection

b=2 → horizontal compression by a factor of ½
no horizontal reflection

h=-4 → translate left 4 units

K=-1 → " down 1 unit

$(x,y)\rightarrow\left(\frac{1}{2}x-4,\ 3y-1\right)$

$y=2^{x}$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

$y=3(2)^{2(x+4)}-1$

	x	y
	-5	-1/4
(-4.5)	-9/2	1/2
	-4	2
(-3.5)	-7/2	5
	-3	11

$3\left(\frac{1}{4}\right)-1$

$\frac{3}{4}-\frac{4}{4}=\frac{-1}{4}$

$3\left(\frac{1}{2}\right)-1$

$\frac{3}{2}-\frac{2}{2}=\frac{1}{2}$

Chapter 7 → Exponential

③ $27^{2x+1} = 9^{x-2}\,(243)^{x}$

$(3^3)^{2x+1} = (3^2)^{x-2}\,(3^5)^{x}$

$3^{6x+3} = 3^{2x-4} \cdot 3^{5x}$

$3^{6x+3} = 3^{2x-4+5x}$

$\cancel{3}^{6x+3} = \cancel{3}^{7x-4}$

$6x+3 = 7x-4$

$3+4 = 7x-6x$

$\boxed{7 = x}$

Test your answer

Ch. 8 → logarithms

4. Rewrite each expression as a single logarithm.

$$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$$

$$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5\left(\frac{x^3(x-1)^{1/2}}{x^2+1}\right)$$

$$\boxed{\log_5 \frac{x^3\sqrt{x-1}}{x^2+1}}$$

$$\log_2\left(\frac{x^2}{y^3\sqrt[4]{z}}\right)$$

$$\log_2 x^2 - \log_2 y^3 - \log_2 z^{1/4}$$

$$\boxed{2\log_2 x - 3\log_2 y - \frac{1}{4}\log_2 z}$$

Chapter 8 → Logarithmic Functions

③ Solve for x:

$\log_6(x-1)+\log_6(x+4)=2$

$\log_6(x-1)(x+4)=2$

$\log_6(x^2+4x-x-4)=2$

$\log_6(x^2+3x-4)=2$ (log form)

$6^2=x^2+3x-4$ (exp form)

$36=x^2+3x-4$

$0=x^2+3x-40$

$8+-5=3$
$8\times-5=-40$

$0=(x+8)(x-5)$

$x+8=0$ | $x-5=0$
$x=-8$ | $\boxed{x=5}$
extraneous

if $x=5$

$\log_6(x-1)+\log_6(x+4)=2$

$\log_6(4)+\log_6(9)$

$\log_6 36$

2

if $x=-8$

$\log_6(x-1)+\log_6(x+4)=2$

$\log_6(-9)+\log_6(-4)$

not possible

Chapter 8 → Log. functions

④ Express as a single logarithm

$$5\log_2 x - \frac{3}{4}\left[4\log_2 x^3 - 12\log_2 x^2\right]$$

$$5\log_2 x - 3\log_2 x^3 + 9\log_2 x^2$$

$$\log_2 x^5 - \log_2 x^9 + \log_2 x^{18}$$

$$\log_2\left(\frac{x^5 \cdot x^{18}}{x^9}\right)$$

$$\log_2\left(\frac{x^{23}}{x^9}\right)$$

$$\log_2 x^{14} \text{ or } 14\log_2 x$$

$\frac{-3}{4} \cdot 4$	$\frac{-3}{4} \cdot -12$
$\frac{-12}{4}$	$\frac{36}{4}$
-3	9

Ch. 8

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$\log_{10}(x+2)(x-1) = 1$

$\log_{10}(x^2-x+2x-2) = 1$

$\log_{10}(x^2+x-2) = 1$ (log)

$10^1 = x^2+x-2$ (exp)

$10 = x^2+x-2$

$0 = x^2+x-12$ $\quad$ $-3 \times 4 = -12$, $-3+4 = 1$ $\quad$ 12: 1×12, 2×6, 3×4

$0 = (x-3)(x+4)$

$x-3=0$	$x+4=0$
$x=3$	$x=-4$ is extraneous

test $x=3$

$\log_{10}(x+2)+\log_{10}(x-1) = 1$ ✓

$\log_{10}5+\log_{10}2$

$\log_{10}10$

1 ✓

test $x=-4$

$\log_{10}(x+2)+\log_{10}(x-1) = 1$

$\log_{10}(-2) + \log_{10}(-5)$

not possible

Ch.7 or Ch.8

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

Base = 1/2 = 0.5 exp = $t/5.3$ Initial Amount = 60

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$M(t) = 60(0.5)^{t/5.3}$ | $y = 60(0.5)^{t/5.3}$

b) What amount will be present in 10.6 years? [2] t = 10.6

$m(t) = 60(0.5)^{10.6/5.3}$

$m(t) = 60(0.5)^2$

$m(t) = 60(0.25) = 15 \text{ mg}$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

① 12.5% of 60mg

0.125 × 60

7.5 mg

$M(t) = 60(0.5)^{t/5.3}$

$\frac{7.5}{60} = \frac{\cancel{60}(0.5)^{t/5.3}}{\cancel{60}}$

$0.125 = (0.5)^{t/5.3}$

$\frac{\log 0.125}{\log 0.5} = 3$

$(\cancel{0.5})^3 = (\cancel{0.5})^{t/5.3}$

$5.3 \cdot 3 = \frac{t}{5.3} \cdot 5.3$

$\boxed{15.9 \text{ years} = t}$

Ch. 4 → Trig Equation

2. Solve for all values of θ in the specified domain.

$\tan^2\theta + \tan\theta = 0$, $0 \le \theta \le 2\pi$ (Radians)

S | A
T | C

$\tan\theta(\tan\theta + 1) = 0$

$\tan\theta = 0$ (Unit Circle)

$\boxed{\theta = 0, \pi, 2\pi}$

$\tan\theta + 1 = 0$

$\tan\theta = -1$ (Special Triangle)

$\bar{\theta} = \tan^{-1}(1)$

$\bar{\theta} = \frac{\pi}{4}$

where is $\tan\theta < 0$

Q2	Q4
$\theta = \pi - \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4}$
$\boxed{\theta = \frac{3\pi}{4}}$	$\boxed{\theta = \frac{7\pi}{4}}$

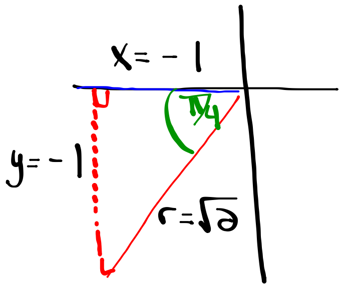
e. $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta < 360°$

Ch.4 → Trig Expression

* $$\frac{5\tan^2 5\pi/4}{6\sin 5\pi/6 + 4\sin 4\pi/3}$$

$\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$

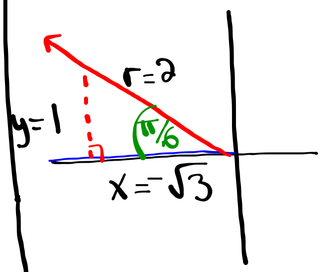
1π



$\tan\frac{5\pi}{4} = \frac{-1}{-1} = 1$

$\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

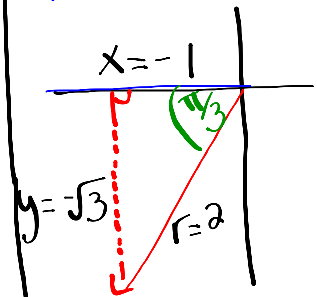
1π



$\sin\frac{5\pi}{6} = \frac{1}{2}$

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

1π



$\sin\frac{4\pi}{3} = \frac{-\sqrt{3}}{2}$

$$\frac{5\tan^2 5\pi/4}{6\sin 5\pi/6 + 4\sin 4\pi/3}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right) + 4\left(\frac{-\sqrt{3}}{2}\right)}$$

$\frac{-4\sqrt{3}}{2} = -2\sqrt{3}$

$$\frac{5}{(3 - 2\sqrt{3})}\frac{(3+2\sqrt{3})}{(3+2\sqrt{3})}$$

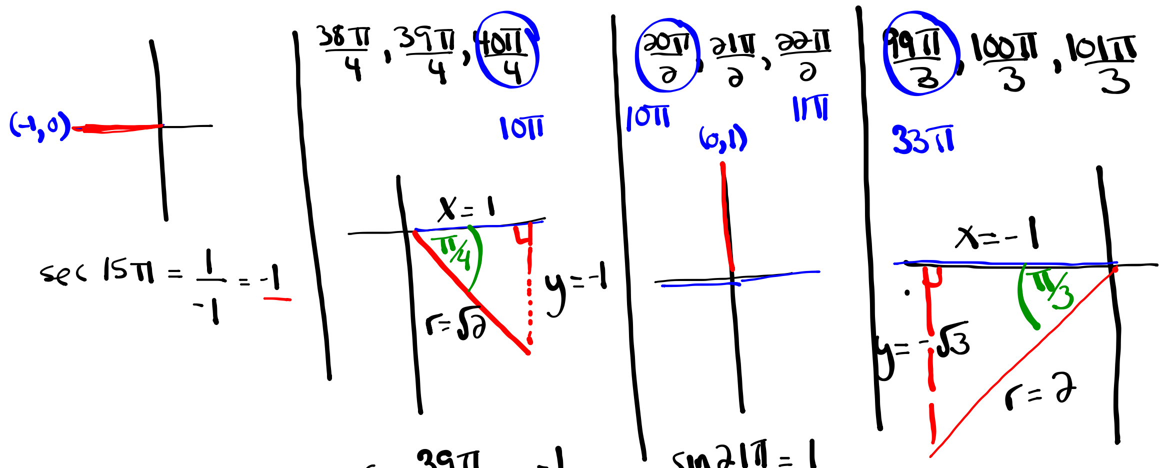
$(-2\sqrt{3})(2\sqrt{3}) = -4\sqrt{9}$

$$\frac{15 + 10\sqrt{3}}{9 + \cancel{6\sqrt{3}} - \cancel{6\sqrt{3}} - 4(3)}$$

$$\frac{15 + 10\sqrt{3}}{-3} \quad \text{or} \quad \frac{-15 - 10\sqrt{3}}{3}$$

Ch. 4

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$



$\sec 15\pi = \frac{1}{-1} = -1$

$\sin\frac{39\pi}{4} = \frac{-1}{\sqrt{2}}$

$\sin\frac{21\pi}{2} = 1$

$\csc\frac{100\pi}{3} = \frac{2}{-\sqrt{3}}$

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

$$(-1) + \cancel{\sqrt{2}}\left(-\frac{1}{\cancel{\sqrt{2}}}\right)(1) - \left(-\frac{2}{\sqrt{3}}\right)^2$$

$$-1 - 1 - \frac{4}{3}$$

$$-2 - \frac{4}{3}$$

$$-\frac{6}{3} - \frac{4}{3}$$

$$-\frac{10}{3}$$

Ch. 5 → Trig Functions

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

max to min = half the P

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

max = 50
min = 30

$k = \frac{50+30}{2} = 40$

Amp = max − k
Amp = 50 − 40
Amp = 10
$a = \pm 10$

$P = 2(1.8 - 0.4)$
$P = 2(1.4)$
$P = 2.8$

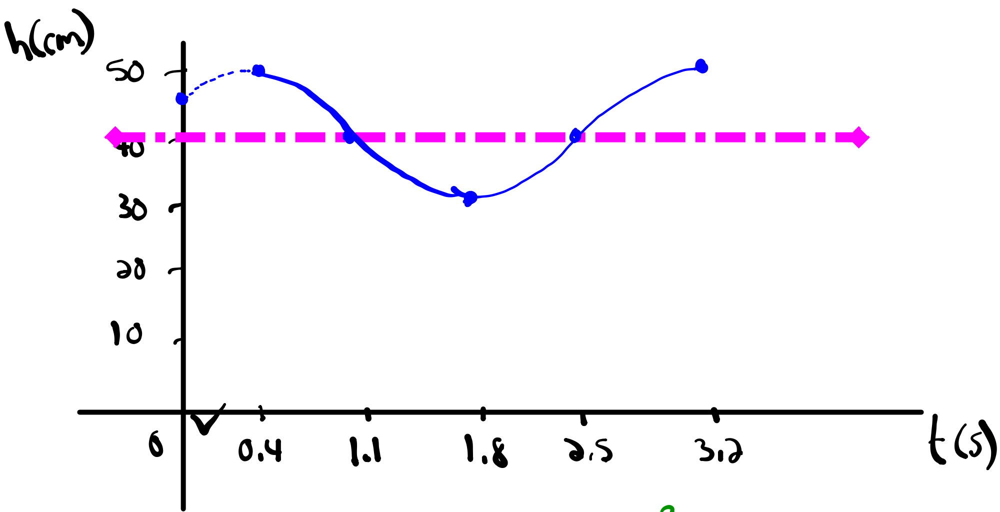
$b = \frac{360}{2.8} = 128.57$

$h = 0.4$

$$y = 10\cos[128.57(17.2 - 0.4)] + 40 = \boxed{50\text{cm}}$$

(b) How high was the weight above the floor when the stopwatch was initially started? (t = 0)

$$y = 10\cos[128.57(0 - 0.4)] + 40 = \boxed{46.2\text{ cm}}$$



Count by $\frac{P}{4} = \frac{2.8}{4} = 0.7$

$$\frac{1}{\sec^2\theta\cot\theta} = \frac{\sin\theta - \sin^3\theta}{\cos\theta}$$

$\boxed{\frac{1}{\sec^2\theta}} \cdot \boxed{\frac{1}{\cot\theta}}$

$\cos^2\theta \quad \tan\theta$

$\cos^{\cancel{2}}\theta \quad \frac{\sin\theta}{\cancel{\cos\theta}}$

$\boxed{\sin\theta\cos\theta}$

$\frac{\sin\theta\boxed{(1-\sin^2\theta)}}{\cos\theta}$

$\frac{\sin\theta(\cos^{\cancel{2}}\theta)}{\cancel{\cos\theta}}$

$\boxed{\sin\theta\cos\theta}$

Extra questions worked out

3 b) $(32)^{-x+1} = \sqrt{256}\left(\frac{1}{8}\right)^{2x}$

$(2^5)^{-x+1} = 16\,(2^{-3})^{2x}$

$2^{-5x+5} = 2^4(2^{-6x})$

$\cancel{2}^{-5x+5} = \cancel{2}^{4-6x}$

$-5x+5 = 4-6x$

$\boxed{x=-1}$

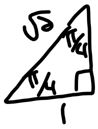
12 a) $\sin\theta = \sin\theta\tan\theta$ $\quad 0 \le \theta \le 2\pi$

Common Factor

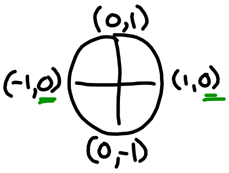
$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$	$\tan\theta - 1 = 0$
$\theta = 0, \pi, 2\pi$	$\tan\theta = 1$
	$\theta_R = \frac{\pi}{4}$



S | A
T | C



Where is $\tan\theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

13 b) $3\sin^2\theta - 2\sin\theta - 1 = 0$ $\quad , 0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$ $\quad \frac{-3}{} \times \frac{1}{} = -3$, $\quad \frac{-3}{} + \frac{1}{} = -2$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$3\sin\theta + 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = \frac{-1}{3}$	$\sin\theta = 1$
$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$	$\theta = 90°$
$\theta_R = 19$	

Unit Circle

Where is sine negative:

S | A
T | C

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

① Amp = 11   P = 16   min = -4

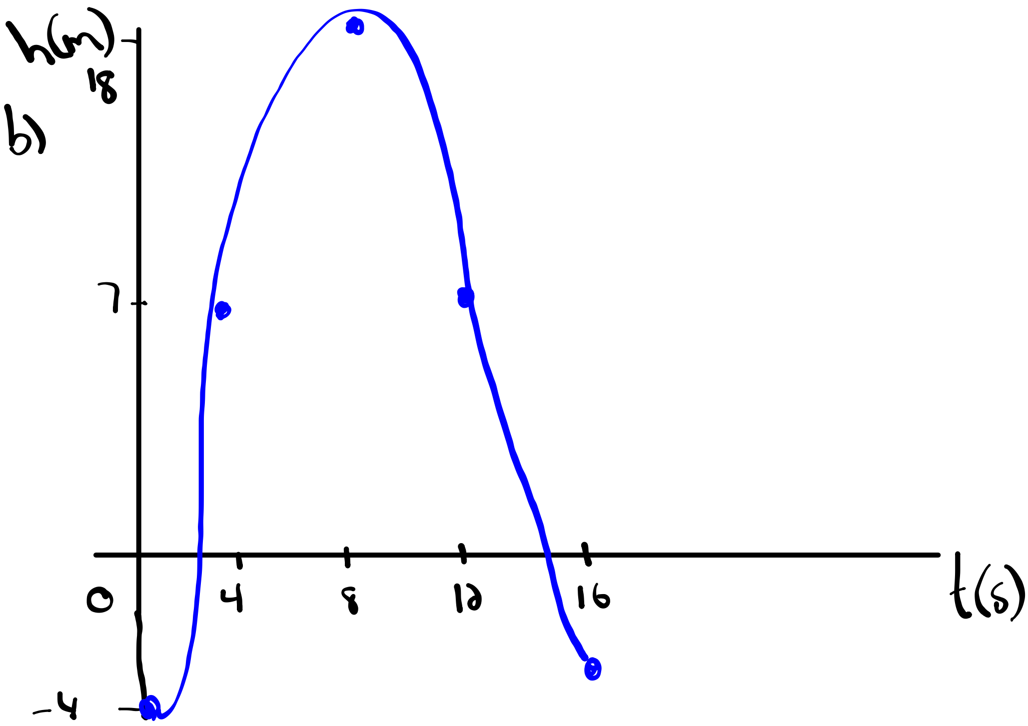
$a = \pm 11$ $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

$k = -4 + 11 = 7$

$h = 0$

a) equation: $y = -11\cos[22.5(x)] + 7$

b)



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(4)

max = 68

min = 24

$K = \frac{68+24}{2} = 46$

Amp = 68-46 = 22

a = ± 22

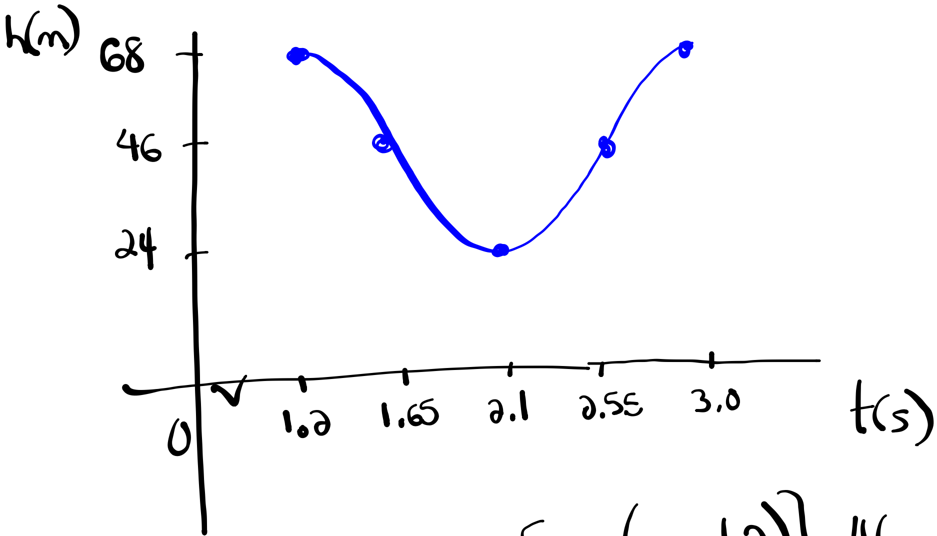
P = 2(2.1-1.2)

P = 1.8

$b = \frac{360}{1.8} = 200$

h = 1.2

$\frac{P}{4} = \frac{1.8}{4} = 0.45$



$y = 22\cos[200(x-1.2)]+46$

⑤ c) $y = \frac{1}{2}\cos(1\theta + \pi) - 4$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

$(x, y) \rightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$\longrightarrow$

x	y	
$-\pi$	$-7/2$	-3.5
$-\frac{\pi}{2}$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5