Questions from Homework

⑥a) $f(x) = \sqrt{x} - \sqrt{1-x} = x^{1/2} - (1-x)^{1/2}$

$F(x) = \dfrac{x^{3/2}}{3/2} - \dfrac{(1-x)^{3/2}}{3/2}(-1) + C$

$F(x) = \dfrac{2}{3}x^{3/2} + \dfrac{2}{3}(1-x)^{3/2} + C$

⑥ b) $f(x) = \dfrac{1}{x} - \dfrac{1}{1-x}$

$F(x) = \ln|x| - \ln|1-x|(-1) + C$

$F(x) = \ln|x| + \ln|1-x| + C$

$F(x) = \ln|x-x^2| + C$

⑥ c) $f(x) = \dfrac{1}{\sqrt{1-x}} + \dfrac{1}{\sqrt{x}}$

$f(x) = (1-x)^{-1/2} + (x)^{-1/2}$

$F(x) = \dfrac{(1-x)^{1/2}}{1/2}(-1) + \dfrac{x^{1/2}}{1/2} + C$

$F(x) = -2\sqrt{1-x} + 2\sqrt{x} + C$ ✓

$F(x) = 2(\sqrt{x} - \sqrt{1-x}) + C$

Warm Up

Determine the general antiderivative of the following:

$$f(x) = 2x^2 - x + 7$$

$$F(x) = \frac{2x^3}{3} - \frac{1x^2}{2} + 7x + C$$

$$f(x) = \cos x - \sin x$$

$$F(x) = \frac{1}{1}\sin 1x - \frac{-1}{1}\cos 1x + C$$

$$F(x) = \sin x + \cos x + C$$

$$f(x) = -3e^{-x} + 6e^{2x}$$

$$F(x) = \frac{-3e^{-1x}}{-1} + \frac{6e^{2x}}{2} + C$$

$$F(x) = 3e^{-x} + 3e^{2x} + C$$

$$f(x) = \frac{2}{x^2} - \frac{5}{x} + x \quad = 2x^{-2} - \frac{5}{x} + x$$

$$F(x) = \frac{2x^{-1}}{-1} - 5\ln|x| + \frac{1}{2}x^2 + C$$

$$F(x) = \frac{-2}{x} - 5\ln x + \frac{1}{2}x^2 + C$$

Differential Equations

An equation that involves the derivative of a function is called a differential equation:

As discussed previously, in applications of calculus it is very common to have a situation where it is required to find a function, given knowledge about its derivatives.

Find all functions g such that: (general antiderivative) indefinite integral

$$g'(x) = 4\sin x - 3x^5 + 6\sqrt[4]{x^3}$$

$$g'(x) = 4\sin x - 3x^5 + 6x^{3/4}$$

$$g(x) = -\frac{4}{1}\cos 1x - \frac{3x^6}{6} + \frac{6x^{7/4}}{7/4} + C$$

$$6 \div \frac{7}{4}$$

$$6 \times \frac{4}{7} = \frac{24}{7}$$

$$g(x) = -4\cos x - \frac{1}{2}x^6 + \frac{24}{7}x^{7/4} + C$$

Identifying a unique solution for an antiderivative

Examples:
Determine the function with the given derivative whose graph satisfies the initial condition provided.

Find f if given f '(x): and f (0) = - 2

$$f'(x) = e^x + \frac{20}{1+x^2}$$

$f(x) = \frac{1}{1}e^{1x} + \frac{20}{1}\tan^{-1} 1x + C$

$f(x) = e^x + 20\tan^{-1}(x) + \underline{C}$

$-2 = e^0 + 20\tan^{-1}(0) + C$

$-2 = 1 + 20(0) + C$

$-2 = 1 + 0 + C$

$\underline{-3 = C}$

$\boxed{f(x) = e^x + 20\tan^{-1}(x) - 3}$

$$f(x) = e^x + 20\tan^{-1} x - 3$$

Find f if given f ''(x): and f (0) = 4, and f (1) = 1

x=0 y=4 x=1 y=1

$$f''(x) = 12x^2 + 6x - 4$$

$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - 4x + C$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + d$

$f(x) = x^4 + x^3 - 2x^2 + cx + d$

$4 = (0)^4 + (0)^3 - 2(0)^2 + c(0) + d$

$4 = d$

$f(x) = x^4 + x^3 - 2x^2 + cx + 4$

$1 = (1)^4 + (1)^3 - 2(1)^2 + c(1) + 4$

$1 = 1 + 1 - 2 + c + 4$

$1 = 4 + c$

$-3 = c$

$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$

$$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

Practice Problems...

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Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Let's look at the following:

$$f'(x) = (x^2 - 3)^5 (2x)$$

$$f'(x) = x^2 \sqrt{x^3 - 1}$$

$$f'(x) = \frac{3x}{\sqrt{1 - 5x^2}}$$

$$f'(x) = \frac{\cos 8x}{(1 + \sin 8x)^4}$$