Correct Homework Sheet

1 a) $f(x) = 3x^{-2} + 3x^3 + 1$

$f'(x) = -6x^{-3} + 9x^2$

b) $y = \dfrac{4x+1}{6-2x^5}$

$$y' = \frac{\overset{f'}{4}\overset{g}{(6-2x^5)} - \overset{g'}{(-10x^4)}\overset{f}{(4x+1)}}{(6-2x^5)^2}$$

c) $f(x) = 3(2x^5 + x - 5)^{10}$

$f'(x) = 30(2x^5 + x - 5)^9 (10x^4 + 1)$

d) $h(x) = (x^2 - x)\sqrt{4-9x} = (x^2-x)(4-9x)^{1/2}$

$$h'(x) = \overset{f'}{(2x-1)}\overset{g}{(4-9x)^{1/2}} + \overset{f}{(x^2-x)}\overbrace{\left(\tfrac{1}{2}\right)(4-9x)^{-1/2}(-9)}^{g'}$$

2 a) $y = \dfrac{2}{x} + \dfrac{3}{5x^3} - 6\sqrt{x} + \sqrt[3]{9x^8} - 8\pi$

$y = 2x^{-1} + \frac{3}{5}x^{-3} - 6x^{1/2} + (9x^8)^{1/3} - 8\pi$

$y' = -2x^{-2} - \frac{9}{5}x^{-4} - 3x^{-1/2} + \frac{1}{3}(9x^8)^{-2/3}(72x^7) - 0$

b) $y = \sqrt[3]{\dfrac{1-x^6}{2+(5x-1)^4}} = \left(\dfrac{1-x^6}{2+(5x-1)^4}\right)^{1/3}$

$$y' = \frac{1}{3}\left(\frac{1-x^6}{2+(5x-1)^4}\right)^{-2/3}\left[\frac{-6x^5(2+(5x-1)^4) - (1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2}\right]$$

c) $g(x) = (x-5)^3(7x^5+2x)^9(4-2x^3)^5$

$g'(x) = 3(x-5)^2(1)(7x^5+2x)^9(4-2x^3)^5 + 9(7x^5+2x)^8(35x^4+2)(x-5)^3(4-2x^3)^5 + 5(4-2x^3)^4(-6x^2)(x-5)^3$

$(7x^5+2x)^9$

2 d) $f(x) = \sqrt{25 + 4(2x-1)^4} = (25 + 4(2x-1)^4)^{1/2}$

$f'(x) = \frac{1}{2}(25 + 4(2x-1)^4)^{-1/2}(16(2x-1)^3(2))$

Correct Homework Sheet

3) a) $y=\sqrt{x^2-5x\sqrt{2x^3+3\sqrt{x}}} = \left[x^2-5x\left(2x^3+3x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$

$y' = \frac{1}{2}\left[x^2-5x\left(2x^3+3x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}\left(2x-\left[5\left(2x^3+3x^{\frac{1}{2}}\right)^{\frac{1}{2}}+5x\left(\frac{1}{2}\right)\left(2x^3+3x^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(6x^2+\frac{3}{2}x^{-\frac{1}{2}}\right)\right]\right)$

Correct Homework Sheet

③ b) $f(x) = \dfrac{8x^3(12x^2-5x)^8}{2-3\sqrt[5]{(1-32x^{10})}} = \dfrac{8x^3(12x^2-5x)^8}{2-3(1-32x^{10})^{1/5}}$

$$f'(x) = \frac{\left[24x^2(12x^2-5x)^8 + 8x^3(8)(12x^2-5x)^7(24x-5)\right]\left[2-3(1-32x^{10})^{1/5}\right] - \left[8x^3(12x^2-5x)^8\right]\left[-\frac{3}{5}(1-32x^{10})^{-4/5}(-320x^9)\right]}{\left[2-3(1-32x^{10})^{1/5}\right]^2}$$

Correct Homework Sheet

③ c) $$f(x) = \frac{\left[x^5 - x\sqrt{4-x^2}\right]^6}{12\sqrt{x}\,(5x^3-8)^7} = \frac{\left[x^5 - x(4-x^2)^{1/2}\right]^6}{12x^{1/2}(5x^3-8)^7} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$f'(x) = \frac{\overbrace{6\left[x^5 - x(4-x^2)^{1/2}\right]^5\left[5x^4 - \left(1(4-x^2)^{1/2} + x\left(\tfrac{1}{2}\right)(4-x^2)^{-1/2}(-2x)\right)\right]}^{f'(x)}\overbrace{\left[12x^{1/2}(5x^3-8)^7\right]}^{g(x)} - \overbrace{\left[x^5 - x(4-x^2)^{1/2}\right]^6}^{f(x)}\overbrace{\left[6x^{-1/2}(5x^3-8)^7 + 12x^{1/2}(7)(5x^3-8)^6(15x^2)\right]}^{g'(x)}}{\left[12x^{1/2}(5x^3-8)^7\right]^2}$$

To be handed in today

$$f(x)=\sqrt[7]{\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}} = \left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{1/7}$$

$$f'(x)=\frac{1}{7}\left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{-6/7}\left[\frac{64x^3\left[4x^5(3x^8+8x-2)\right]^5-(9+16x^4)(5)\left[4x^5(3x^8+8x-2)\right]^4\cdot\left[(20x^4)(3x^8+8x-2)+4x^5(24x^7+8)\right]}{\left[4x^5(3x^8+8x-2)\right]^{10}}\right]$$

Review

① a) $f(x) = \sqrt{x-5}$ $\quad f(x+h) = \sqrt{x+h-5}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \frac{(\sqrt{x+h-5} + \sqrt{x-5})}{(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x}+h\cancel{-5} - (\cancel{x}\cancel{-5})}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})} = \frac{1}{2\sqrt{x-5}}$$

② $f(x) = \dfrac{3}{\sqrt{x}} = \dfrac{3}{x^{1/2}} = 3x^{-1/2}$

$$f'(x) = -\frac{3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}} = \frac{-3}{2\sqrt{x^3}}$$

④ b) $y = \dfrac{\sqrt{x}}{3+x^2} = \dfrac{x^{1/2}}{3+x^2}$ $\quad \begin{matrix} f(x) \\ g(x) \end{matrix}$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - (2x)(x^{1/2})}{(3+x^2)^2}$$

$$y' = \frac{\frac{(3+x^2)}{\cancel{2x^{1/2}}}\cancel{2x^{1/2}} - \frac{2x^{3/2}}{1} \cdot 2x^{1/2}}{(3+x^2)^2 \; 2x^{1/2}} \qquad \text{CD: } 2x^{1/2}$$

$$y' = \frac{3+x^2-4x^2}{2\sqrt{x}\,(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

Review

① c) $y=\dfrac{(2x+1)^2}{(x^4-x+1)^2}\quad \begin{matrix} f(x) \\ g(x) \end{matrix}$

$$\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

$$y'=\frac{2(2x+1)^1(2)(x^4-x+1)^2-(2x+1)^2(2)(x^4-x+1)^1(4x^3-1)}{(x^4-x+1)^4}$$

$$y'=\frac{4(2x+1)(x^4-x+1)^2-2(4x^3-1)(2x+1)^2(x^4-x+1)}{(x^4-x+1)^4}$$

$$y'=\frac{2(2x+1)(x^4-x+1)\left[\overset{2x^4-2x+2}{2(x^4-x+1)}\overset{-}{-}\overset{(8x^4+4x^3-2x-1)}{(4x^3-1)(2x+1)}\right]}{(x^4-x+1)^4}$$

$$y'=\frac{2(2x+1)\cancel{(x^4-x+1)}(-6x^4-4x^3+3)}{(x^4-x+1)^{\cancel{4}\,3}}$$

Example 1

Let $F(x) = f(g(x))$ $F'(x) = f'(g(x)) \cdot g'(x)$

If $f(2) = 3$, $f'(2) = \underline{5}$, $g(1) = \underline{2}$ and $g'(1) = \underline{4}$ find $F'(1)$.

$$F'(1) = f'(\underline{g(1)}) \cdot g'(1)$$
$$= \underline{f'(2)} \cdot \underline{g'(1)}$$
$$= 5 \cdot 4$$
$$= 20$$

Question #6 ex. 2.4

Given
$f(2) = 3$
$f'(2) = 5$
$g(2) = -1$
$g'(2) = -4$

find $(fg)'(2) = f'(2)g(2) + f(2)g'(2)$
$$= (5)(-1) + (3)(-4)$$
$$= -5 - 12$$
$$= -17$$

find $\left(\frac{f}{g}\right)'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2}$
$$= \frac{(5)(-1) - (3)(-4)}{(-1)^2}$$
$$= \frac{-5 + 12}{1}$$
$$= 7$$

Example 2

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

$\frac{dy}{du} = 10u^9 + 5u^4$ $\frac{du}{dx} = -6x$

when $x = 1$
$u = 1 - 3(1)^2 = -2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\left.\frac{dy}{dx}\right]_{x=1} = (10u^9 + 5u^4)(-6x)$
$= (10(-2)^9 + 5(-2)^4)(-6(1))$
$= (-5120 + 80)(-6)$
$= (-5040)(-6)$
$= 30240$

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

$y = (1-3x^2)^{10} + (1-3x^2)^5 + 2$

$\frac{dy}{dx} = 10(1-3x^2)^9(-6x) + 5(1-3x^2)^4(-6x)$

$\left.\frac{dy}{dx}\right]_{x=1} = 10(-2)^9(-6) + 5(-2)^4(-6)$

$= 30720 - 480$

$= 30240$

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#3-10

3 Find $\left.\frac{dy}{dx}\right]_{x=4}$ if $y=u^2-2u^5$ and $u=x-\sqrt{x}$

$\frac{dy}{du}=2u-10u^4$

$\frac{du}{dx}=1-\frac{1}{2}x^{-1/2}$

$\frac{du}{dx}=1-\frac{1}{2\sqrt{x}}$

when $x=4$ $u=2$

$$\left.\frac{dy}{dx}\right]_{x=4}=\left[\frac{dy}{du}\right]\left[\frac{du}{dx}\right]$$

$$=\left[2u-10u^4\right]\left[1-\frac{1}{2\sqrt{x}}\right]$$

$$=\left[2(2)-10(2)^4\right]\left[1-\frac{1}{2\sqrt{4}}\right]$$

$$=(\overset{-39}{\cancel{-156}})\left(\frac{3}{\cancel{4}_1}\right)$$

$$=\boxed{-117}$$

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(4) Find $\frac{dy}{dt}\Big]_{t=1} \longrightarrow$ when $t=1$: $r=\frac{1+1}{2(1)+1}=\frac{2}{3}$

$y=\sqrt{1+r^2}=(1+r^2)^{1/2}$

$\frac{dy}{dr}=\frac{1}{2}(1+r^2)^{-1/2}(2r)$

$\frac{dy}{dr}=\frac{\cancel{2}r}{\cancel{2}(1+r^2)^{1/2}}=\boxed{\frac{r}{\sqrt{1+r^2}}}$

$r=\frac{t+1}{2t+1}$

$\frac{dr}{dt}=\frac{1(2t+1)-2(t+1)}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{2t+1-2t-2}{(2t+1)^2}=\boxed{\frac{-1}{(2t+1)^2}}$

$\frac{dy}{dt}\Big]_{t=1}=\left[\frac{r}{\sqrt{1+r^2}}\right]\left[\frac{-1}{(2t+1)^2}\right]$

$=\left[\frac{2/3}{\sqrt{1+(\frac{2}{3})^2}}\right]\left[\frac{-1}{(2(1)+1)^2}\right]$

$=\left[\frac{2/3}{\sqrt{1+\frac{4}{9}}}\right]\left[\frac{-1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{13/9}}\right]\left[-\frac{1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{13}/\sqrt{9}}\right]\left[-\frac{1}{9}\right]$

$=\left[\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\sqrt{13}}\right]\left[-\frac{1}{9}\right]$

$=\boxed{\frac{-2}{9\sqrt{13}}}$

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6 e) $F(x) = \dfrac{x}{\sqrt{2x+3}} = x(2x+3)^{-1/2}$

$F'(x) = 1(2x+3)^{-1/2} + x\left(-\frac{1}{2}\right)(2x+3)^{-3/2}(2)$

$F'(x) = (2x+3)^{-1/2} - x(2x+3)^{-3/2}$

$F'(x) = (2x+3)^{-3/2}\left[(2x+3) - x\right]$

$F'(x) = \dfrac{x+3}{(2x+3)^{3/2}} = \dfrac{x+3}{\sqrt{(2x+3)^3}}$

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9. $F(x) = f(g(x))$ ← Composite Function

Given:

$g(2) = 4$

$g'(2) = 3$

$f'(4) = 5$

$f(2) = -1$

$f'(2) = 2$

find $F'(2)$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'(g(2)) \cdot g'(2)$

$F'(2) = f'(4) \cdot g'(2)$

$F'(2) = 5 \cdot 3$

$F'(2) = 15$