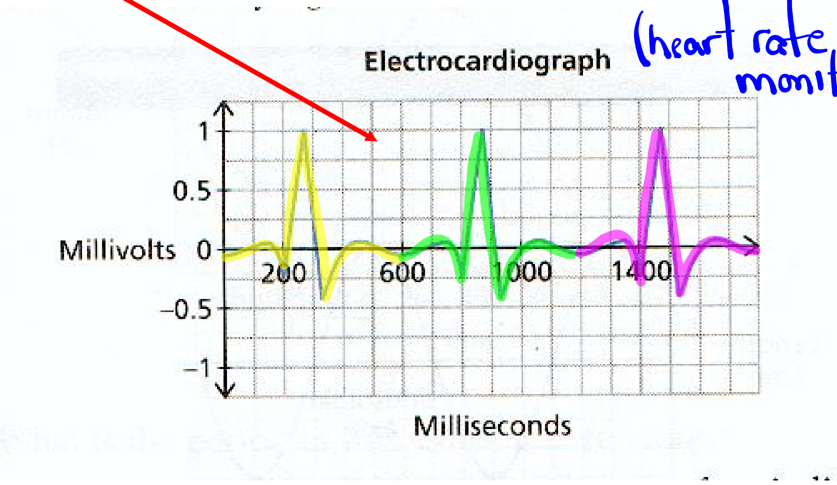
Sinusoidal Relations (Trig Graphs)

y = sin x
y = cos x

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(a function that repeats)

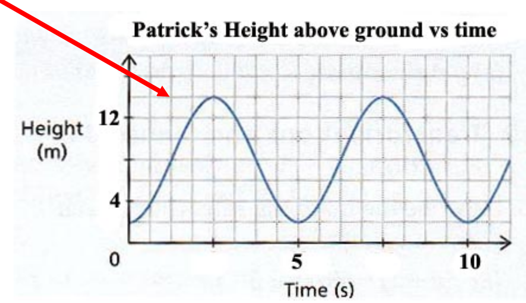
Example of periodic behavior



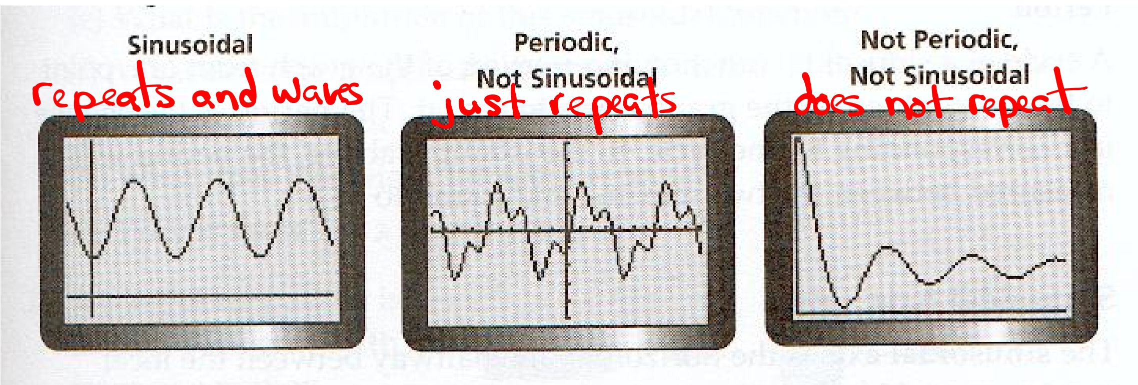
Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

(Repeats and looks like a smooth wave).

Example of sinusoidal behavior



These illustrations should summarize periodic and sinusoidal...

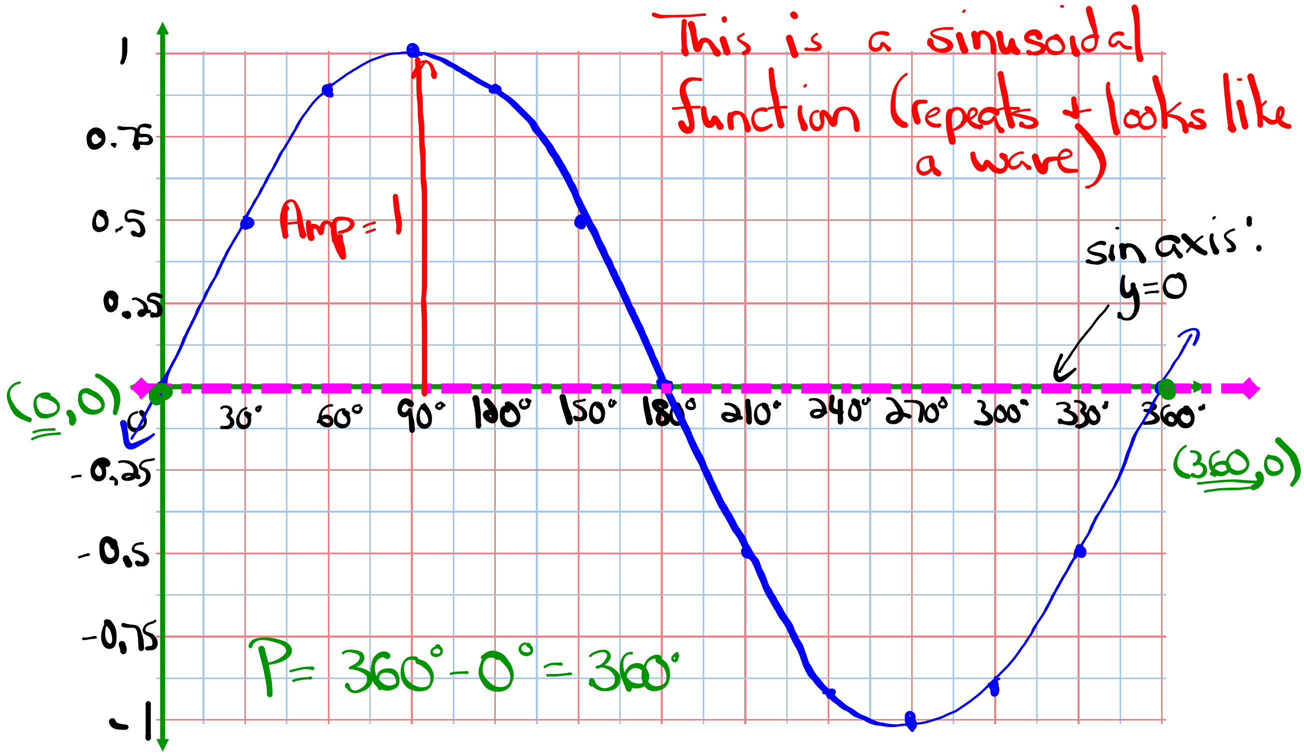


Let's examine the graph of $y = \sin\theta$

$y = \sin x$

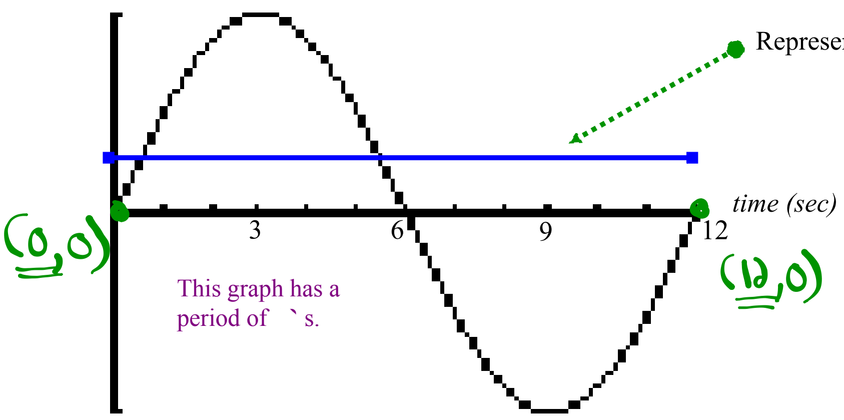
θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Now plot the above points...



Vocabulary of Sinusoidal Functions

I. Period:
The change in x corresponding to one cycle. (one repitition)



P=12-0=12

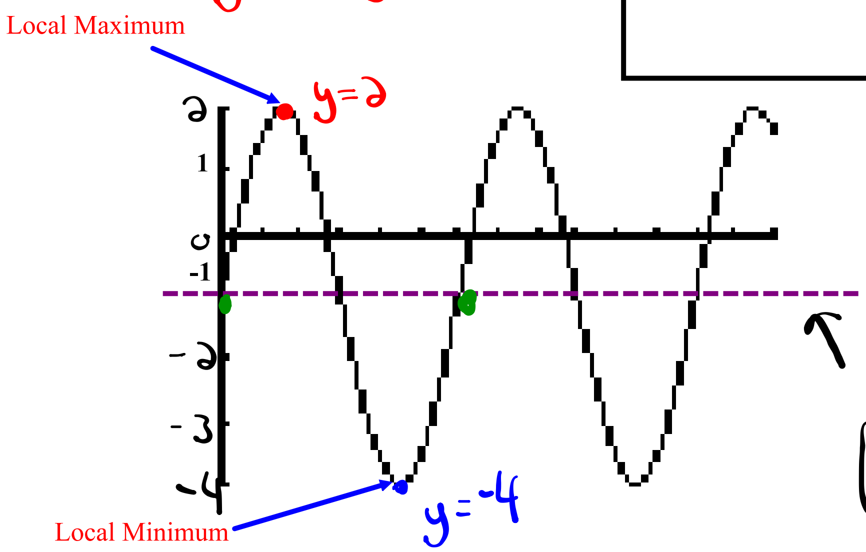
II. Sinusoidal Axis:
The horizontal line halfway between the local maximum and local minimum.

highest (y) lowest (y)

$$\boxed{\text{Sinusoidal Axis} = \frac{\text{Max} + \text{Min}}{2}}$$

$$= \frac{2 + (-4)}{2}$$

$$= \frac{-2}{2} = -1$$



Equation of sinusoidal axis is $y =$ -1

$\boxed{k = -1}$

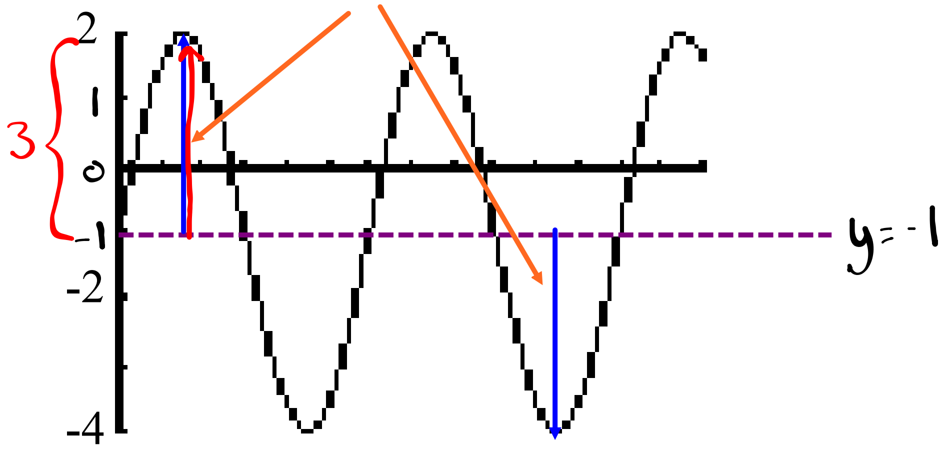
III. Amplitude:

The vertical distance from the sinusoidal axis to a local maximum or local minimum.

Amplitude = |a|

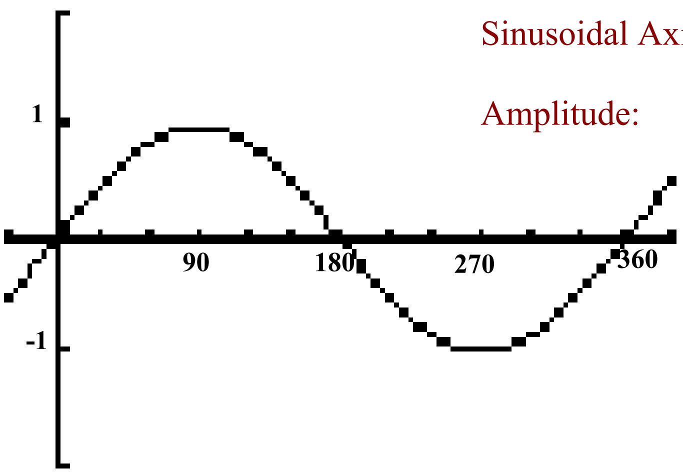
Amp is always positive

Amplitude would equal 3



Summarize...

Here is the graph of $y = \sin\theta$



Period :

Sinusoidal Axis:

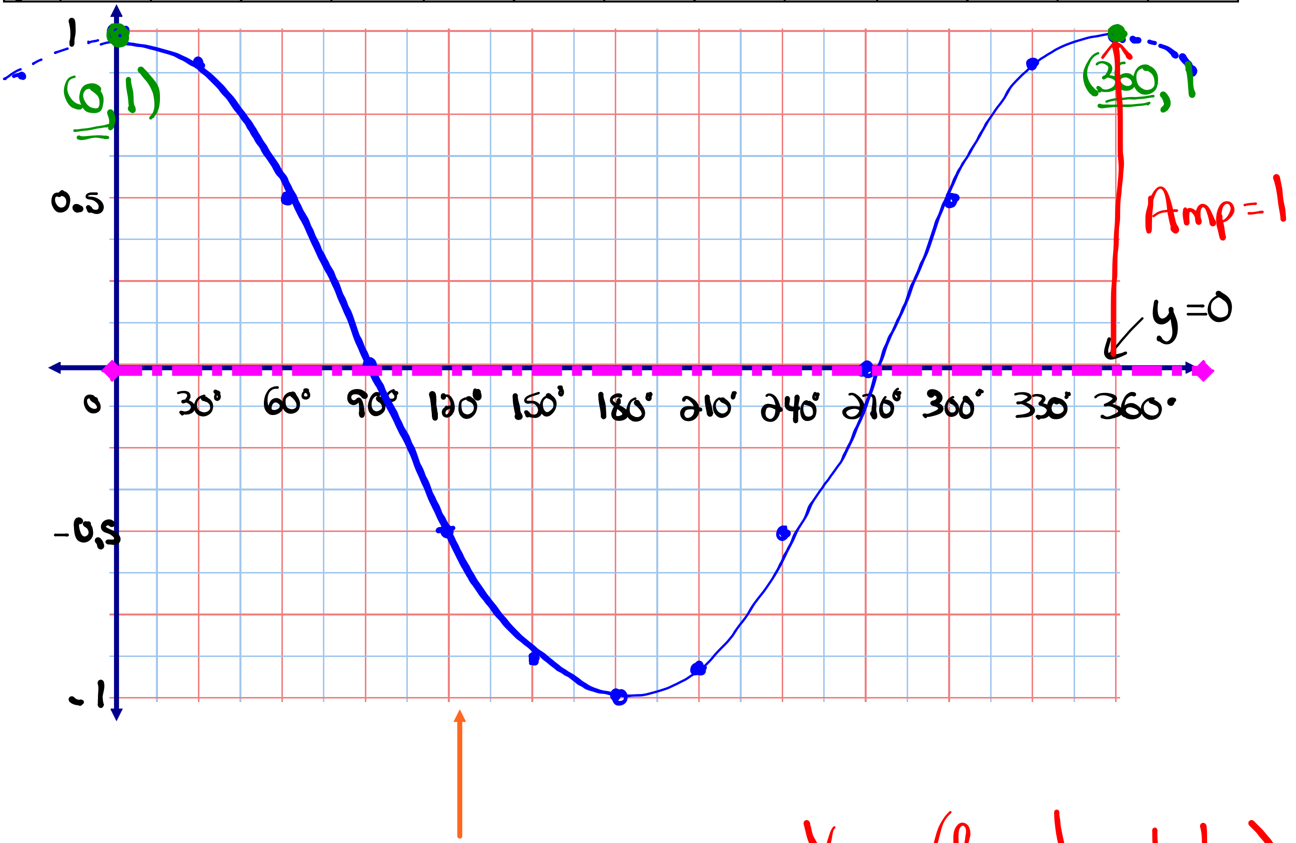
Amplitude:

What about $y = \cos\theta$?

$y = \cos x$

Complete the table of values and sketch below

θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Is this a sinusoidal function? Yes (Repeats + looks like waves)

What about the period, sinusoidal axis, and amplitude?

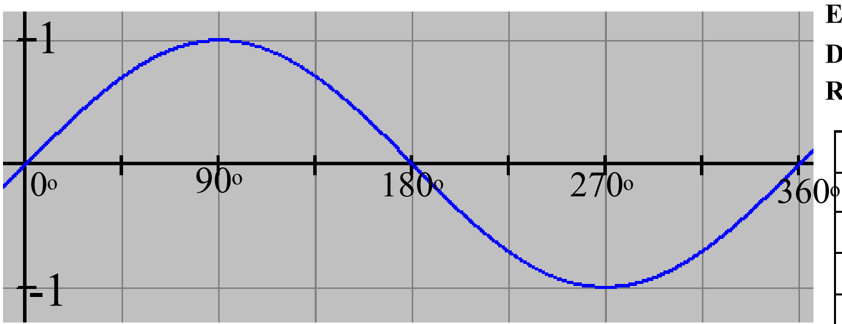
Period = $360° - 0° = 360°$

sinusoidal axis = $\frac{\text{Max} + \text{Min}}{2} = \frac{1 + (-1)}{2} = \frac{0}{2} = 0 \; (y = 0)$

Amplitude = 1

Basic Trig Graphs (Base Functions)

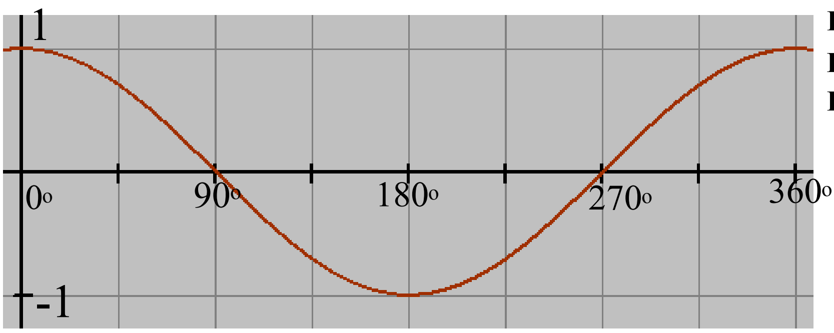
y = sin θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R}
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	0
90°	1
180°	0
270°	–1
360°	0

y = cos θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R}
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	1
90°	0
180°	–1
270°	0
360°	1

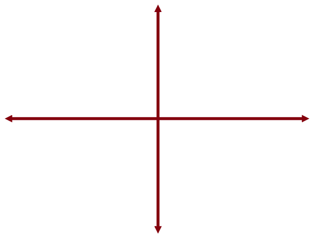
Transformations of the Sinusoidal Function

Recall...

$$y = -2(x-3)^2 + 4$$

Vertex $\Rightarrow$

Sketch $\Rightarrow$



Now, let's look at a sinusoidal function...

$$y = -2\sin\left[3(\theta - 60°)\right] - 1$$

$a = -2 \rightarrow$ reflected in the x-axis and vertically stretched by a factor of 2 (Amp = 2)

$b = 3 \rightarrow$ horizontally stretched by a factor of $\frac{1}{3}$.

$* \; P = \frac{360°}{b} = \frac{360°}{3} = 120°$

$h = 60° \rightarrow$ translated 60° right (Phase Shift)

$k = -1 \rightarrow$ " 1 unit down

$*$ Sin axis: $y = -1$

Mapping Rule: $(x, y) \rightarrow \left[\frac{1}{3}x + 60°, -2y - 1\right]$

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$\rightarrow$

x	y
60°	-1
90°	-3
120°	-1
150°	1
180°	-1

Equations in Standard Form

$$y = a\sin[b(x-c)]+d \quad \text{or} \quad y = a\cos[b(x-h)]+k$$

a = ***Amplitude*** → influences how tall the sine curve is. (always positive)

$b = \frac{360°}{P}$ → influences how often the pattern repeats. $\left(P = \frac{360°}{b}\right)$

Period

c = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift. (Phase Shift)

- If C is positive → Shift Left
- If C is negative → Shift Right

} Inside Brackets

d = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down
- equal to the sinusoidal axis:
  ↳ equation of sinusoidal axis: $y=d$

Example:

$2y+5 = -6\sin\left(\frac{1}{3}x-30°\right)-3$ (Subtract 5 from both sides)

$\frac{2y}{2} = \frac{-6\sin\left(\frac{1}{3}x-30°\right)}{2}-\frac{8}{2}$ (Divide by 2)

$y = -3\sin\left(\frac{1}{3}x-30°\right)-4$ (Factor out a $\frac{1}{3}$)

$y = -3\sin\left[\frac{1}{3}(x-90°)\right]-4$

$a=-3$ $b=\frac{1}{3}$ $h=90°$ $k=-4$

$Amp=3$ $P=\frac{360°}{\frac{1}{3}}=1080°$ equation of sin axis: $y=-4$

g) $y+5 = -2\sin\left(4x+\frac{\pi}{3}\right)$ -5 -5

$y = -2\sin\left(4x+\frac{\pi}{3}\right)-5$

$y = -2\sin\left[4\left(x+\frac{\pi}{12}\right)\right]-5$

$\frac{\pi}{3} \div 4$

$\frac{\pi}{3} \times \frac{1}{4} = \frac{\pi}{12}$

Homework

Page 233 #1-9

ex: $2y - 5 = -4\cos[3x - 90°] - 7$ $\quad +5 \quad +5$

$$\frac{2y}{2} = \frac{-4\cos[3x - 90°]}{2} \; \frac{-2}{2}$$

$$y = -2\cos[3x - 90°] - 1$$

$$y = -2\cos[3(x - 30°)] - 1$$

a = -2 (Amp = 2) Vertically stretched by a factor of 2 and reflected in x-axis

b = 3 horizontally stretched by a factor of $\frac{1}{3}$

h = 30° translated 30° right

K = -1 " 1 unit down

Questions from Homework

6. Match each function with its graph.

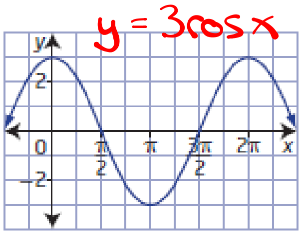
a) $y = 3\cos x$ $a = 3$

b) $y = \cos 3x$ $b = 3 \rightarrow b = \frac{2\pi}{3}$

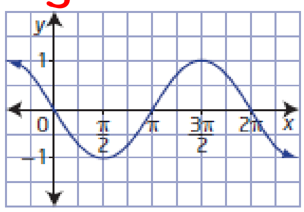
c) $y = -\sin x$ $a = -1$

d) $y = -\cos x$ $a = -1$

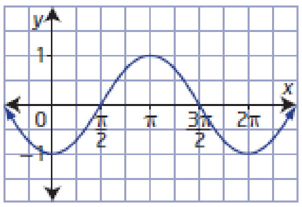
A $y = 3\cos x$



C $y = -\sin x$

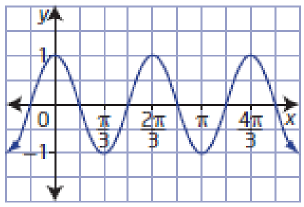


B



$y = -\cos x$

D



$y = \cos 3x$

From Sheet:

h) $\cancel{2}\cdot\frac{1}{\cancel{2}}(y+2) = 2\cdot 3\cos(x-90^\circ)$

$y+2 = 6\cos(x-90^\circ)$ -2 -2

$y = \underline{6}\cos(x-\underline{90^\circ}) - 2$

$a = 6$ $h = 90^\circ$ equation of sin axis: $y = -2$

$b = 1$ $k = -2$ $P = \frac{360^\circ}{b} = \frac{360^\circ}{1} = 360^\circ$

$y = a\cos[b(x-h)] + k$

① d) $y - 5 = 6\cos[\frac{1}{3}(x - \frac{\pi}{2})] - 2$

$y = 6\cos[\frac{1}{3}(x - \frac{\pi}{2})] + 3$

$a = 6$ $h = \frac{\pi}{2}$ equation of sin. axis: $y = 3$

$b = \frac{1}{3}$ $k = 3$ $P = \frac{2\pi}{b} = 2\pi \div \frac{1}{3} = 2\pi \cdot \frac{3}{1} = 6\pi$

g) $y + 5 = -2\sin(4x + \frac{\pi}{3})$

$y = -2\sin(4x + \frac{\pi}{3}) - 5$ (Factor out a 4)

$y = -2\sin[4(x + \frac{\pi}{12})] - 5$

$\frac{\pi}{3} \div 4$

$\frac{\pi}{3} \times \frac{1}{4} = \frac{\pi}{12}$

$a = -2$ $h = -\frac{\pi}{12}$ equation of sin. axis: $y = -5$

$b = 4$ $k = -5$ $P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

Standard Form $\longrightarrow y = a\sin\left[b(x-h)\right]+k$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.
2. Amplitude: The amplitude of the graph will be equal to | a | .
3. Period: The period of the graph will be equal to $\frac{360°}{b}$ or $\frac{2\pi}{b}$
4. Horizontal Phase Shift: The graph will shift h units to the right. (Think Opposite)
5. Vertical Translation: The graph will shift k units up.

Mapping Notation: $(x, y) \rightarrow \left(\frac{1}{b}x+h, ay+k\right)$

Transformations of Sinusoidal Functions

Example: $f(\theta) = -2\sin\left[3(\theta+30°)\right]-2$

$a=-2$ $b=3$ $h=-30°$ $k=-2$

$Amp = 2$ $P = \frac{360°}{3} = 120°$

$max = k + Amp = -2+2 = 0$

$min = k - Amp = -2-2 = -4$

Domain	$\{\theta \mid \theta \in R\}$
Range	$\{y \mid -4 \le y \le 0, y \in R\}$
Reflection	in the x-axis (a<0)
Amplitude	2
Horizontal Phase Shift	30° left
Vertical Translation	2 units down
Period	120°

EXAMPLE #1

Now let's sketch a graph of $y = 3\cos[2(\theta - 135°)] + 2$

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

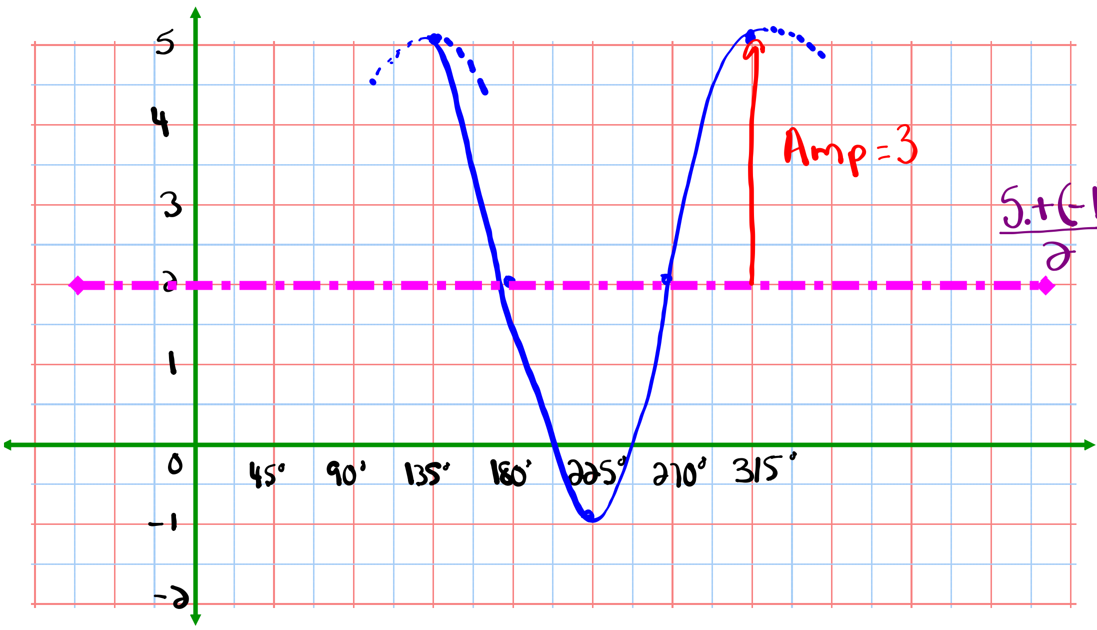
$a = 3$ $b = 2$ $h = 135°$ $k = 2$

$y = \cos x$ $(x, y) \rightarrow \left(\frac{1}{2}x + 135°, 3y + 2\right)$

θ	y
0°	1
90°	0
180°	−1
270°	0
360°	1

New points after mapping

θ	y
135°	5
180°	2
225°	−1
270°	2
315°	5



$\frac{5 + (-1)}{2} = 2$

DOMAIN	$\{x \mid x \in R\}$
RANGE	$\{y \mid -1 \leq y \leq 5, y \in R\}$
AMPLITUDE	3
PERIOD	$\frac{360°}{2} = 180°$
PHASE SHIFT	135° Right
VERTICAL TRANSLATION	2 Up
EQUATION OF SINUSOIDAL AXIS	$y = 2$

State ***a, b, h, k, and P*** from the following sinusoidal equations:

$$2y+6 = 4\sin(4x+\frac{\pi}{2})-2$$

$-6 \qquad -6$

$$\frac{2y}{2} = \frac{4\sin(4x+\frac{\pi}{2})}{2} - \frac{8}{2}$$

$$y = 2\sin(4x+\frac{\pi}{2}) - 4$$

Factor out the 4

$$y = 2\sin[4(x+\frac{\pi}{8})] - 4$$

$$\frac{\pi}{2} \div 4$$

$$\frac{\pi}{2} \times \frac{1}{4} = \frac{\pi}{8}$$

$a = 2 \quad b = 4 \quad h = -\frac{\pi}{8} \quad k = -4$

$\text{Amp} = 2 \quad P = \frac{2\pi}{4} = \frac{\pi}{2}$

sin axis: $y = -4$

Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6}{3}\cos(3x-\pi) - \frac{9}{3}$$

$$y = -2\cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

$a = -2$ $b = 3$ $h = \frac{\pi}{3}$ $k = -3$

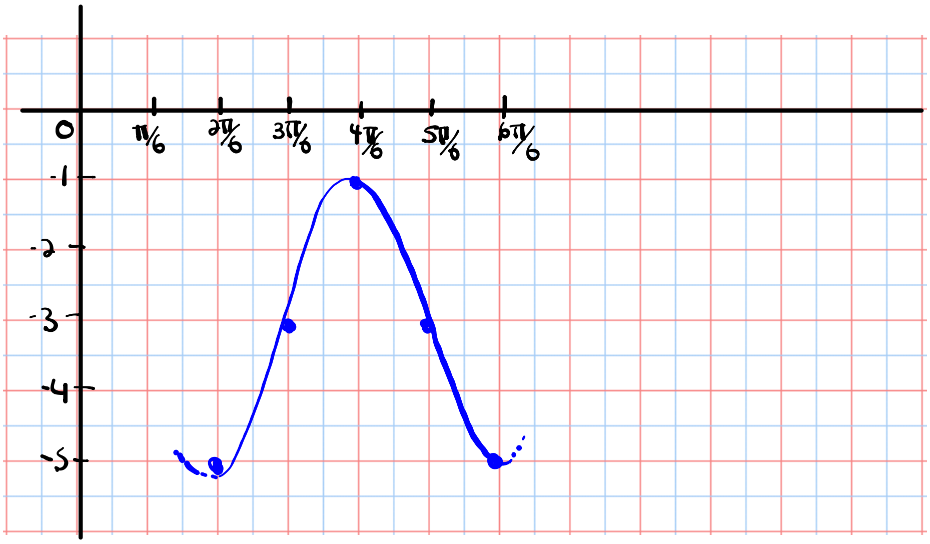
$y = \cos x$

$(x,y) \to \left[\frac{1}{3}x + \frac{\pi}{3}, -2y-3\right]$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

New points after mapping

	x	y
$2\pi/6$	$\pi/3$	-5
$3\pi/6$	$\pi/2$	-3
$4\pi/6$	$2\pi/3$	-1
$5\pi/6$	$5\pi/6$	-3
$6\pi/6$	π	-5



DOMAIN	$\{x \mid x \in R\}$
RANGE	$\{y \mid -5 \le y \le -1, y \in R\}$
AMPLITUDE	2
PERIOD	$\frac{2\pi}{3}$
PHASE SHIFT	$\pi/3$ right
VERTICAL TRANSLATION	3 down
EQUATION OF SINUSOIDAL AXIS	$y = -3$

Given the function $y = -2\sin(2\theta - 30°) + 3$

$y = -2\sin[2(\theta - 15°)] + 3$

$a = -2$ $b = 2$ $h = 15°$ $K = 3$

Remember...Put in standard form first!!

Complete the chart below and sketch the graph of this function.

Mapping:

$(x, y) \to \left(\frac{1}{2}x + 15, -2y + 3\right)$

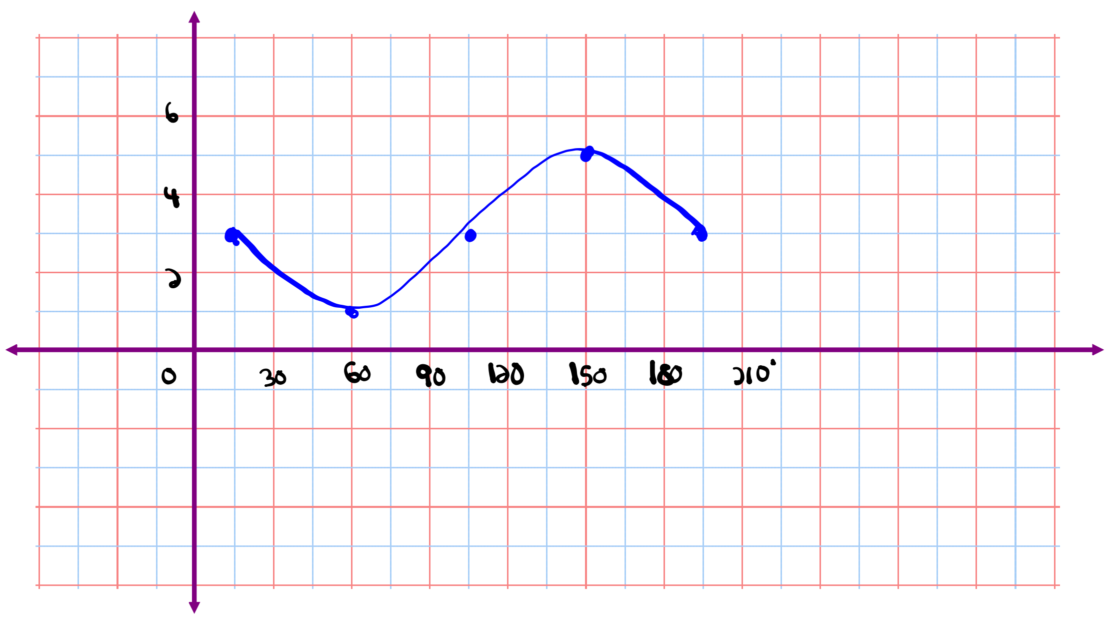
DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid 1 \leq y \leq 5, y \in R\}$
AMPLITUDE	2
PERIOD	$\frac{360°}{2} = 180°$
PHASE SHIFT	15° right
VERTICAL TRANSLATION	3 up
EQUATION OF SINUSOIDAL AXIS	$y = 3$

$y = \sin\theta$

θ	y
0	0
90	1
180	0
270	-1
360	0

New points after mapping →

θ	y
15°	3
60°	1
105°	3
150°	5
195°	3



Hopefully you are not too puzzled for this one...

$$2\cdot\frac{1}{2}(y+1)=2\cdot 3\cos\left(\frac{1}{2}\theta-90^\circ\right)+2\cdot 2$$

Remember...Put in standard form first!!

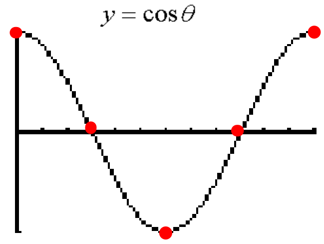
$$y+1-1=6\cos\left[\frac{1}{2}(\theta-180^\circ)\right]+4-1$$

$$y=6\cos\left[\frac{1}{2}(\theta-180^\circ)\right]+3$$

$90^\circ \div \frac{1}{2}$
$90^\circ \times 2$
180°

$a=6$ $b=\frac{1}{2}$ $h=180^\circ$ $k=3$

Remember what the graph of cosine looks like ??



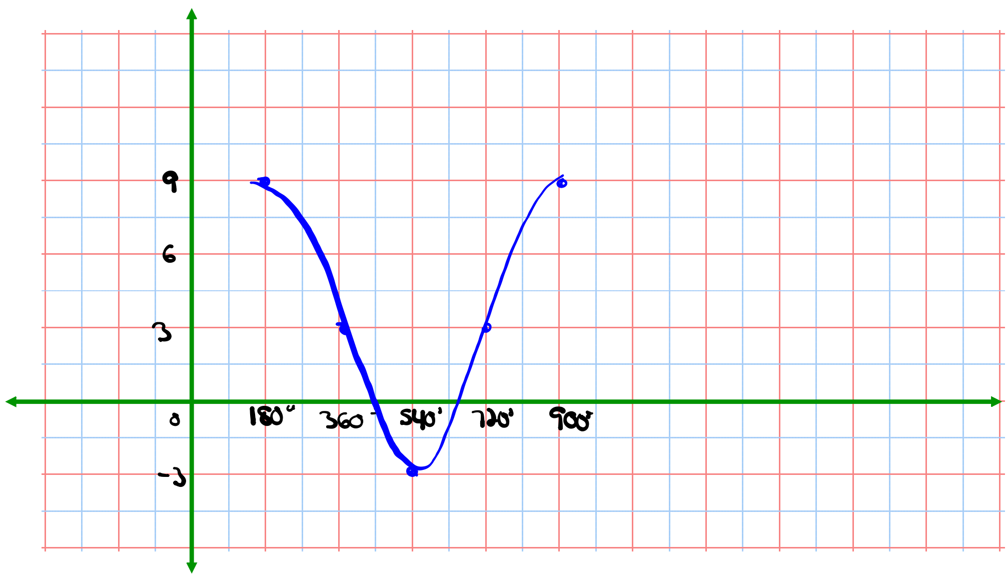
Mapping:
$(x,y)\rightarrow(2x+180^\circ,\ 6y+3)$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
180°	9
360°	3
540°	-3
720°	3
900°	9

DOMAIN	$\{x \mid x \in R\}$
RANGE	$\{y \mid -3 \le y \le 9, y \in R\}$
AMPLITUDE	6
PERIOD	$\frac{360^\circ}{1/2}=720^\circ$
PHASE SHIFT	180° right
VERTICAL TRANSLATION	3 up
EQUATION OF SINUSOIDAL AXIS	$y=3$



Example...

Graph the equation $y = -3\sin(2\theta + \pi) + 1$ using mapping notation.

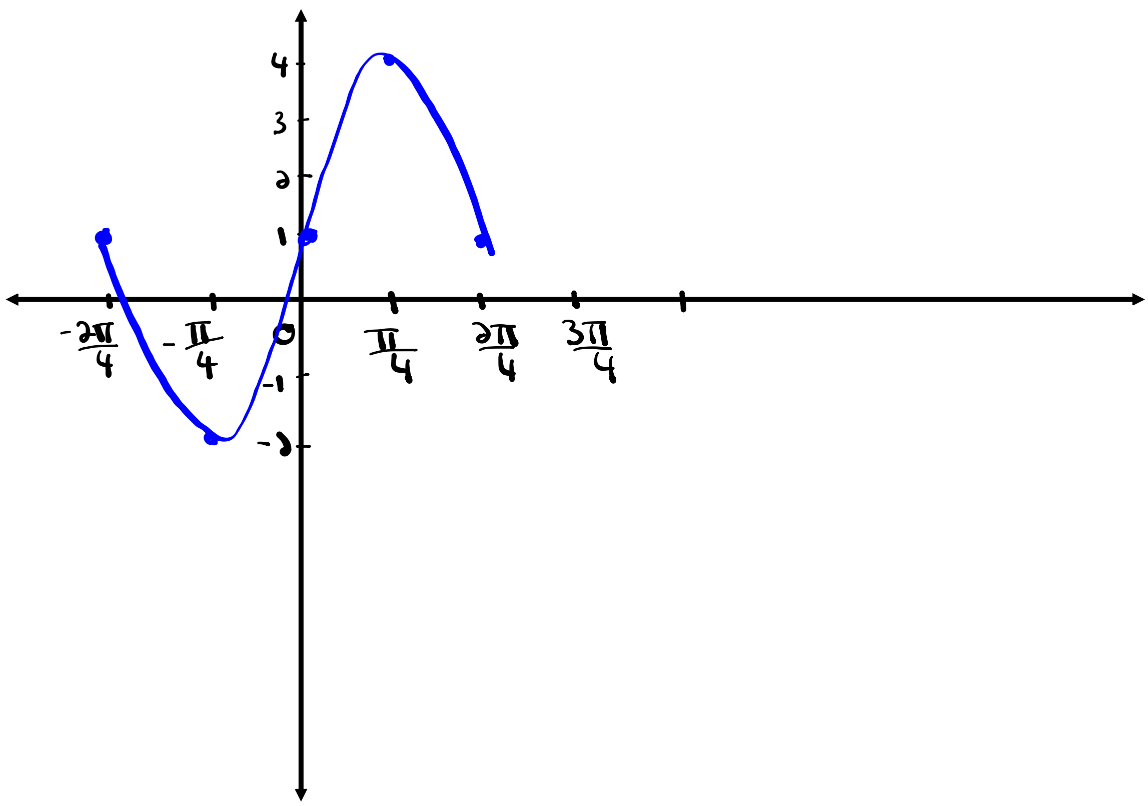
$y = -3\sin\left[2\left(\theta + \frac{\pi}{2}\right)\right] + 1$

$a = -3$ $b = 2$ $h = -\frac{\pi}{2}$ $k = 1$

$y = \sin x$ $(x, y) \rightarrow \left(\frac{1}{2}x - \frac{\pi}{2}, -3y + 1\right)$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

	x	y
$-\frac{2\pi}{4}$	$-\pi/2$	1
$-\frac{\pi}{4}$	$-\pi/4$	-2
$\frac{0\pi}{4}$	0	1
$\frac{\pi}{4}$	$\pi/4$	4
$\frac{2\pi}{4}$	$\pi/2$	1



Homework

Worksheet # 1 - 8

Worksheet - Sketching Sinusoidal Relations

Solutions to the homework

① $y = 3\cos(x) - 2$

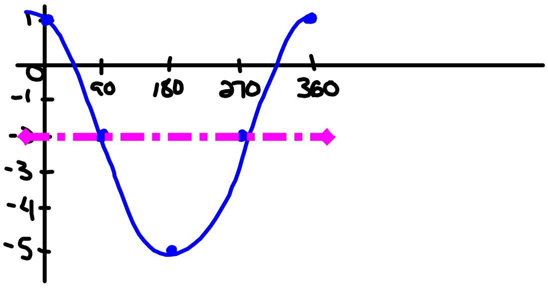
$A = 3 \quad b = 1 \quad C = 0 \quad D = -2 \quad P = 360^\circ$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\rightarrow$

x	y
0	1
90	-2
180	-5
270	-2
360	1



② $y = -\sin\left(2x - \frac{\pi}{6}\right)$

$y = -\sin\left[2\left(x - \frac{\pi}{12}\right)\right]$

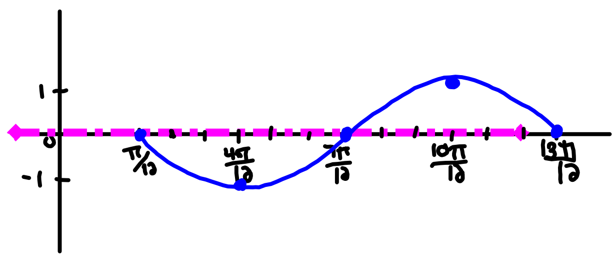
$A = 1 \quad b = 2 \quad C = \frac{\pi}{12} \quad D = 0 \quad P = \pi$

$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\rightarrow$

x	y
$\pi/12$	0
$\pi/3$	-1
$\frac{7\pi}{12}$	0
$5\pi/6$	1
$13\pi/12$	0



③ $y = 4\sin(3x - 180^\circ) + 2$

$y = 4\sin[3(x - 60^\circ)] + 2$

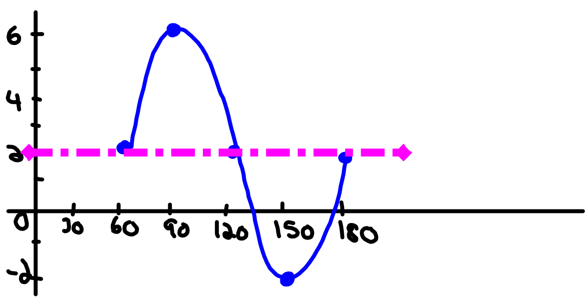
$A = 4 \quad b = 3 \quad c = 60 \quad D = 2 \quad P = 120^\circ$

$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

$\rightarrow$

x	y
60	2
90	6
120	2
150	-2
180	2



⑤ $2y+3 = -4\sin\left(4x-\frac{\pi}{3}\right)-3$

$2y = -4\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-6$

$y = -2\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-3$

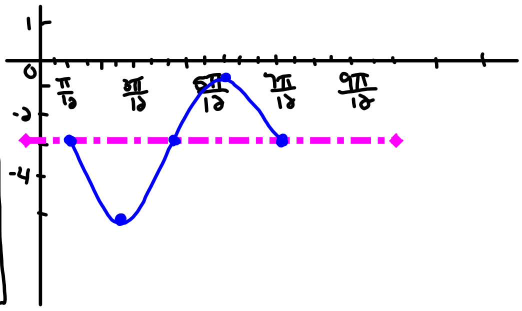
$A=2 \quad b=4 \quad C=\frac{\pi}{12} \quad D=-3 \quad P=\frac{\pi}{2}$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\rightarrow$

x	y
$\pi/12$	-3
$\frac{5\pi}{24}$	-5
$\pi/3$	-3
$11\pi/24$	-1
$\frac{7\pi}{12}$	-3



---

⑥ $\cancel{2}\ \frac{y-1}{\cancel{2}} = {}^{2\cdot}2\cos(\theta+45°) + {}^{2}0$

$y-1 = 4\cos(\theta+45°)+0\ {+1}$

$\boxed{y = 4\cos(\theta+45°)+1}$

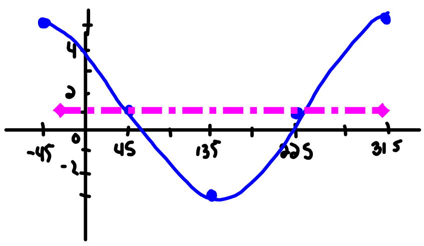
$A=4 \quad b=1 \quad C=-45 \quad D=1 \quad P=360$

$y=\cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\rightarrow$

x	y
-45	5
45	1
135	-3
225	1
315	5



---

① $\frac{1}{2}y-1 = \sin\left[2\left(x+\frac{\pi}{4}\right)\right]$

$2\cdot\ \frac{1}{2}y = 2\cdot\ \sin\left[2\left(x+\frac{\pi}{4}\right)\right]+1\cdot 2$

$y = 2\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+2$

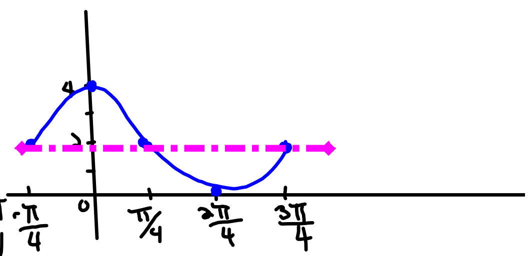
$A=2 \quad b=2 \quad C=-\frac{\pi}{4} \quad D=2 \quad P=\pi$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\rightarrow$

x	y
$-\pi/4$	2
0	4
$\pi/4$	2
$\pi/2$	0
$\frac{3\pi}{4}$	2



⑧ $y = -4\cos(3x + 90^\circ) - 2$

$y = -4\cos[3(x+30)] - 2$

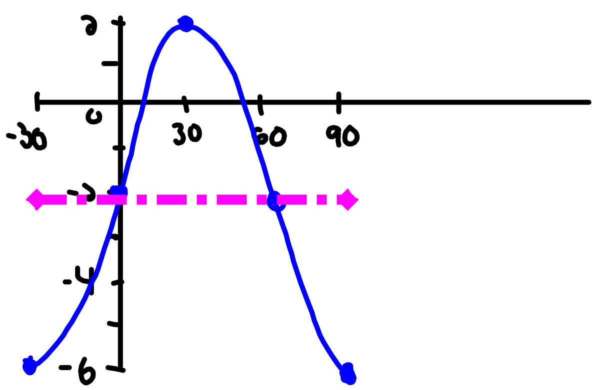
$A = 4 \quad b = 3 \quad c = -30 \quad D = -2 \quad P = 120$

$y = \cos x$

x	y
0	1
90	0
180	1
270	0
360	1

→

x	y
-30	-6
0	-2
30	2
60	-2
90	-6



Extra Practice

Worksheet - Sketching Trigonometric Functions.doc

Attachments

- worksheet-sketching in radian measure.doc
- Worksheet - Finding the Equation.doc
- Worksheet - Sketching Trigonometric Functions.doc
- Worksheet Solns - Sketching Sinusoidal Relations.doc
- Worksheet - Sketching Sinusoidal relations (sept06).pdf
- Bonus Soln - Fox Population.doc
- Worksheet Solns - Applications of Sinusoidal Relations.doc
- Review - Practice Test for Sinusoidal Functions.doc
- Review - Trigonometric Functions(3)(4).doc