

Radical Functions and Transformations

Focus on...

- investigating the function $y = \sqrt{x}$ using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$ and $y = 4\sqrt[3]{5+x}$ are radical functions.

Example 1

Graph Radical Functions Using Tables of Values

Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

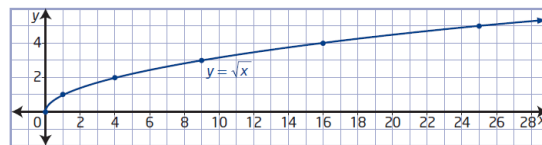
- a) $y = \sqrt{x}$ b) $y = \sqrt{x-2}$ c) $y = \sqrt{x} - 3$

- a) For the function $y = \sqrt{x}$, the radicand x must be greater than or equal to zero, $x \geq 0$.

D: $x \geq 0$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

How can you choose values of x that allow you to complete the table without using a calculator?



The graph has an endpoint at $(0, 0)$ and continues up and to the right. The domain is $\{x \mid x \geq 0, x \in \mathbb{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathbb{R}\}$. $[0, \infty)$

- b) For the function $y = \sqrt{x-2}$, the value of the radicand must be greater than or equal to zero.

D: $x-2 \geq 0$
 $x \geq 2$

$h=2$

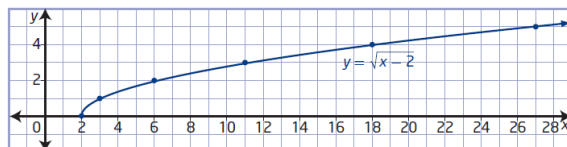
$(x,y) \rightarrow (x+2,y)$

x	y
2	0
3	1
6	2
11	3
18	4
27	5

How is this table related to the table for $y = \sqrt{x}$ in part a)?

translated 2 units right

How does the graph of $y = \sqrt{x-2}$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 2, x \in \mathbb{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathbb{R}\}$. $[2, \infty)$ $[0, \infty)$

- c) The radicand of $y = \sqrt{x} - 3$ must be non-negative.

$k=-3$

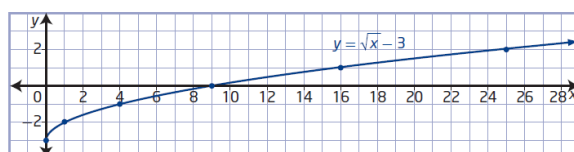
D: $x \geq 0$

$(x,y) \rightarrow (x,y-3)$

x	y
0	-3
1	-2
4	-1
9	0
16	1
25	2

translated 3 units down

How does the graph of $y = \sqrt{x} - 3$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 0, x \in \mathbb{R}\}$ and the range is $\{y \mid y \geq -3, y \in \mathbb{R}\}$.

$[0, \infty)$

$[-3, \infty)$

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a , b , h , and k . The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x -axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y -axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

$$y = a\sqrt{b(x-h)} + k$$

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Example 2

Graph Radical Functions Using Transformations

Sketch the graph of each function using transformations. Compare the domain and range to those of $y = \sqrt{x}$ and identify any changes.

a) $y = 3\sqrt{-(x - 1)}$

b) $y - 3 = -\sqrt{2x}$

a) $y = \underline{3}\sqrt{\underline{-}(x-1)}$ $y = a\sqrt{b(x-h)} + k$

$a=3 \rightarrow$ vertical stretch by a factor of 3.

$b=-1 \rightarrow$ no horizontal stretch but there is a horizontal reflection in the y-axis

$h=1 \rightarrow$ translate 1 unit to the right.

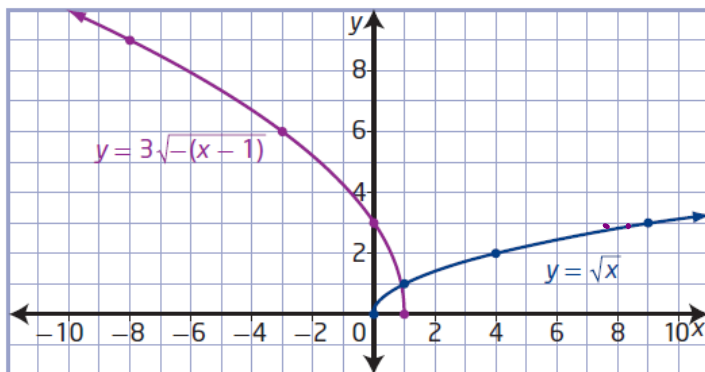
$k=0 \rightarrow$ no vertical translation

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \rightarrow \left(\frac{1}{-1}x+1, 3y+0 \right)$

x	y
1	0
0	3
-3	6
-8	9
-15	12
-24	15



D: $-(x-1) \geq 0$

$-x+1 \geq 0$

$-x \geq -1$

$x \leq 1$

$\{x | x \leq 1, x \in \mathbb{R}\}$

$(-\infty, 1]$

R: $\{y | y \geq 0, y \in \mathbb{R}\}$

$[0, \infty)$

b) $y - 3 = -\sqrt{2x}$

$y = -\sqrt{2x} + 3$

$a = -1 \rightarrow$ no vertical stretch but vertical reflection in x-axis

$b = 2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}$.

$h = 0 \rightarrow$ no horizontal translation

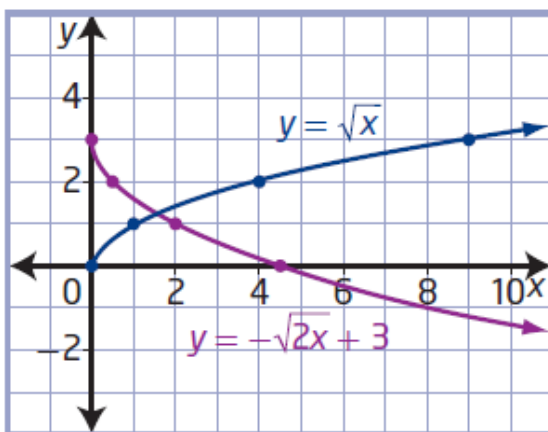
$k = 3 \rightarrow$ translated 3 units up.

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x, y) \rightarrow (\frac{1}{2}x + 0, -|y + 3|)$

x	y
0	3
0.5	2
2	1
4.5	0
8	-1
12.5	-2



D: $x \geq 0$
 $x \geq 0$

$\{x | x \geq 0, x \in \mathbb{R}\}$

$[0, \infty)$

R: $\{y | y \leq 3, y \in \mathbb{R}\}$

$(-\infty, 3]$

Homework

#2-5 on page 72-73

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a) $f(x) = \sqrt{-x} - 3$

b) $r(x) = 3\sqrt{x+1}$

c) $p(x) = -\sqrt{x-2}$

d) $y - 1 = -\sqrt{-4(x-2)}$

e) $m(x) = \sqrt{\frac{1}{2}x + 4}$

f) $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

$$y = \left(\frac{1}{3}\right)\sqrt{-(x+2)} - 1 \quad y = a\sqrt{b(x-h)} + k$$

$a = \frac{1}{3} \rightarrow$ vertical stretch by a factor of $\frac{1}{3}$

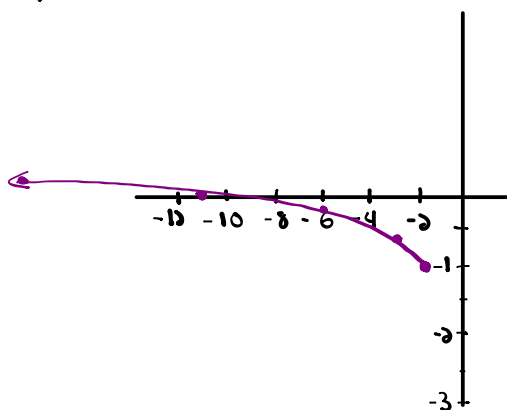
$b = -1 \rightarrow$ no horizontal stretch but there is a horizontal reflection in the y-axis

$h = -2 \rightarrow$ horizontal translation 2 units left.

$k = -1 \rightarrow$ vertical translation 1 unit down

$y = \sqrt{x}$	
x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \rightarrow \left[\frac{1}{3}x-2, \frac{1}{3}y-1\right]$	
x	y
-2	-1
-3	$-\frac{2}{3}$ or $-0.\bar{6}$
-6	$-\frac{1}{3}$ or $-0.\bar{3}$
-11	0
-18	$\frac{1}{3}$ or $0.\bar{3}$
-27	$\frac{2}{3}$ or $0.\bar{6}$



D: $\{x | x \leq -2, x \in \mathbb{R}\}$
or $(-\infty, -2]$

R: $\{y | y \geq -1, y \in \mathbb{R}\}$
or $[-1, \infty)$

5. Sketch the graph of each function using transformations. State the domain and range of each function.

- a) $f(x) = \sqrt{-x} - 3$
- b) $r(x) = 3\sqrt{x+1}$
- c) $p(x) = -\sqrt{x-2}$
- d) $y - 1 = -\sqrt{-4(x-2)}$
- e) $m(x) = \sqrt{\frac{1}{2}x + 4}$
- f) $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

d) $y - 1 = -\sqrt{-4(x-2)}$ $y = a\sqrt{b(x-h)} + k$
 $y = \underline{-1}\sqrt{\underline{-4}(x-\underline{2})} + \underline{1}$

$a = -1 \rightarrow$ no vertical stretch but there is a vertical reflection in the x-axis.

$b = -4 \rightarrow$ a horizontal stretch by a factor of $\frac{1}{4}$ and a horizontal reflection in the y-axis

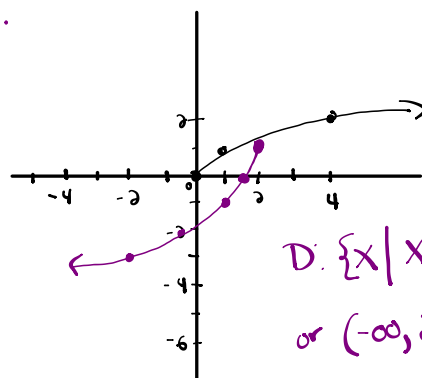
$h = 2 \rightarrow$ a horizontal translation 2 units right

$k = 1 \rightarrow$ a vertical translation 1 unit up

$y = \sqrt{x}$ $(x, y) \rightarrow \left(\frac{1}{-4}x + 2, -|y| + 1\right)$

x	y
0	0
1	1
4	2
9	3
16	4

x	y
2	1
(1.75) 24	0
1	-1
(-0.25) -14	-2
-2	-3



$D: \{x \mid x \leq 2, x \in \mathbb{R}\}$

or $(-\infty, 2]$

$R: \{y \mid y \leq 1, y \in \mathbb{R}\}$

or $(-\infty, 1]$

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a) $f(x) = \sqrt{-x} - 3$

b) $r(x) = 3\sqrt{x+1}$

c) $p(x) = -\sqrt{x-2}$

d) $y - 1 = -\sqrt{-4(x-2)}$

e) $m(x) = \sqrt{\frac{1}{2}x} + 4$

f) $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

e) $m(x) = 1\sqrt{\frac{1}{2}x} + 4$

$a=1 \rightarrow$ no vertical stretch or reflection

$b=\frac{1}{2} \rightarrow$ horizontal stretch by a factor of 2 and no horizontal reflection

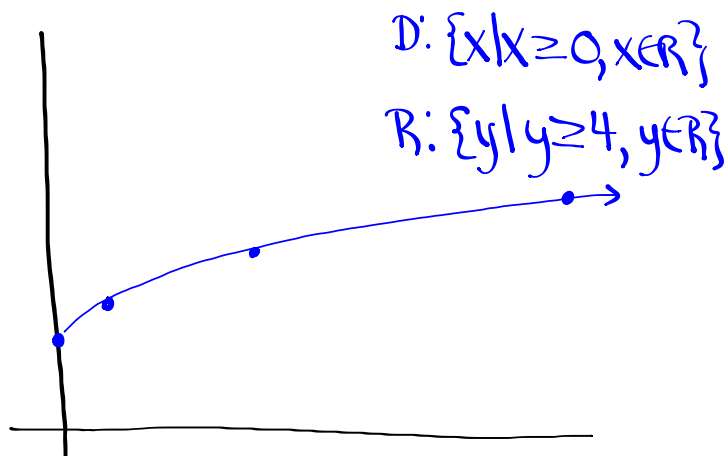
$h=0 \rightarrow$ no horizontal translation

$k=4 \rightarrow$ vertical translation 4 units up.

$$(x, y) \rightarrow (2x, y+4)$$

x	y
0	0
1	1
4	2
9	3

x	y
0	4
2	5
8	6
18	7



$$y - 4 = -2\sqrt{-3x - 9} + 4$$

$$y = -2\sqrt{-3x - 9} + 8$$

$$y = \underline{-2}\sqrt{\underline{-3}(x + \underline{3})} + \underline{8}$$

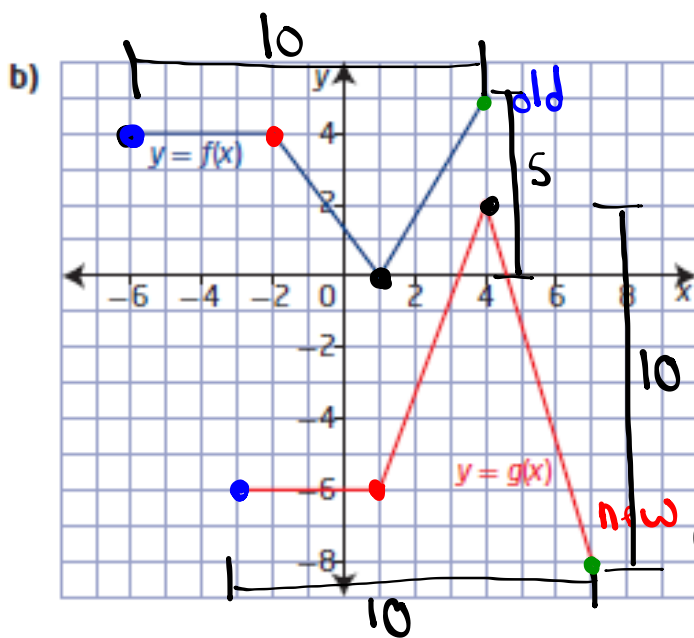
$$a = -2$$

$$b = -3$$

$$h = -3$$

$$k = 8$$

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



① Reflection: vertical in x-axis ($a < 0$)

② VSF: $\frac{10}{5} = 2$ $a = -2$

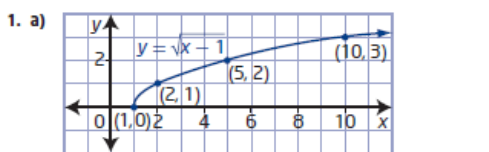
③ HSF: $\frac{10}{10} = 1$ $b = 1$

④ VT: $(1, 0) \rightarrow (4, 2)$ $k = 2$

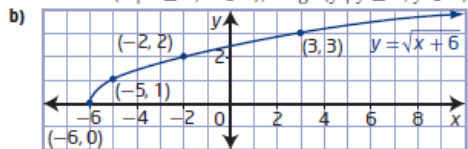
⑤ HT: $(-6, 4) \rightarrow (-3, -6)$ $h = 3$

⑥ $y = -2f(1(x-3)) + 2$

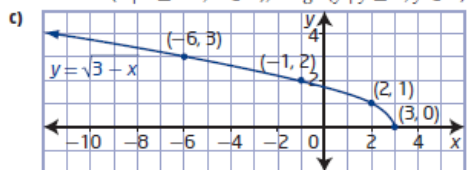
2.1 Radical Functions and Transformations, pages 72 to 77



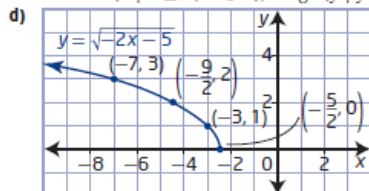
domain $\{x \mid x \geq 1, x \in \mathbb{R}\}$, range $\{y \mid y \geq 0, y \in \mathbb{R}\}$



domain $\{x \mid x \geq -6, x \in \mathbb{R}\}$, range $\{y \mid y \geq 0, y \in \mathbb{R}\}$

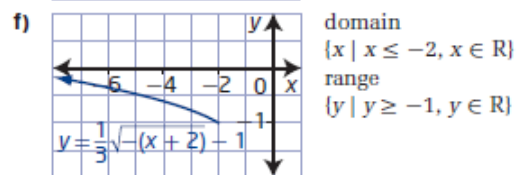
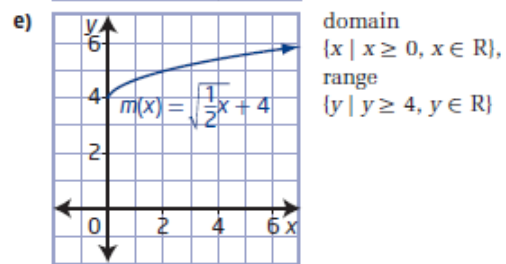
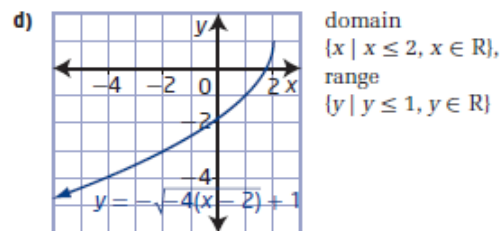
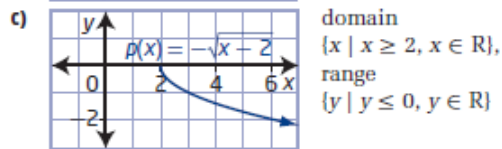
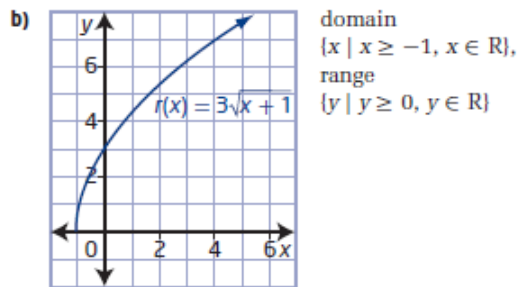
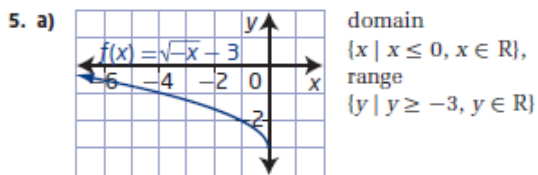


domain $\{x \mid x \leq 3, x \in \mathbb{R}\}$, range $\{y \mid y \geq 0, y \in \mathbb{R}\}$



domain $\{x \mid x \leq -\frac{5}{2}, x \in \mathbb{R}\}$,
range $\{y \mid y \geq 0, y \in \mathbb{R}\}$

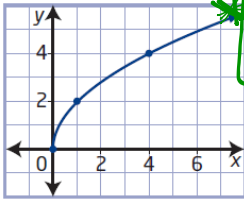
2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
 $h = 9 \rightarrow$ horizontal translation 9 units right
 domain $\{x \mid x \geq 9, x \in \mathbb{R}\}$, range $\{y \mid y \geq 0, y \in \mathbb{R}\}$
 b) $b = -1 \rightarrow$ reflected in y-axis
 $k = 8 \rightarrow$ vertical translation up 8 units
 domain $\{x \mid x \leq 0, x \in \mathbb{R}\}$, range $\{y \mid y \geq 8, y \in \mathbb{R}\}$
 c) $a = -1 \rightarrow$ reflected in x-axis
 $b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
 domain $\{x \mid x \geq 0, x \in \mathbb{R}\}$, range $\{y \mid y \leq 0, y \in \mathbb{R}\}$
 d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
 $h = -6 \rightarrow$ horizontal translation 6 units left
 $k = -4 \rightarrow$ vertical translation 4 units down
 domain $\{x \mid x \geq -6, x \in \mathbb{R}\}$,
 range $\{y \mid y \geq -4, y \in \mathbb{R}\}$
3. a) B b) A c) D d) C
4. a) $y = 4\sqrt{x+6}$ b) $y = \sqrt{8x} - 5$
 c) $y = \sqrt{-(x-4)} + 11$ or $y = \sqrt{-x+4} + 11$
 d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$



Example 3

Determine a Radical Function From a Graph

Mayleen is designing a symmetrical pattern. She sketches the curve shown and wants to determine its equation and the equation of its reflection in each quadrant. The graph is a transformation of the graph of $y = \sqrt{x}$. What are the equations of the four functions Mayleen needs to work with?



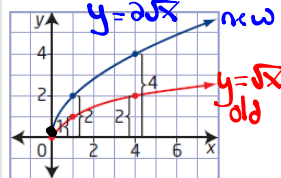
A radical function that involves a stretch can be obtained from either a vertical stretch or a horizontal stretch. Use an equation of the form $y = a\sqrt{x}$ or $y = \sqrt{bx}$ to represent the image function for each type of stretch

Method 1: Compare Vertical or Horizontal Distances

Superimpose the graph of $y = \sqrt{x}$ and compare corresponding distances to determine the factor by which the function has been stretched.

View as a Vertical Stretch ($y = a\sqrt{x}$)

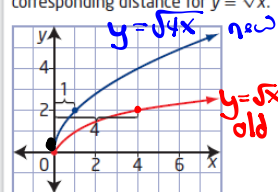
Each vertical distance is 2 times the corresponding distance for $y = \sqrt{x}$.



This represents a vertical stretch by a factor of 2, which means $a = 2$. The equation $y = 2\sqrt{x}$ represents the function.

View as a Horizontal Stretch ($y = \sqrt{bx}$)

Each horizontal distance is $\frac{1}{4}$ the corresponding distance for $y = \sqrt{x}$.



This represents a horizontal stretch by a factor of $\frac{1}{4}$, which means $b = 4$. The equation $y = \sqrt{4x}$ represents the function.

Express the equation of the function as either $y = 2\sqrt{x}$ or $y = \sqrt{4x}$.

① reflections: None

② VSF: $\frac{2}{1} = 2$ $a=2$

③ HSF: Skip if $y = \sqrt{x}$

④ HT: $(0,0) \rightarrow (0,0)$ $h=0$

⑤ VT: $(0,0) \rightarrow (0,0)$ $k=0$

⑥ $y = 2\sqrt{1(x-0)} + 0$

$y = 2\sqrt{x}$

① reflections: none

② HSF: $\frac{1}{4}$ $b=4$

③ VSF: skip if $y = \sqrt{x}$

④ HT: $(0,0) \rightarrow (0,0)$ $h=0$

⑤ VT: $(0,0) \rightarrow (0,0)$ $k=0$

⑥ $y = 1\sqrt{4(x-0)} + 0$

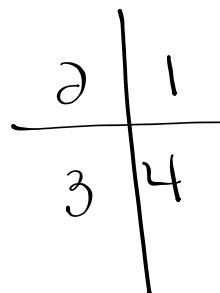
$y = \sqrt{4x}$

$y = 2\sqrt{x}$ (Quad 1)

$y = 2\sqrt{-x}$ (Quad 2)

$y = -2\sqrt{-x}$ (Quad 3)

$y = -2\sqrt{x}$ (Quad 4)



Homework

#6, 9, 10

(Page 73)

$$\textcircled{6} \quad f(x) = \frac{1}{4} \sqrt{5x}$$

$$f(x) = \frac{1}{4} \sqrt{5x}$$

$$b) \quad y = a\sqrt{x}$$

$$\text{and} \quad y = \sqrt{bx}$$

$$y = \frac{1}{4} \cdot \sqrt{5} \sqrt{x}$$

$$y = \sqrt{\frac{1}{4} \cdot \frac{1}{4} \cdot 5x}$$

$$y = \frac{\sqrt{5}}{4} \sqrt{x}$$

$$y = \sqrt{\frac{5x}{16}}$$

$$a = \frac{\sqrt{5}}{4}$$

$$b = \frac{5}{16}$$

