Questions From Homework

The sum of two numbers is 12. Find the numbers so that their product is a maximum?

A rectangle has a perimeter of 150cm. What length and width should it have so that its area is a maximum?

A rectangle has a perimeter of 200cm. What length and width should it have so that its area is a maximum?

(x, y)

Find the point on the graph of $y = 2x + 6$ that is the minimum distance from the point (1, 2). $x_1 = 1$ $y_1 = 2$

Remember ***d is smallest when d^2 is smallest***

$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$

$d = \sqrt{(x-1)^2 + (y-2)^2}$

$d = \sqrt{(x-1)^2 + (2x+6-2)^2}$

$d = \sqrt{(x-1)^2 + (2x+4)^2}$

$d = \sqrt{x^2 - 2x + 1 + 4x^2 + 16x + 16}$

$d = \sqrt{5x^2 + 14x + 17} = (5x^2 + 14x + 17)^{1/2}$

$f(x) = 5x^2 + 14x + 17$

$f'(x) = 10x + 14$

$f'(x) = 2(5x + 7)$

CV: $2(5x+7) = 0$

$5x + 7 = 0$

$5x = -7$

$x = -\frac{7}{5}$

$-$ $+$

(−2) $-\frac{7}{5}$ (0)

if $x = -\frac{7}{5}$:

$y = 2x + 6$

$y = 2\left(-\frac{7}{5}\right) + 6$

$y = -\frac{14}{5} + \frac{30}{5}$

$y = \frac{16}{5}$

$\left(-\frac{7}{5}, \frac{16}{5}\right)$ is the point

Remember ***d is smallest when d^2 is smallest ...*** here is the proof!

$$d = \left[(x-1)^2 + (2x+4)^2\right]^{1/2}$$

$$d' = \frac{1}{2}\left[(x-1)^2+(2x+4)^2\right]^{-1/2}\left[2(x-1)(1)+2(2x+4)(2)\right]$$

$$= \frac{1}{2}\left[(x-1)^2+(2x+4)^2\right]^{-1/2}\left[2x-2+8x+16\right]$$

$$= \frac{1}{2}\left[(x-1)^2+(2x+4)^2\right]^{-1/2}(10x+14)$$

$$= \frac{5x+7}{\sqrt{(x-1)^2+(2x+4)^2}}$$ ← denominator is always positive

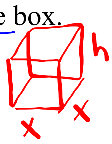
So $d' = 0$ when $5x+7=0$

$$5x = -7$$

$$x = \frac{-7}{5}$$ ← is the only critical number

Surface Area closed top $A = 2x^2 + 4xh$

If 2700 cm² of material is available to make a box with square base and an open top, find the largest possible volume of the box.



Let x = *the length of the base*
Let h = *the height*

$A = x^2 + 4xh$

$2700 = x^2 + 4xh$

$2700 - x^2 = 4xh$

$\frac{2700 - x^2}{4x} = h$

We want to maximize the *volume*.

$V = lwh$

$V = x^2 h$

We want to eliminate h from the *volume* function and we do so by finding a relationship between x *and* h. We use the area of the available material

(i) $V = x^2 h$

$V = x^2 \left(\frac{2700 - x^2}{4x} \right)$

$V = \frac{2700x^2 - x^4}{4x}$

$V = \frac{2700x - x^3}{4}$

$V = 675x - \frac{1}{4}x^3$

$V' = 675 - \frac{3}{4}x^2$

$0 = 675 - \frac{3}{4}x^2$

$4 \cdot \frac{3}{4}x^2 = 675 \cdot 4$

$\frac{3x^2}{3} = \frac{2700}{3}$

$x^2 = 900$

$x = \pm 30$

$x = 30\text{cm}$

(ii) $h = \frac{2700 - x^2}{4x}$

$h = \frac{2700 - (30)^2}{4(30)} = \frac{1800}{120} = 15\text{cm}$

(iii) $V = x^2 h$

$V = (30)^2 (15)$

$V = 13500 \text{ cm}^3$

Find the points on the parabola $y = 6 - x^2$ that are closest to the point (0, 3)

Homework