

$$\textcircled{3} \quad y = \frac{(2x+5)^2}{3x} \quad \begin{array}{l} f(x) \\ g(x) \end{array}$$

$$y' = \frac{2(2x+5)'(2)(3x) - 3(2x+5)^2}{(3x)^2}$$

$$y' = \frac{12x(2x+5) - 3(2x+5)^2}{9x^2}$$

$$y' = \frac{3(2x+5) \left[ 4x - (2x+5) \right]}{9x^2} \quad \text{Common Factor}$$

$$y' = \frac{\cancel{3}(2x+5)(2x-5)}{\cancel{3}9x^2} = \frac{(2x+5)(2x-5)}{3x^2}$$

$$\textcircled{4} \quad f(x) = \frac{x+2}{(x-3)^3} \quad \begin{array}{l} f(x) \\ g(x) \end{array}$$

$$f'(x) = \frac{1(x-3)^3 - (x+2)(3)(x-3)^2(1)}{[(x-3)^3]^2}$$

$$f'(x) = \frac{(x-3)^3 - 3(x+2)(x-3)^2}{(x-3)^6}$$

$$f'(x) = \frac{(x-3)^2 \left[ (x-3) - 3(x+2) \right]}{(x-3)^6}$$

$$f'(x) = \frac{\cancel{(x-3)^2}(-2x-9)}{\cancel{(x-3)^6}^4} = \frac{-2x-9}{(x-3)^4} = -\frac{2x+9}{(x-3)^4}$$

$$\textcircled{4} \quad f(x) = \frac{x+2}{(x-3)^3} \quad \begin{array}{l} f(x) \\ g(x) \end{array}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

$$f'(x) = \frac{1(x-3)^3 - (x+2)(3)(x-3)^2(1)}{[(x-3)^3]^2}$$

$$f'(x) = \frac{(x-3)^3 - 3(x+2)(x-3)^2}{(x-3)^6} \quad \text{common factor}$$

$$f'(x) = \frac{(x-3)^2 \left[ \overset{x-3-3x-6}{(x-3) - 3(x+2)} \right]}{(x-3)^6}$$

$$\frac{(x-3)^2}{(x-3)^6} = (x-3)^{-4}$$

$$f'(x) = \frac{\cancel{(x-3)}^2 (-2x-9)}{(x-3)^4}$$

$$f'(x) = \frac{-2x-9}{(x-3)^4} = -\frac{(2x+9)}{(x-3)^4}$$

$$\textcircled{5} \quad g(x) = \frac{6x^2}{(2x+1)^2}$$

$$g'(x) = \frac{12x(2x+1)^2 - 6x^2(2)(2x+1)'(2)}{[(2x+1)^2]^2}$$

$$g'(x) = \frac{12x(2x+1)^2 - 24x^2(2x+1)}{(2x+1)^4}$$

$$g'(x) = \frac{12x \cancel{(2x+1)} \left[ \overset{2x+1-2x}{(2x+1)-2x} \right]}{(2x+1)^{\cancel{4}_3}} = \frac{12x}{(2x+1)^3}$$

$$\textcircled{1} f(x) = \underbrace{(3x+2)^3}_{f(x)} \underbrace{(2x-1)^2}_{g(x)}$$

□

$$f'(x) = \underbrace{3(3x+2)^2(3)}_{f'(x)} \underbrace{(2x-1)^2}_{g(x)} + \underbrace{(3x+2)^3}_{f(x)} \underbrace{(2)(2x-1)^1(2)}_{g'(x)}$$

$$f'(x) = \underline{9(3x+2)^2(2x-1)^2} + \underline{4(3x+2)^3(2x-1)}$$

$$f'(x) = \underline{(3x+2)^2(2x-1)} \left[ \overset{18x-9}{\underline{9(2x-1)}} + \overset{+12x+8}{\underline{4(3x+2)}} \right]$$

$$f'(x) = (3x+2)^2(2x-1)(30x-1)$$

## Quiz

#5  $g(x) = (2x^2 + 5x)(3x + 4)$  Find equation of tangent line @  $(\underline{-1}, \underline{-3})$

(i) Find derivative

$$g(x) = (2x^2 + 5x)(3x + 4)$$

$$g'(x) = (4x + 5)(3x + 4) + 3(2x^2 + 5x)$$

(iii)  $y - y_1 = m(x - x_1)$

$$y - (-3) = -8(x - (-1))$$

$$y + 3 = -8(x + 1)$$

$$y + 3 = -8x - 8$$

$$y = -8x - 11$$

(ii) Find  $m$  by filling in  $x$ :

$$g'(-1) = [4(-1) + 5][3(-1) + 4] + 3[2(-1) + 5]$$

$$g'(-1) = (1)(1) + 3(2 - 5)$$

$$g'(-1) = 1 + 3(-3)$$

$$g'(-1) = 1 - 9 = \underline{-8}$$

$$m = -8$$

Quiz

#6

$y = \frac{x}{x-1}$  ← quotient

parallel to  $x + 4y = 1$

(i) Find  $y'$ :

$$y' = \frac{1(x-1) - x(1)}{(x-1)^2}$$

$$y' = \frac{x-1-x}{(x-1)^2}$$

$$y' = \frac{-1}{(x-1)^2}$$

(ii) Find m:  $\frac{4y}{4} = \frac{-x+1}{4} + \frac{1}{4}$

$$y = \frac{-1}{4}x + \frac{1}{4}$$

$$m = -\frac{1}{4}$$

(iii) Set slope equal to  $y'$

$$-\frac{1}{4} = \frac{-1}{(x-1)^2}$$

$$\frac{-(x-1)^2}{-1} = \frac{-4}{-1}$$

$$(x-1)^2 = 4$$

(iv) Find  $y$ :

•  $x = 3$

$$y = \frac{(3)}{(3)-1} = \frac{3}{2}$$

$(3, \frac{3}{2})$

•  $x = -1$

$$y = \frac{(-1)}{(-1)-1} = \frac{-1}{-2} = \frac{1}{2}$$

$(-1, \frac{1}{2})$

$$x^2 - x - x + 1 = 4$$

$$x^2 - 2x + 1 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\begin{array}{l|l} x-3=0 & x+1=0 \\ x=3 & x=-1 \end{array}$$

$$\textcircled{6} \quad y = \left( \frac{x^2+1}{x+1} \right)^9$$

$$y' = 9 \left( \frac{x^2+1}{x+1} \right)^8 \left[ \frac{2x^2+2x - x^2 - 1}{(x+1)^2} \right]$$

$$y' = 9 \frac{(x^2+1)^8}{(x+1)^8} \frac{(x^2+2x-1)}{(x+1)^2}$$

$$y' = \frac{9(x^2+1)^8(x^2+2x-1)}{(x+1)^{10}}$$

Review Sheet.

$$\textcircled{1} \text{ a) } f(x) = \sqrt{x-5} \quad f(x+h) = \sqrt{x+h-5}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h-5} - \sqrt{x-5}) (\sqrt{x+h-5} + \sqrt{x-5})}{h (\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x+h-5 - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})} = \frac{1}{\sqrt{x-5} + \sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}$$



## Review Sheet

$$\textcircled{1} b) f(x) = \frac{2x-2}{x+3}$$

$$f(x+h) = \frac{2(x+h)-2}{(x+h)+3}$$

$$f(x+h) = \frac{2x+2h-2}{x+h+3}$$

$$\text{CO: } (x+3)(x+h+3)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{2x+2h-2}{x+h+3} - \frac{2x-2}{x+3}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)} \leftarrow \text{Expand}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + \cancel{6h} - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - \cancel{2h} - \cancel{6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{8h}{h(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

$$f'(x) = \frac{8}{(x+3)^2}$$

Review Sheet

$$\textcircled{2} \text{ b) } y = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$$

$$3 \cdot -\frac{1}{2} = -\frac{3}{2}$$

$$-\frac{1}{2} - \frac{2}{2} = -\frac{3}{2}$$

$$y' = \frac{-3}{2} x^{-3/2} = \frac{-3}{2x^{3/2}} = \frac{-3}{2\sqrt{x^3}}$$

$$y = \frac{3}{x^{1/2}} \quad (\text{quotient Rule})$$

$$y' = \frac{0(x^{1/2}) - 3\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2})^2}$$

$$y' = \frac{-\frac{3}{2}x^{-1/2}}{x}$$

$$y' = \frac{-\frac{3}{2x^{1/2}} \cdot 2x^{1/2}}{x \cdot 2x^{1/2}}$$

$$\text{CD: } 2x^{1/2}$$

$$x' \cdot x^{1/2} = x^{2/2+1/2}$$

$$y' = \frac{-3}{2x^{3/2}}$$

Review Sheet

$$\textcircled{3} \text{ a) } y = \underbrace{(3x^2 - 2)}_{f(x)} \underbrace{(4x + 5)}_{g(x)}$$

$$y' = 6x(4x + 5) + 4(3x^2 - 2)$$

$$y' = 24x^2 + 30x + 12x^2 - 8$$

$$y' = 36x^2 + 30x - 8$$

## Review Sheet

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{3+x^2} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$y' = \frac{1 \cdot x^{-1/2} (3+x^2) - 2x(x^{1/2})}{(3+x^2)^2}$$

$$\left[ \begin{array}{l} x^1 \cdot x^{1/2} \\ = x^{3/2} \end{array} \right.$$

$$y' = \frac{1 (3+x^2) - 2x^{3/2}}{2x^{1/2} (3+x^2)^2}$$

$$y' = \frac{\cancel{2x^{1/2}} \frac{3+x^2}{\cancel{2x^{1/2}}} - \frac{2x^{3/2}}{1}}{2x^{1/2} (3+x^2)^2} \quad \text{CO: } 2x^{1/2}$$

$$y' = \frac{3+x^2 - 4x^2}{2x^{1/2} (3+x^2)^2} = \boxed{\frac{3-3x^2}{2\sqrt{x} (3+x^2)^2}}$$

## Review Sheet

$$\textcircled{6} \text{ b) } y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$$

$$y' = -8(x-1)^{-3/2} \quad (1)$$

$$y' = -8(x-1)^{-3/2}$$

$$y' = \frac{-8}{(x-1)^{3/2}} = \frac{-8}{\sqrt{(x-1)^3}}$$

$$y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} \quad \begin{array}{l} f(x) \\ g(x) \end{array}$$

$$\left( \frac{1}{2} - \frac{2}{2} = -\frac{1}{2} \right)$$

$$y' = \frac{\cancel{0(x-1)^{1/2}} - 16\left(\frac{1}{2}\right)(x-1)^{-1/2}}{\left[(x-1)^{1/2}\right]^2} \quad (1)$$

$$y' = \frac{-8(x-1)^{-1/2}}{(x-1)}$$

$$y' = \frac{-8}{(x-1)(x-1)^{1/2}} = \frac{-8}{(x-1)^{3/2}}$$

$$1 + \frac{1}{2} = \frac{3}{2}$$

$$\textcircled{2} \quad \text{c) } g(x) = \overset{f(x)}{(x-5)^3} \overset{g(x)}{(7x^5+2x)^9} \overset{h(x)}{(4-2x^3)^5}$$

$$g'(x) = f'(x)g(x)h(x) + g'(x)f(x)h(x) + h'(x)f(x)g(x)$$

$$g'(x) = (3)(x-5)^2(1)(7x^5+2x)^9(4-2x^3)^5 + \dots$$

$$\textcircled{3} \text{ b) } f(x) = \frac{8x^3(12x^2 - 5x)^8}{2 - 3(1 - 32x^{10})^{1/5}}$$

$$f'(x) = \frac{\begin{array}{l} \text{derivative of numerator} \\ [24x^2(12x^2 - 5x)^8 + 8x^3(8)(12x^2 - 5x)(24x - 5)] \end{array} \begin{array}{l} \text{denominator} \\ [2 - 3(1 - 32x^{10})^{1/5}] \end{array} - \begin{array}{l} \text{numerator} \\ [8x^3(12x^2 - 5x)^8] \end{array} \begin{array}{l} \text{derivative of the denominator} \\ [-\frac{3}{5}(1 - 32x^{10})^{-4/5}(-320x^9)] \end{array}}{\begin{array}{l} \text{denominator squared.} \\ [2 - 3(1 - 32x^{10})^{1/5}]^2 \end{array}}$$

$$[2 - 3(1 - 32x^{10})^{1/5}]^2$$

denominator squared.

Chain:

$$y = (x^2 + 1)^9$$

$$y' = 9(x^2 + 1)^8 (2x)$$

Quotient:

$$y = \frac{x^2 + 1}{x + 1}$$

$$y' = \frac{2x(x+1) - 1(x^2+1)}{(x+1)^2}$$



Chain Rule:

$$\textcircled{4} \quad G(x) = \sqrt{x^4 - x + 1} = (x^4 - x + 1)^{1/2}$$

$$G'(x) = \frac{1}{2} (x^4 - x + 1)^{-1/2} (4x^3 - 1)$$

$$G'(x) = \frac{1}{2(x^4 - x + 1)^{1/2}} \cdot (4x^3 - 1)$$

$$G'(x) = \frac{4x^3 - 1}{2\sqrt{x^4 - x + 1}}$$

$$\textcircled{8} \quad y = \frac{4}{\sqrt{9-x^2}} = \frac{4}{(9-x^2)^{1/2}} = 4(9-x^2)^{-1/2}$$

$$y' = -2(9-x^2)^{-3/2} (-2x)$$

$$y' = 4x(9-x^2)^{-3/2}$$

$$y' = \frac{4x}{(9-x^2)^{3/2}} = \frac{4x}{\sqrt{(9-x^2)^3}}$$

$$\textcircled{10} \quad y = \sqrt{x + \sqrt{x}} = (x + \sqrt{x})^{1/2} = (x + x^{1/2})^{1/2}$$

$$y' = \frac{1}{2} (x + x^{1/2})^{-1/2} \left(1 + \frac{1}{2}x^{-1/2}\right)$$

$$y' = \left[ \frac{1}{2(x + \sqrt{x})^{1/2}} \right] \left[ \frac{1}{1} + \frac{1}{2\sqrt{x}} \right]$$

← Add by finding common denom

$$y' = \left[ \frac{1}{2\sqrt{x + \sqrt{x}}} \right] \left[ \frac{2\sqrt{x} + 1}{2\sqrt{x}} \right]$$

$$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}} \quad \text{or} \quad \frac{2\sqrt{x} + 1}{4\sqrt{x^3 + x^{3/2}}}$$

Review:

$$\textcircled{4} \quad g(x) = (x^2 - 3x + 4)(2x^2 + 4x)$$

$$g'(x) = (x^2 - 3x + 4)(4x + 4) + (2x - 3)(2x^2 + 4x)$$

$$g'(x) = 4x^3 + 4x^2 - 12x^2 - 12x + 16x + 16 + 4x^3 + 8x^2 - 6x^2 - 12x$$

$$g'(x) = 8x^3 - 6x^2 - 8x + 16$$

## Differentiation Rules:

$$\textcircled{6} \quad y = (1+x)^{10} \quad x = \underline{0} \quad y = \underline{1} \quad (0, 1)$$

$$* \quad y = (1+0)^{10} = \underline{1}$$

① Find Derivative

$$y' = 10(1+x)^9 (1)$$

$$y' = 10(1+x)^9$$

② Find Slope (X=0)

$$y'(0) = 10(1+0)^9$$

$$= 10(1)^9$$

$$= 10$$

↑  
m

③ Equation

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 10(x - 0)$$

$$y - 1 = 10x$$

$$0 = 10x - y + 1$$

③ Sum + Difference Rule:

$$a) \quad y = x^{10} + 20x^5 - 12\sqrt[4]{x^3} + 30$$

$$= x^{10} + 20x^5 - 12x^{3/4} + 30$$

$$y' = 10x^9 + 100x^4 - 9x^{-1/4} + 0$$

$$= 10x^9 + 100x^4 - \frac{9}{x^{1/4}}$$

$$\textcircled{8} \quad y = \frac{4}{\sqrt{9-x^2}} = \frac{4}{(9-x^2)^{1/2}} = 4(9-x^2)^{-1/2}$$

$$\begin{aligned} y' &= -2(9-x^2)^{-3/2}(-2x) \\ &= 4x(9-x^2)^{-3/2} \\ &= \frac{4x}{(9-x^2)^{3/2}} \quad \text{or} \quad \frac{4x}{\sqrt{(9-x^2)^3}} \end{aligned}$$

$$\textcircled{4} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$b) \quad f(x) = \frac{2x-1}{4x} \quad \Bigg| \quad f(x+h) = \frac{2(x+h)-1}{4(x+h)} \\ = \frac{2x+2h-1}{4x+4h}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{2x+2h-1}{4x+4h} - \frac{2x-1}{4x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{h(4x)(4x+4h)} \\ &= \lim_{h \rightarrow 0} \frac{8x^2 + 8xh - 4x - (8x^2 + 8xh - 4x - 4h)}{h(4x)(4x+4h)} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{8x^2} + \cancel{8xh} - \cancel{4x} - \cancel{8x^2} - \cancel{8xh} + \cancel{4x} + 4h}{h(4x)(4x+4h)} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{4h}}{\cancel{h}(4x)(\cancel{4x+4h})} = \frac{4}{(4x)^2} = \frac{4}{16x^2} = \frac{1}{4x^2} \end{aligned}$$

Chain Rule:

$$\textcircled{10} \quad y = \sqrt{x + \sqrt{x}} = (x + \sqrt{x})^{1/2} = (x + x^{1/2})^{1/2}$$

$$y' = \frac{1}{2} (x + x^{1/2})^{-1/2} \left( 1 + \frac{1}{2} x^{-1/2} \right)$$

$$y' = \left[ \frac{1}{2(x + x^{1/2})^{1/2}} \right] \left[ \frac{1}{1} + \frac{1}{2x^{1/2}} \right]$$

$$y' = \left[ \frac{1}{2\sqrt{x + \sqrt{x}}} \right] \left[ \frac{2\sqrt{x} + 1}{2\sqrt{x}} \right]$$

$$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}} \quad \text{or} \quad \frac{2x^{1/2} + 1}{4x^{1/2}(x + x^{1/2})^{1/2}}$$

Ex : 2.5

$$\textcircled{6} \quad y = \frac{x^2}{2x+5}$$

$$y' = \frac{(2x+5)(2x) - x^2(2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2 + 10x}{(2x+5)^2} = \frac{2x(x+5)}{(2x+5)^2}$$

$$\frac{0}{1} \rightarrow \frac{2x(x+5)}{(2x+5)^2}$$

$$2x(x+5) = 0$$

$$\begin{array}{l|l} 2x=0 & x+5=0 \\ x=0 & x=-5 \end{array}$$

$$\begin{array}{l} x=0 \\ y = \frac{(0)^2}{2(0)+5} \end{array}$$

$$= \frac{0}{5}$$

$$= 0$$

$$(0, 0)$$

$$\begin{array}{l} x = -5 \\ y = \frac{(-5)^2}{2(-5)+5} \end{array}$$

$$= \frac{25}{-5}$$

$$= -5$$

$$(-5, -5)$$