Questions from Homework

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time.*

Related Rates

1. Draw a diagram
2. List what is given in differentiation notation $\frac{da}{dt}$, $\frac{dv}{dt}$, etc.
3. List what is to be found in differentiation notation.
4. Find an appropriate equation that relates the variables in steps 2 and 3.
5. Differentiate with respect to time.
6. Substitute the values given and solve for the unknown.

Areas and Volumes

The length of a square is 4m and is increasing at a rate of 1.25m/min. How fast is the *area* of the square increasing?

Hint!
write down what is given
find an equation that relates the two quantities

Given:

$l = 4m$

$\frac{dl}{dt} = 1.25m/min$

$\frac{dA}{dt} = ?$

$A = l^2$

$\frac{dA}{dt} = 2l\frac{dl}{dt}$

$\frac{dA}{dt} = 2(4)(1.25)$

$\frac{dA}{dt} = 10m^2/min$

Suppose you tossed a stone into a lake. A circular ripple starts and moves outward with its radius increasing at a rate of 5cm/sec. How fast is the *area* of the circle increasing after 3 seconds? ***(Hint: what would the radius be at 3 seconds?)***

Given:

$\frac{dr}{dt} = 5cm/sec$

$\frac{dA}{dt} = ?$

when $t = 3$ sec

$r = 5cm/s \times 3s$

$r = 15cm$

Area of a circle

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(15)(5)$

$\frac{dA}{dt} = 150\pi \, cm^2/sec$

$\frac{dA}{dt} = 471.24 \, cm^2/sec$

Volumes/Surface Areas of Spheres

A *spherical* snowball is melting in such a way that its volume is *decreasing* at a rate of 1 cm³/min. At what rate is the radius of the snowball decreasing if the original radius is 5 cm?

Hint!
write down what is given
find an equation that relates the two quantities

Given:

$\frac{dV}{dt} = -1 \text{ cm}^3/\text{min}$

$\frac{dr}{dt} = ?$

$r = 5\text{cm}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-1 = 4\pi(5)^2 \frac{dr}{dt}$$

$$-1 = 100\pi \frac{dr}{dt}$$

$$-\frac{1}{100\pi} \text{ cm/min} = \frac{dr}{dt}$$

$$-0.00318 \text{ cm/min} = \frac{dr}{dt}$$

A beach ball is being inflated so that its surface area is*increasing* at a rate of 100 cm²/sec. Find the rate at which the radius is increasing if the original radius is 2 cm?

Given:

$\frac{dA}{dt} = 100 cm^2/sec$

$\frac{dr}{dt} = ?$

$r = 2cm$

$$A = 4\pi r^2$$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$100 = 8\pi (2) \frac{dr}{dt}$

$100 = 16\pi \frac{dr}{dt}$

$\frac{100}{16\pi} cm/sec = \frac{dr}{dt}$

$\frac{25}{4\pi} cm/sec = \frac{dr}{dt} = 1.99 cm/sec$

Homework