

Warm-up

Evaluate:

① $(81)^{0.75}$
 $(81)^{\frac{3}{4}}$ ← exponent
 ← index
 $(\sqrt[4]{81})^3$
 $(3)^3$
 (27)

② $\left(\frac{16}{81}\right)^{-5/4}$
 ← exponent
 $\left(\frac{81}{16}\right)^{5/4}$ ← index
 $\left(\sqrt[4]{81}\right)^5$
 $\left(\sqrt[4]{16}\right)^5$
 $(3)^5$
 $(2)^5$
 $\left(\frac{243}{32}\right)$

Important Rules to Remember !!

Exponent Laws

Product of powers:	$a^m \cdot a^n = a^{m+n}$
Quotient of powers:	$a^m \div a^n = a^{m-n}, a \neq 0$
Power of a power:	$(a^m)^n = a^{mn}$
Power of a product:	$(ab)^m = a^m b^m$
Power of a quotient:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

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$$5^2 \times 5^{11}$$

$$= 5^{2+11}$$


$$= 5^{13}$$

$$= 1\,220\,703\,125$$

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$$\frac{5^8}{5^6}$$


$$5^8 \div 5^6$$

$$= 5^{8-6}$$

$$= 5^2$$

$$= 25$$

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$$(5^8)^3$$

$$= 5^{8(3)}$$

$$= 5^{24}$$

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$$(xy)^3$$

$$= x^3 y^3$$

$$(x^2y)^3$$

$$= x^{2(3)} y^{1(3)}$$

$$= x^6 y^3$$

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$$\left(\frac{x^1}{y^1}\right)^2$$

$$\frac{x^{1(2)}}{y^{1(2)}} = \frac{x^2}{y^2}$$

$$\left(\frac{x^8}{y^6}\right)^2$$

$$\frac{x^{8(2)}}{y^{6(2)}} = \frac{x^{16}}{y^{12}}$$

Simplify by Writing as a Single Power

$$\text{a) } 0.3^{-3} \cdot 0.3^5$$

$$= 0.3^{-3+5}$$

$$= 0.3^2$$

Simplify by Writing as a Single Power

$$\begin{aligned} \text{a) } & 0.3^{-3} \cdot 0.3^5 \\ &= 0.3^{(-3) + 5} \\ &= 0.3^2 \end{aligned}$$

$$\text{b) } \frac{b^3 \times b^5}{b^7}$$

Simplify the numerator first!!

$$= \frac{b^{3+(5)}}{b^7}$$

$$= \frac{b^{-2}}{b^7}$$

$$= b^{-2-7}$$

$$= b^{-9}$$

$$= \left(\frac{1}{b}\right)^9$$

$$= \frac{(1)^9}{(b)^9} .$$

$$= \left(\frac{1}{b^9}\right)$$

b) $\frac{b^3 \times b^5}{b^7}$ Simplify the numerator first!!

$$= \frac{b^{-2}}{b^7}$$

$$= b^{-2-7}$$

$$= b^{-9}$$

$$= \frac{1}{b^9}$$

ALWAYS
express your answer
with a
positive exponent!!

$$c) \frac{(a^5 \times a^3)^{-2}}{a^{-2}}$$

$$= \frac{(a^{5+(-3)})^{-2}}{a^{-2}}$$

$$= \frac{(a^2)^{-2}}{a^{-2}}$$

$$= \frac{a^{2(-2)}}{a^{-2}}$$

$$= \frac{a^{-4}}{a^{-2}}$$

$$= a^{-4-(-2)}$$

$$= a^{-2}$$

$$= \left(\frac{1}{a}\right)^2$$

$$= \left(\frac{1}{a^2}\right)$$

$$\begin{aligned} \mathbf{c)} \quad & \frac{(\mathbf{a^5 \times a^3})^{-2}}{\mathbf{a^{-2}}} \\ &= \frac{(\mathbf{a^2})^{-2}}{\mathbf{a^{-2}}} \\ &= \frac{\mathbf{a^{-4}}}{\mathbf{a^{-2}}} \\ &= \mathbf{a^{-4 - (-2)}} \\ &= \mathbf{a^{-4 + 2}} \\ &= \mathbf{a^{-2}} \\ &= \mathbf{1/a^2} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & \frac{(1.4^3)(1.4^4)}{1.4^{-2}} \\ & = \frac{1.4^7}{1.4^{-2}} \\ & = 1.4^{7-(-2)} \\ & = 1.4^9 \end{aligned}$$

A red arrow points from the handwritten text "3+4" to the exponent 7 in the second step of the calculation.

$$\mathbf{d)} \quad \frac{(1.4^3)(1.4^4)}{1.4^{-2}}$$

Use the product of powers law. **(ADD)**

$$= \frac{1.4^{3+4}}{1.4^{-2}}$$

$$= \frac{1.4^7}{1.4^{-2}}$$

Use the quotient of powers law. **(SUBTRACT)**

$$= 1.4^{7 - (-2)}$$

$$= 1.4^9$$

$$e) \left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3$$

$$\left(-\frac{3}{2} \right)^{-4 \times 2} \cdot \left(-\frac{3}{2} \right)^{2 \times 3}$$

$$= \left(-\frac{3}{2} \right)^{-8} \cdot \left(-\frac{3}{2} \right)^6$$

$$= \left(-\frac{3}{2} \right)^{-8+6}$$

$$= \left(-\frac{3}{2} \right)^{-2}$$

$$= \left(\frac{2}{-3} \right)^2$$

$$= \frac{(2)^2}{(-3)^2}$$

$$= \frac{4}{9}$$

$$\mathbf{e)} \quad \left[\left(-\frac{3}{2} \right)^{-4} \right]^2 \cdot \left[\left(-\frac{3}{2} \right)^2 \right]^3$$

First use the power of a power law:

For each power, multiply the exponents.

$$= \left(-\frac{3}{2} \right)^{-8} \cdot \left(-\frac{3}{2} \right)^6$$

Then use the product of powers law. **(ADD)**

$$= \left(-\frac{3}{2} \right)^{-2}$$

Write with a positive exponent.

$$= \left(-\frac{2}{3} \right)^2$$

$$\mathbf{f)} \quad \left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$$

$$\mathbf{f)} \quad \left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$$

Use the product of powers law. (ADD)

$$= \left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3} + \frac{5}{3}}} \right)^6$$

$$= \left(\frac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}} \right)^6$$

Use the quotient of powers law. (SUBTRACT)

$$= \left(7^{\frac{2}{3} - \frac{6}{3}} \right)^6$$

$$= \left(7^{-\frac{4}{3}}\right)^6$$

Use the power of a power law. (MULTIPLY)

$$= 7^{\left(-\frac{4}{3}\right)(6)}$$

$$= 7^{-\frac{24}{3}}$$

$$= 7^{-8}$$

Write with a positive exponent.

$$= \frac{1}{7^8}$$

CHECK YOUR UNDERSTANDING

Simplify by writing as a single power. Explain your reasoning.

a) $0.8^2 \cdot 0.8^{-7}$

b) $\left[\left(-\frac{4}{5} \right)^2 \right]^{-3} \div \left[\left(-\frac{4}{5} \right)^4 \right]^{-5}$

c) $\frac{(1.5^{-3})^{-5}}{1.5^5}$

d) $\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

$$\begin{aligned} \text{a)} \quad & 0.8^2 \cdot 0.8^{-7} \\ & = \mathbf{0.8^{-5}} \end{aligned}$$

$$\begin{aligned} \text{b) } & \left[\left(-\frac{4}{5} \right)^2 \right]^{-3} \div \left[\left(-\frac{4}{5} \right)^4 \right]^{-5} \\ & \left(-\frac{4}{5} \right) \div \left(-\frac{4}{5} \right) \end{aligned}$$

$$\text{c) } \frac{(1.5^{-3})^{-5}}{1.5^5}$$

$$\text{d) } \frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$$