Sinusoidal Relations (Trig Graphs)

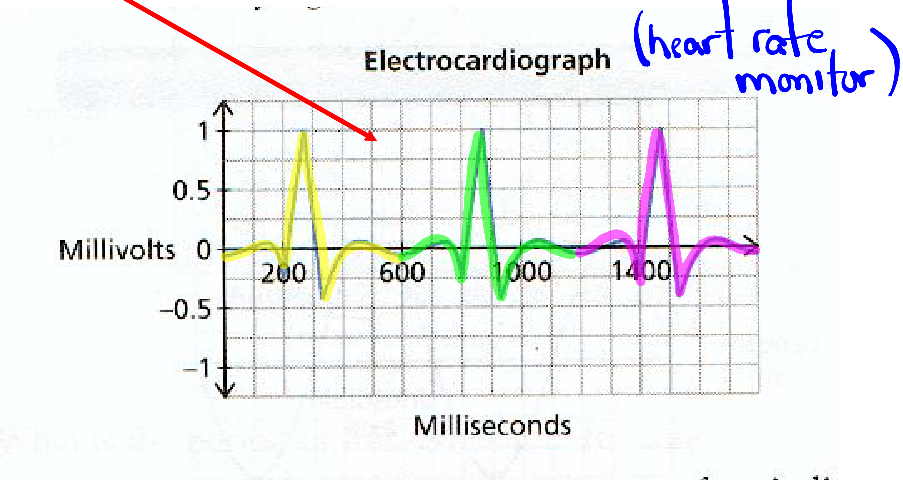
$y = \sin x$
$y = \cos x$

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(a function that repeats)

Example of periodic behavior

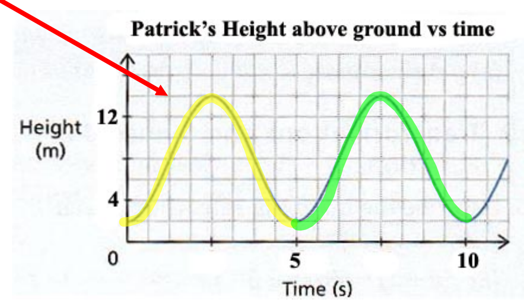
(heart rate monitor)



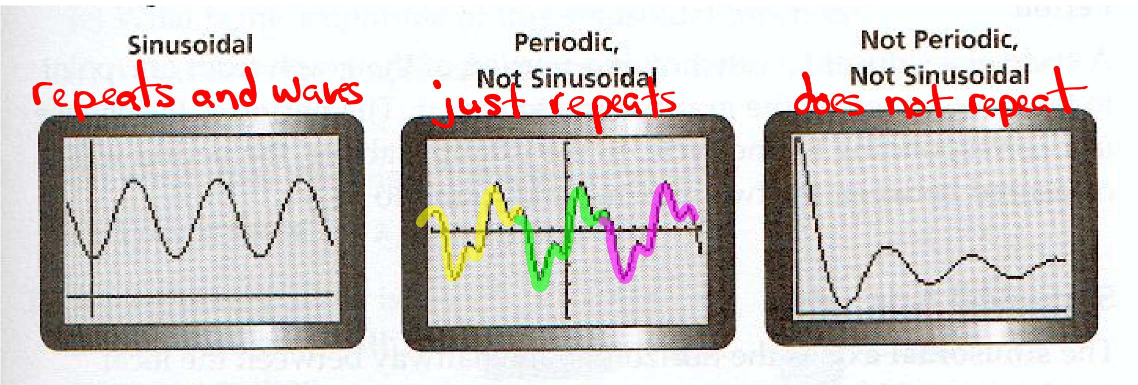
Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

(Repeats and looks like a smooth wave).

Example of sinusoidal behavior



These illustrations should summarize periodic and sinusoidal...

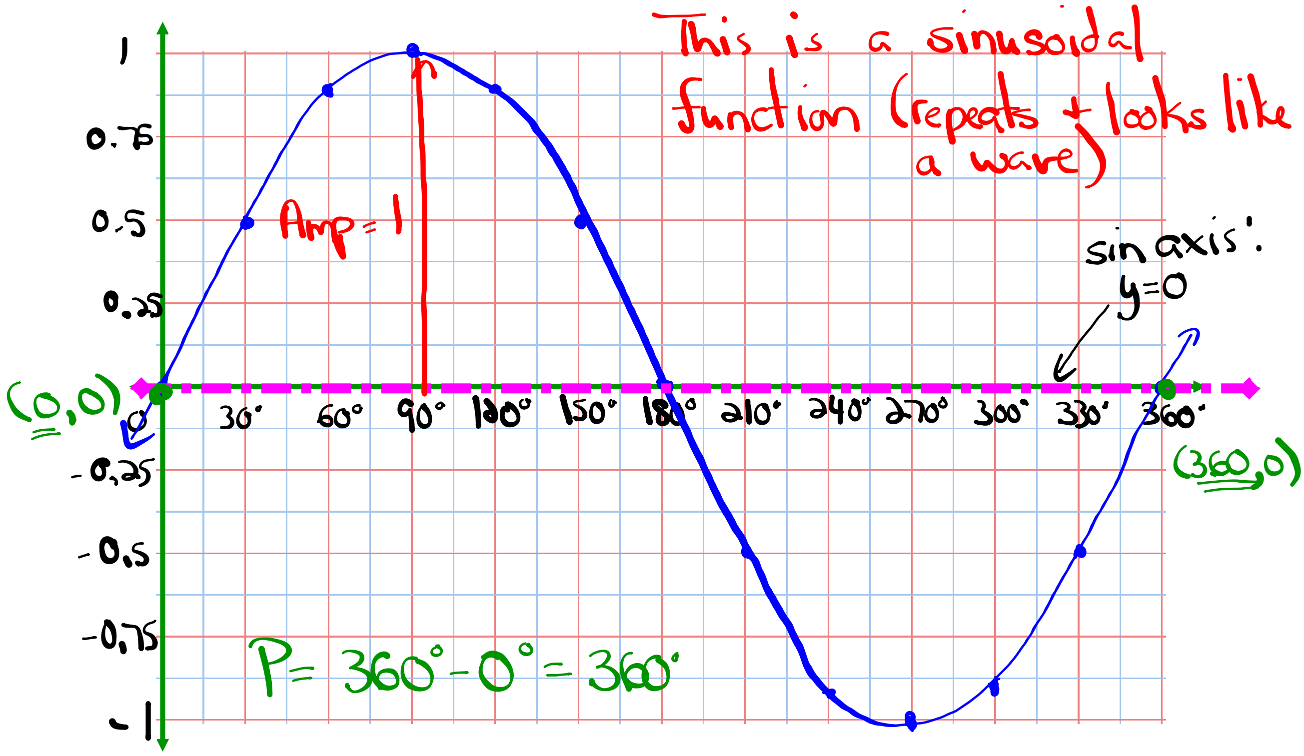


Let's examine the graph of $y = \sin\theta$

$y = \sin x$

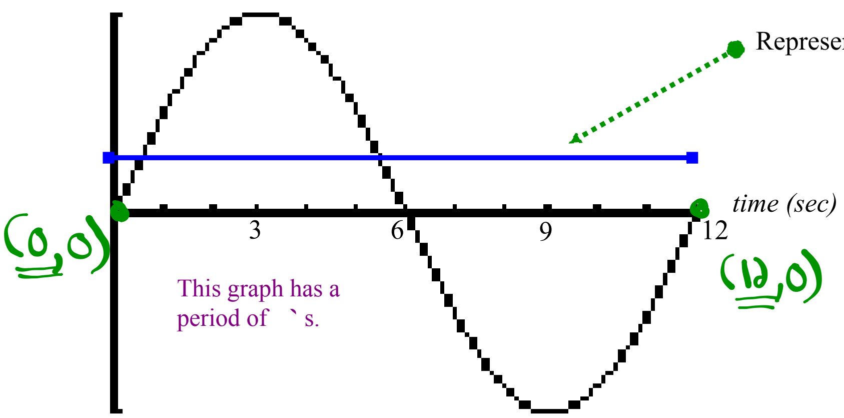
θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Now plot the above points...



Vocabulary of Sinusoidal Functions

I. Period:
The change in x corresponding to one cycle. (one repitition)



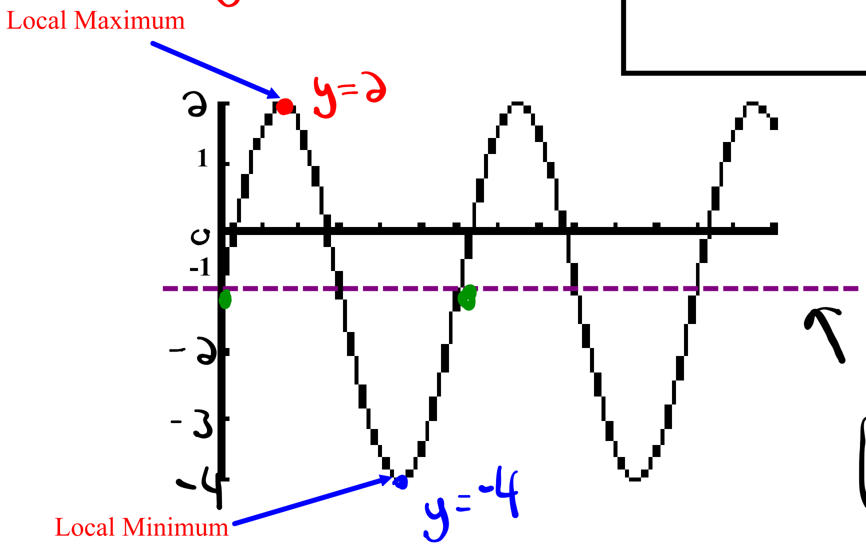
$P = 12 - 0 = 12$

II. Sinusoidal Axis:
The horizontal line halfway between the local maximum and local minimum.

highest (y) lowest (y)

$$\boxed{\text{Sinusoidal Axis} = \frac{\text{Max} + \text{Min}}{2}}$$

$$= \frac{2 + (-4)}{2} = \frac{-2}{2} = -1$$



Sinusoidal Axis

Equation of sinusoidal axis is $y = -1$

$\boxed{k = -1}$

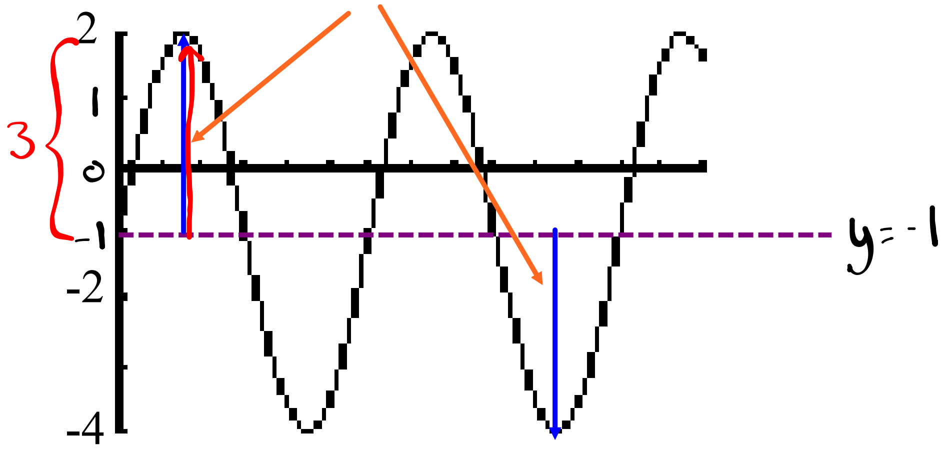
III. Amplitude:

The vertical distance from the sinusoidal axis to a local maximum or local minimum.

Amplitude = |a|

Amp is always positive

Amplitude would equal 3



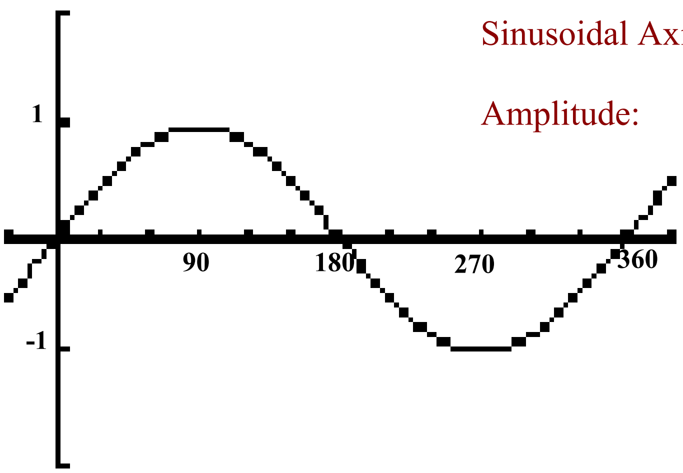
Summarize...

Here is the graph of $y = \sin\theta$

Period :

Sinusoidal Axis:

Amplitude:

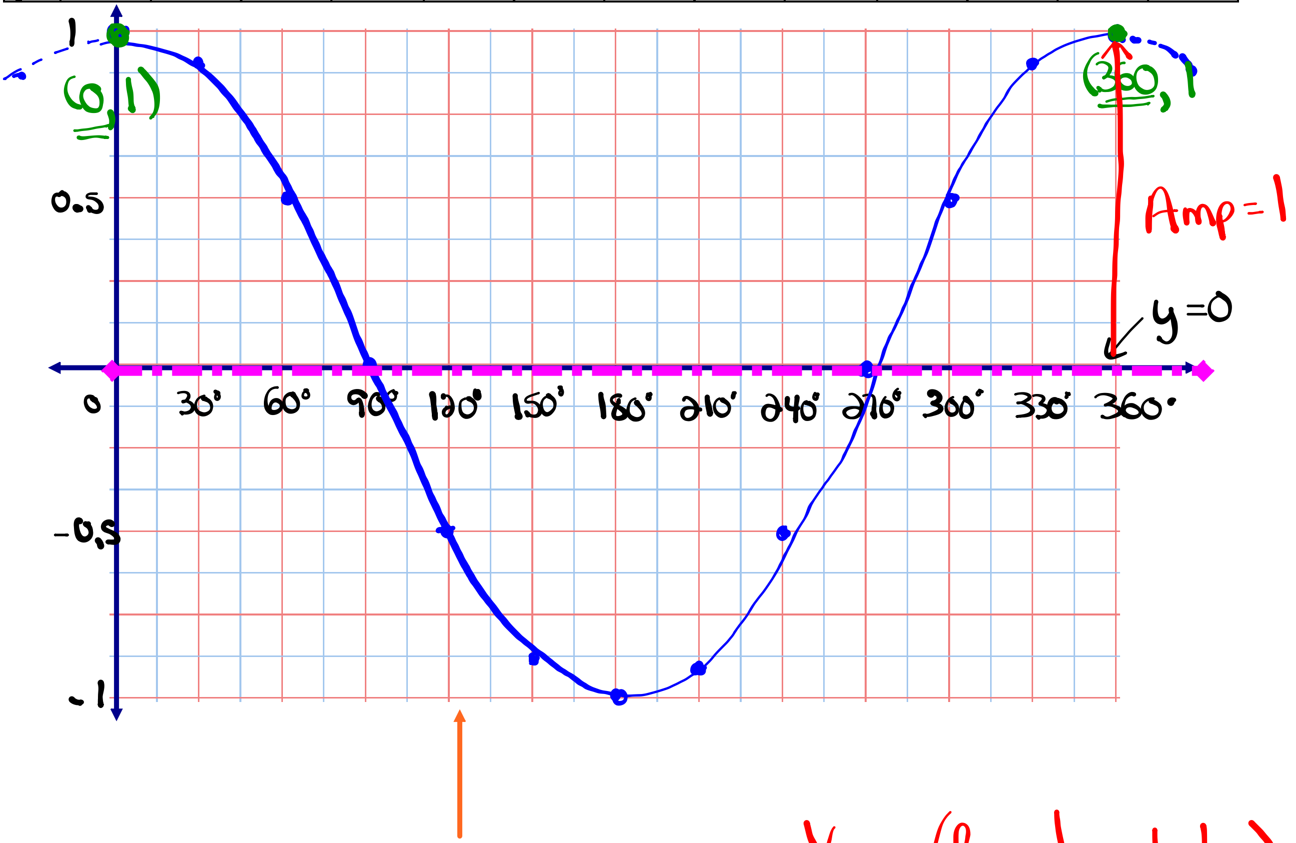


What about $y = \cos\theta$?

$y = \cos x$

Complete the table of values and sketch below

θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Is this a sinusoidal function? Yes (Repeats + looks like waves)

What about the period, sinusoidal axis, and amplitude?

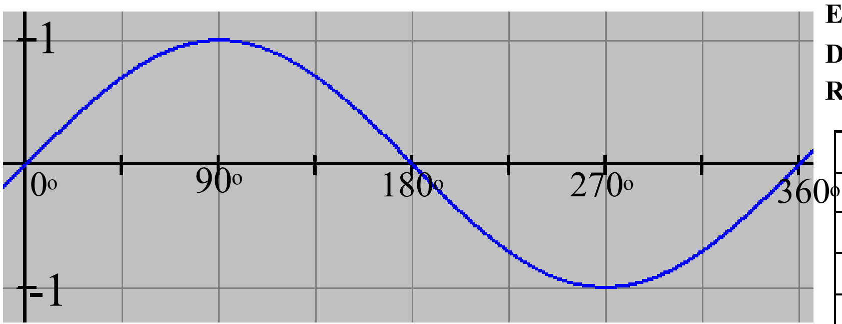
Period $= 360° - 0° = 360°$

sinusoidal axis $= \frac{Max + Min}{2} = \frac{1+(-1)}{2} = \frac{0}{2} = 0$ $(y=0)$

Amplitude $= 1$

Basic Trig Graphs

y = sin θ

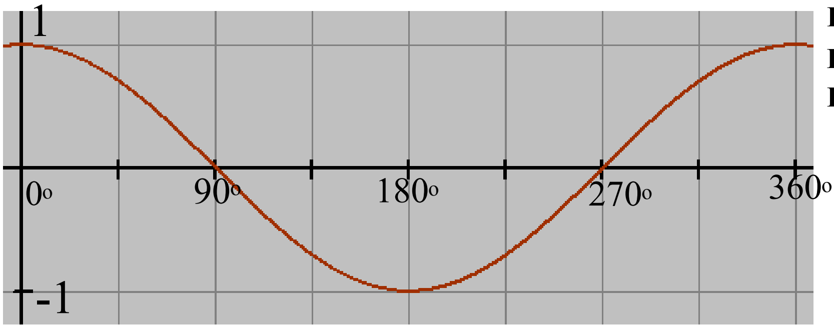


Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	0
90°	1
180°	0
270°	−1
360°	0

---

y = cos θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	1
90°	0
180°	−1
270°	0
360°	1

Transformations of the Sinusoidal Function

Recall...

$$y = -2(x-3)^2 + 4$$

reflection in the x-axis

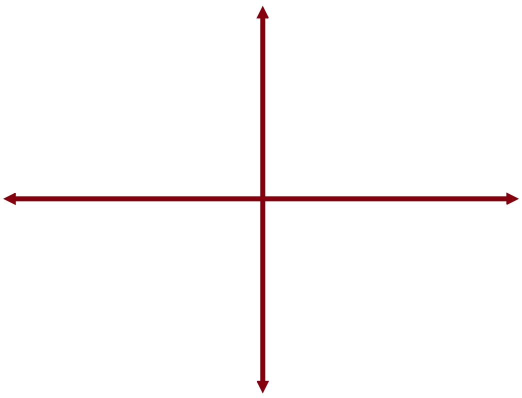
vertical stretch factor

horizontal translation

vertical translation

Vertex $\Rightarrow$

Sketch $\Rightarrow$



Now, let's look at a sinusoidal function...

$$y = \underline{-2}\,\sin\left[\underline{3}(\theta - \underline{60^\circ})\right] \underline{-1}$$

Reflection

Amplitude (v. stretch)

Horizontal Stretch

Vertical Translation

Equations in Standard Form

$$y = a\sin[b(x-c)]+d \quad \text{or} \quad y = a\cos[b(x-h)]+k$$

$|a|$ = ***Amplitude*** → influences how tall the sine curve is.

$b = \frac{360^\circ}{P}$ → influences how often the pattern repeats. $\left(P = \frac{360^\circ}{b}\right)$
where P is the period

h = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift. (Phase Shift)

- If h is positive → Shift Left
- If h is negative → Shift Right

} Inside Brackets

k = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If k is positive → Shift Up
- If k is negative → Shift Down
- equal to the sinusoidal axis:

Example:

$2y+5 = -6\sin\left(\frac{1}{3}x-30^\circ\right)-3$ (Subtract 5 from both sides)

$\frac{2y}{2} = \frac{-6\sin\left(\frac{1}{3}x-30^\circ\right)}{2}\frac{-8}{2}$ (Divide by 2)

$y = -3\sin\left(\frac{1}{3}x-30^\circ\right)-4$ (Factor out a $\frac{1}{3}$)

$y = -3\sin\left[\frac{1}{3}(x-90^\circ)\right]-4$

$a = -3$ $\quad b = \frac{1}{3}$ $\quad h = 90^\circ$ $\quad k = -4$

Amp $= 3$ $\quad P = \frac{360^\circ}{1/3} = 1080^\circ$ $\quad$ equation of sinusoidal axis: $y = -4$

3. $3 \cdot \frac{y+4}{3} = 3 \cdot -2\cos\left[3\theta - \frac{\pi}{2}\right] + 1 \cdot 3$ (Multiply/Divide only a + K)

$y+4 = -6\cos\left[3\theta - \frac{\pi}{2}\right] + 3$ $\quad -4 \quad -4$

$y = -6\cos\left[3\theta - \frac{\pi}{2}\right] - 1$

$y = -6\cos\left[3\left(\theta - \frac{\pi}{6}\right)\right] - 1$

$-\frac{\pi}{2} \div 3$

$-\frac{\pi}{2} \times \frac{1}{3} = -\frac{\pi}{6}$

$a = -6$ $\quad b = 3$ $\quad h = \frac{\pi}{6}$ $\quad K = -1$

$Amp = 6$ $\quad P = \frac{2\pi}{b}$ $\quad$ equation of sin axis: $y = -1$

$P = \frac{2\pi}{3}$

$(\theta, y) \rightarrow \left[\frac{1}{3}\theta + \frac{\pi}{6}, -6y - 1\right]$

$y = \cos\theta \longrightarrow$

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

θ	y
$\pi/6$	-7
$2\pi/6$	-1
$3\pi/6$	5
$4\pi/6$	-1
$5\pi/6$	-7

Homework

Page 233 #1-9

ex: $2y - 5 \overset{+5}{=} -4\cos[3x - 90^\circ] - 7^{\,+5}$

$$\frac{2y}{2} = \frac{-4}{2}\cos[3x - 90^\circ] \frac{-2}{2}$$

$$y = \underline{\underline{-2}}\cos[\underline{\underline{3}}(x - \underline{\underline{30^\circ}})] - \underline{\underline{1}}$$

$a = -2 \rightarrow$ Vertically stretched by a factor of 3 and reflected in the x-axis

$b = 3 \rightarrow$ horizontally stretched by a factor of $1/3$ and no reflection in the y-axis.

$h = 30^\circ \rightarrow$ Shift 30° right.

$K = -1 \rightarrow$ Shift 1 unit down.

Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $f(\theta) = a\sin k(\theta - c) + d$

1. Reflection: If a < 0 the graph will be reflected in the *x*-axis.
2. Amplitude: The amplitude of the graph will be equal to | a | .
3. Period: The period of the graph will be equal to $\frac{360°}{k}$
4. Horizontal Phase Shift: The graph will shift "c" units to the right. (Think Opposite)
5. Vertical Translation: The graph will shift "d" units up.

Mapping Notation: $(x, y) \rightarrow \left(\frac{1}{k}\theta + c, ay + d\right)$

Transformations of Sinusoidal Functions

Example: $f(\theta) = -2\sin 3(\theta + 30°) - 2$

Domain	
Range	
Reflection	
Amplitude	
Horizontal Phase Shift	
Vertical Translation	
Period	

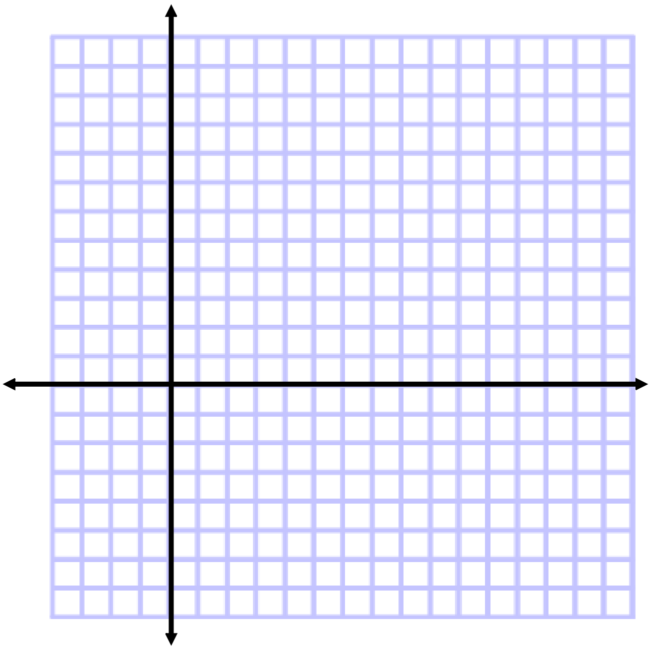
EXAMPLE #1

Now let's sketch a graph of $f(\theta) = -2\sin 3(\theta + 30°) - 2$

" THINK: RST "

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*



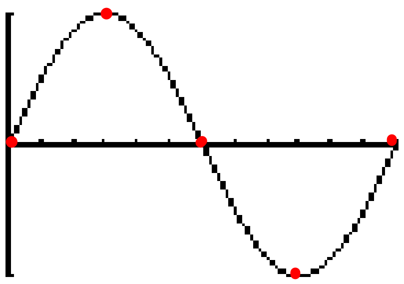
DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$f(\theta) = -2\sin 3(\theta + 30°) - 2$$

$y = \sin\theta$

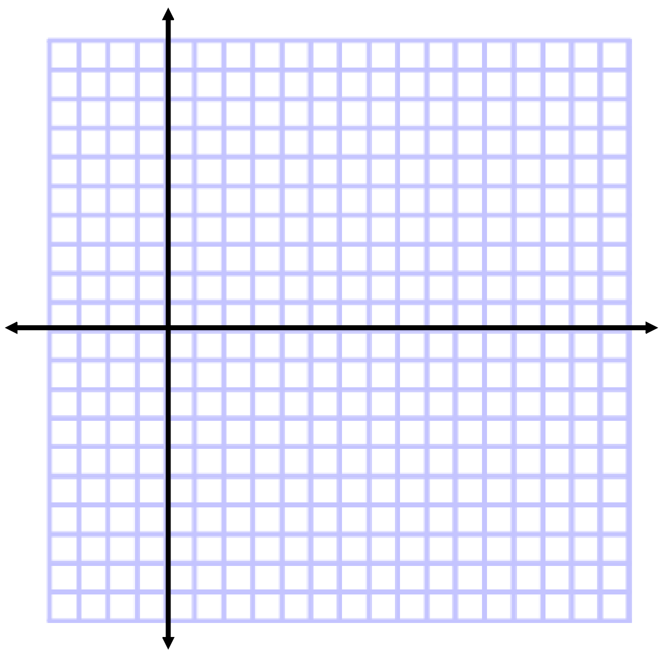


Mapping:

ϑ	y
0	
90	
180	
270	
360	

New points after mapping

ϑ	y



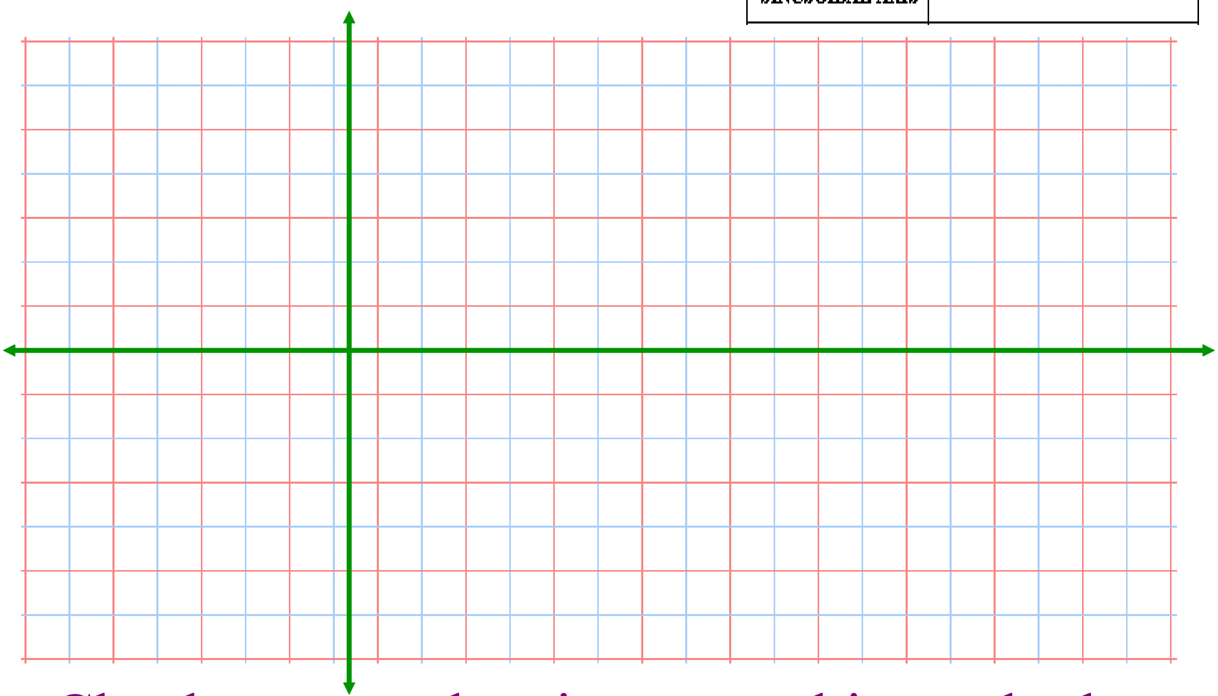
EXAMPLE #2

Now let's sketch a graph of $y = 3\cos\left[2(\theta - 135°)\right] + 2$

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	



Check our graph using a graphing calculator

Attachments

- worksheet-sketching in radian measure.doc
- Worksheet - Finding the Equation.doc
- Worksheet - Sketching Trigonometric Functions.doc
- Worksheet Solns - Sketching Sinusoidal Relations.doc
- Worksheet - Sketching Sinusoidal relations (sept06).pdf
- Bonus Soln - Fox Population.doc
- Worksheet Solns - Applications of Sinusoidal Relations.doc
- Review - Practice Test for Sinusoidal Functions.doc
- Review - Trigonometric Functions(3)(4).doc