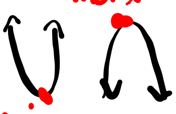
Questions from homework

Quadratic 

④ a) $y = x^2 + 6x + 11$

$y' = 2x + 6$

$y' = 2(x+3)$

$0 = 2(x+3)$

CV: $x + 3 = 0$

$x = -3$

min

$-$ $+$

(-4) -3 (0)

min $y = (-3)^2 + 6(-3) + 11$

$y = 9 - 18 + 11$

$y = 2$ $(-3, 2)$

The First Derivative Test

If f has a local maximum or minimum at c, then c must be a critical value of f (Fermat's Theorem), but not all critical numbers give rise to a maximum or minimum. For instance, recall that 0 is a critical number of the function $y = x^3$ but this function has no maximum or minimum at a critical number.

One way of solving this is suggested by the figure below.
If f is increasing to the left of a critical number c and decreasing to the right of c, then f has a local max at c.
If f is decreasing to the left of a critical number c and increasing to the right of c, then f has a local min at c.

The First Derivative Test

Let c be a critical number of a continuous function f.

1. If $f'(x)$ changes from positive to negative at c, then f has a local max at c.

2. If $f'(x)$ changes from negative to positive at c, then f has a local min at c.

3 If $f'(x)$ does not change signs at c, then f has no max or min at c.

Example 1

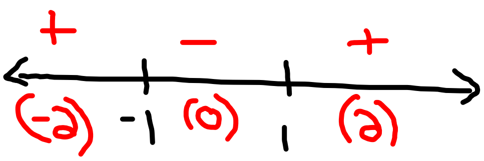
Find the local maximum and minimum values of
$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(x) = 3(x^2 - 1)$

$f'(x) = 3(x+1)(x-1)$

CV: $x = \pm 1$



$f(-1) = -1 + 3 + 1 = 3$

$(-1, 3)$ local max

$f(1) = 1 - 3 + 1 = -1$

$(1, -1)$ local min

Example 2

Find the local maximum and minimum values of
$g(x) = x^4 - 4x^3 - 8x^2 - 1$. Use this information to sketch the graph of g.

$g'(x) = 4x^3 - 12x^2 - 16x$

$g'(x) = 4x(x^2 - 3x - 4)$

$g'(x) = 4x(x-4)(x+1)$

$0 = 4x(x-4)(x+1)$

CV: $x = -1, 0, 4$

min max min

$- \quad + \quad - \quad +$

(-2) -1 (-1/2) 0 (1) 4 (5)

Decreasing on $(-\infty, -1)$ and $(0, 4)$

Increasing on $(-1, 0)$ and $(4, \infty)$

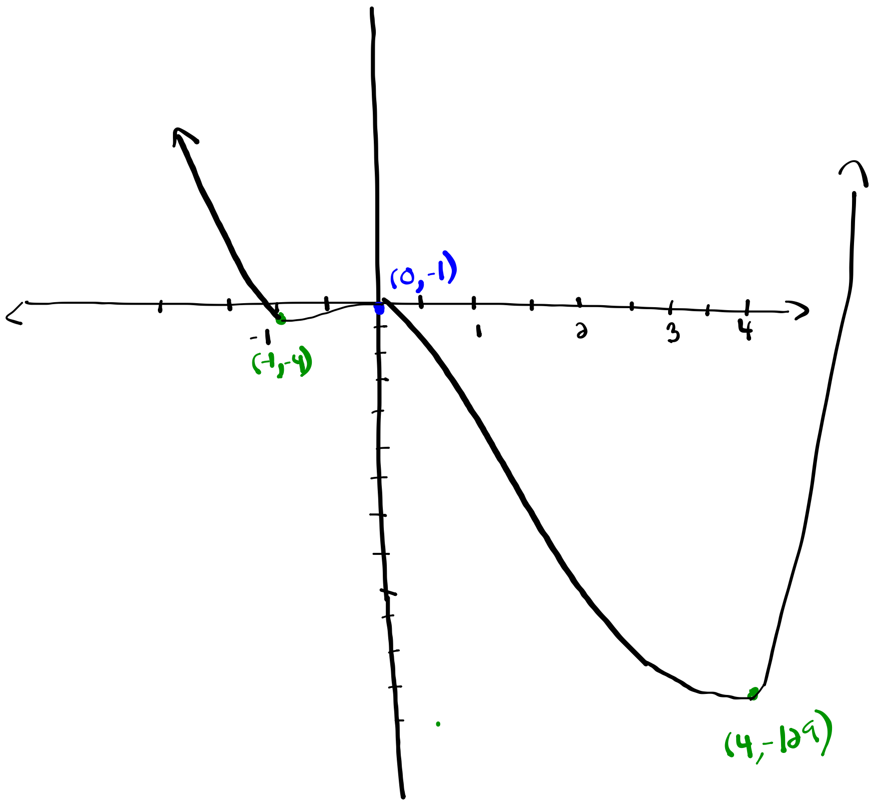
$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f(-1) = (-1)^4 - 4(-1)^3 - 8(-1)^2 - 1 = 1 + 4 - 8 - 1 = -4$ $\quad(-1, -4)$

$f(0) = (0)^4 - 4(0)^3 - 8(0)^2 - 1 = 0 - 0 - 0 - 1 = -1$ $\quad(0, -1)$

$f(4) = (4)^4 - 4(4)^3 - 8(4)^2 - 1 = 256 - 256 - 128 - 1 = -129$

$(4, -129)$

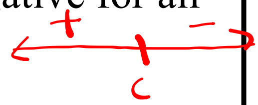


The First Derivative Test

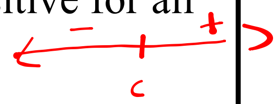
(for absolute extreme values)

Let c be a critical number of a continuous function f.

1. If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum value.



2. If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum value.



Homework

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