

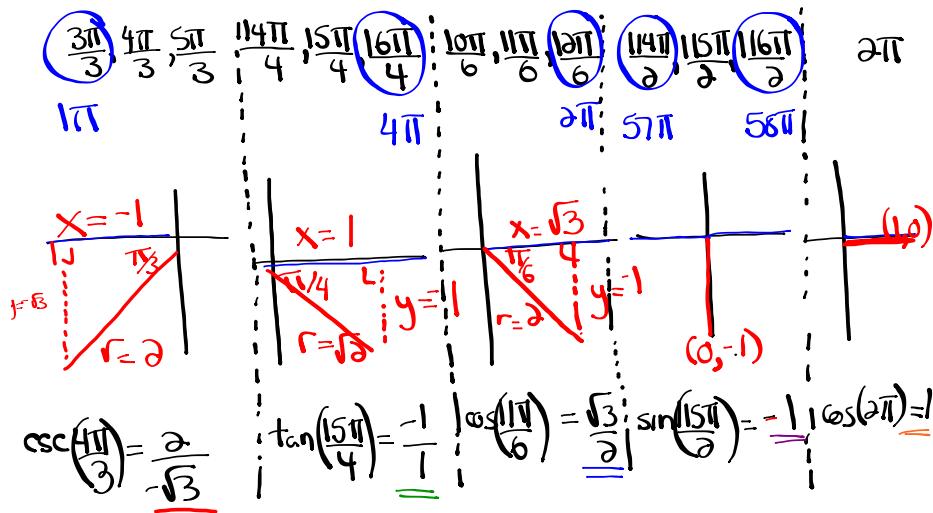
Questions from Homework

$$-\frac{13\pi}{6} + \frac{24\pi}{6} = \frac{11\pi}{6}$$

$$-14\pi + 16\pi = 2\pi$$

$$\textcircled{5} \quad \csc^2\left(\frac{4\pi}{3}\right) \tan\left(\frac{15\pi}{4}\right) + \cos\left(-\frac{13\pi}{6}\right) - \sin\left(\frac{115\pi}{2}\right) + \cos(-14\pi)$$

$$\csc^2\left(\frac{4\pi}{3}\right) \tan\left(\frac{15\pi}{4}\right) + \cos\left(\frac{11\pi}{6}\right) - \sin\left(\frac{115\pi}{2}\right) + \cos(2\pi)$$



$$\boxed{\csc^2\left(\frac{4\pi}{3}\right)} \boxed{\tan\left(\frac{15\pi}{4}\right)} + \boxed{\cos\left(\frac{11\pi}{6}\right)} - \boxed{\sin\left(\frac{115\pi}{2}\right)} + \boxed{\cos(2\pi)}$$

$$\left(-\frac{2}{\sqrt{3}}\right)^2 (-1) + \left(\frac{\sqrt{3}}{2}\right) - (-1) + (1)$$

$$\left(\frac{4}{3}\right)(-1) + \frac{\sqrt{3}}{2} + 1 + 1$$

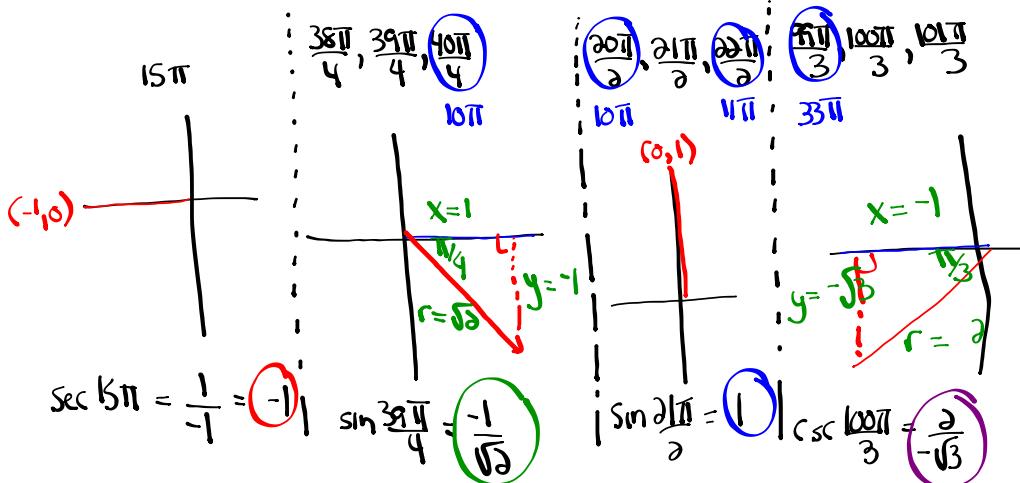
$$-\frac{4}{3} + \frac{\sqrt{3}}{2} + \frac{2}{1}$$

$$-\frac{8}{6} + \frac{3\sqrt{3}}{6} + \frac{12}{6}$$

$$\boxed{\frac{4+3\sqrt{3}}{6}} \quad \text{or} \quad \frac{3\sqrt{3}+4}{6}$$

Questions from Homework

$$\textcircled{6} \quad \sec 15\pi + \sqrt{2} \sin \frac{39\pi}{4} \sin \frac{21\pi}{\delta} - \csc^3 \frac{100\pi}{3}$$



$$\begin{aligned} & \underline{\sec 15\pi} + \sqrt{2} \sin \frac{39\pi}{4} \sin \frac{21\pi}{\delta} - \csc^3 \frac{100\pi}{3} \\ & (-1) + \sqrt{2} \left(-\frac{1}{\sqrt{2}}\right)(1) - \left(-\frac{2}{\sqrt{3}}\right)^3 \end{aligned}$$

$$-1 + \sqrt{2} \left(-\frac{1}{\sqrt{2}}\right)(1) - \frac{4}{3}$$

$$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$$

$$-1 - 1 - \frac{4}{3}$$

$$-\frac{3}{3} - \frac{3}{3} - \frac{4}{3}$$

$$\boxed{-\frac{10}{3}}$$

Questions from Homework

$$\textcircled{5} \quad \frac{\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{30\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(\frac{5\pi}{6}\right)}$$

$$\begin{array}{l|l} * \quad \begin{array}{l} -\frac{15\pi}{4} + \frac{4\pi}{1} \\ -\frac{15\pi}{4} + \frac{16\pi}{4} \\ \hline \frac{\pi}{4} \end{array} & \begin{array}{l} -\frac{31\pi}{6} + \frac{6\pi}{1} \\ -\frac{31\pi}{6} + \frac{36\pi}{6} \\ \hline \frac{5\pi}{6} \end{array} \end{array}$$

$$\textcircled{1} \quad \begin{array}{l} \textcircled{2\pi/4}, \boxed{\pi/4}, 2\pi/4 \\ 0 \\ \text{Diagram: } r = \sqrt{2}, j = 1, x = 1, \theta = \pi/4 \end{array}$$

$$\tan\frac{\pi}{4} = \frac{1}{1} = \underline{\underline{1}}$$

$$\textcircled{2} \quad \begin{array}{l} \textcircled{300\pi/3}, \boxed{30\pi/3}, 302\pi/3 \\ 100\pi \\ \text{Diagram: } r = 2, j = \sqrt{3}, x = 1, \theta = 30\pi/3 \end{array}$$

$$\sec\frac{30\pi}{3} = \frac{2}{1} = \underline{\underline{2}}$$

$$\textcircled{3} \quad \begin{array}{l} \textcircled{24\pi/2}, \frac{25\pi}{2}, \boxed{26\pi/2} \\ 12\pi \\ (5, 1) \\ \text{Diagram: } \theta = 24\pi/2 \end{array}$$

$$\csc\frac{25\pi}{2} = \frac{1}{1} = \underline{\underline{1}}$$

$$\textcircled{4} \quad \begin{array}{l} \textcircled{4\pi/6}, \boxed{5\pi/6}, \textcircled{6\pi/6} \\ \text{III} \\ \text{Diagram: } r = 2, j = -1, x = -\sqrt{3}, \theta = 5\pi/6 \end{array}$$

$$\cot\frac{5\pi}{6} = \frac{-\sqrt{3}}{1} = \underline{\underline{-\sqrt{3}}}$$

$$\frac{\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{30\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(\frac{5\pi}{6}\right)}$$

$$\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$(1-\sqrt{3})(1+\sqrt{3})$$

$$(1+\sqrt{3})^2$$

$$\frac{3+3\sqrt{3}}{1+3\sqrt{3}-3}$$

$$\frac{3+3\sqrt{3}}{-2} \quad \text{or} \quad \boxed{\frac{-3-3\sqrt{3}}{2}}$$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

Solve: $\sin \theta = 0.9659$, $-360^\circ < \theta < 720^\circ$

Domain in degrees

...Pre-Calculus 110

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$$\sin \theta = 0.9659$$

① Find $\bar{\theta}$

② Where is $\sin \theta > 0$

S A
T C

$$\bar{\theta} = \sin^{-1}(0.9659) \quad \text{③ Find } \theta$$

$$\bar{\theta} = 75^\circ$$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180^\circ - \bar{\theta}$
$\theta = 75^\circ$	$\theta = 180^\circ - 75^\circ = 105^\circ$
$\theta = 75^\circ - 360^\circ = -285^\circ$	$\theta = 105^\circ - 360^\circ = -255^\circ$
$\theta = 75^\circ + 360^\circ = 435^\circ$	$\theta = 105^\circ + 360^\circ = 465^\circ$

(Approx.)

Solve: $\sec \theta = -1.3054$, $-2\pi \leq \theta \leq 2\pi$ (Radians) $-6.28 \leq \theta \leq 6.28$

$$\cos \theta = \frac{1}{-1.3054}$$

$$\cos \theta = -0.7660$$

① Find $\bar{\theta}$

$$\bar{\theta} = \cos^{-1}(0.7660)$$

$$\bar{\theta} = 0.7 \text{ rads}$$

ignore negative

③ Find θ ② Where is $\cos \theta < 0$

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

Warm-up

(Approx.)

Ex': $\tan \theta = -0.8504$, $-360^\circ \leq \theta \leq 360^\circ$ (Degrees)

① Find $\bar{\theta}$

$$\bar{\theta} = \tan^{-1}(0.8504) \leftarrow \text{ignore negative}$$

$$\bar{\theta} = 40.4^\circ$$

② Where is $\tan \theta < 0$ ③ Find θ

Q2

$$\theta = 180^\circ - \bar{\theta}$$

$$\theta = 180^\circ - 40.4^\circ = 139.6^\circ$$

Q4

$$\theta = 360^\circ - \bar{\theta}$$

$$\theta = 360^\circ - 40.4^\circ = 319.6^\circ$$

$$\theta = 139.6^\circ - 360^\circ = -219.4^\circ \quad \theta = 319.6^\circ - 360^\circ = -40.4^\circ$$

(Exact)

$$-\frac{1}{\sqrt{2}} = -0.7071$$

Ex. $\sqrt{2} \cos \theta + 1 = 0, -360^\circ \leq \theta \leq 720^\circ$ (Degrees)

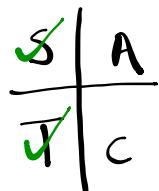
$$\frac{\cancel{\sqrt{2}} \cos \theta}{\cancel{\sqrt{2}}} = -\frac{1}{\sqrt{2}}$$

$$\cos \theta = -\frac{1}{\sqrt{2}}$$

① Find $\bar{\theta}$

$$\bar{\theta} = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) \quad (\text{Triangle } \#1) \quad \cos \theta = \frac{a}{h} = \frac{x}{r}$$

$$\bar{\theta} = 45^\circ$$

② Where is $\cos \theta < 0$ ③ Find θ

Q2	Q3
$\theta = 180^\circ - \bar{\theta}$	$\theta = 180^\circ + \bar{\theta}$
$\theta = 180^\circ - 45^\circ = 135^\circ$	$\theta = 180^\circ + 45^\circ = 225^\circ$
$\theta = 135^\circ - 360^\circ = -225^\circ$	$\theta = 225^\circ - 360^\circ = -135^\circ$
$\theta = 135^\circ + 360^\circ = 495^\circ$	$\theta = 225^\circ + 360^\circ = 585^\circ$

(Exact)

Ex. $\sin x + 1 = 0, -2\pi \leq x \leq 4\pi$ (Radians) $-\frac{4\pi}{\partial} \leq x \leq \frac{8\pi}{\partial}$

$$\sin x = -1$$

① Find x (Unit Circle)

$$x = \frac{3\pi}{\partial}$$

$$x = \frac{3\pi}{\partial} - \frac{2\pi}{1} = \frac{3\pi}{\partial} - \frac{4\pi}{\partial} = -\frac{\pi}{\partial}$$

$$x = \frac{3\pi}{\partial} + \frac{2\pi}{1} = \frac{3\pi}{\partial} + \frac{4\pi}{\partial} = \frac{7\pi}{\partial}$$

Your Turn

Solve each trigonometric equation in the specified domain.

- a) $3 \cos \theta - 1 = \cos \theta + 1, -2\pi \leq \theta \leq 2\pi$
 b) $4 \sec x + 8 = 0, 0^\circ \leq x < 360^\circ$

$$\text{(Exact)} \\ \text{a) } 3\cos\theta - 1 = \cos\theta + 1, \quad -2\pi \leq \theta \leq 2\pi \quad (\text{Radians})$$

$$3\cos\theta - \cos\theta = 1 + 1$$

$$\frac{2\cos\theta}{\theta} = \frac{2}{\theta}$$

$$\cos\theta = 1$$

① Find θ (unit circle)

$$\theta = 0, 2\pi$$

$$\theta = 0 - 2\pi = -2\pi$$

$$\text{b) } 4\sec x + 8 = 0, \quad 0^\circ \leq x < 360^\circ$$

$$4\sec x = -8$$

$$\sec x = -2$$

$$\cos x = -\frac{1}{2}$$

① Find \bar{x}

$$\bar{x} = \cos^{-1}\left(\frac{1}{2}\right) \quad (\text{Triangle #2})$$

$$\bar{x} = 60^\circ$$

② Where is $\cos x < 0$



③ Find x

$Q2$ $x = 180^\circ - \bar{x}$ $x = 180^\circ - 60^\circ = 120^\circ$	$Q3$ $x = 180^\circ + \bar{x}$ $x = 180^\circ + 60^\circ = 240^\circ$
---	---

Homework

Finish worksheet and Page 211 #1-5

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Worksheet Solutions

① $\sin \theta = -\frac{\sqrt{3}}{2}$ where is $\sin \theta < 0$ 

$$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = 180^\circ + 60^\circ = 240^\circ \quad \theta = 360^\circ - 60^\circ = 300^\circ$$

$$\bar{\theta} = 60^\circ \quad 240^\circ \pm 360^\circ n, \text{n} \in \mathbb{N} \quad 300^\circ \pm 360^\circ n, \text{n} \in \mathbb{N}$$

② $\cos \theta = -\frac{1}{2}$ where is $\cos \theta < 0$ 

$$\bar{\theta} = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 180^\circ - 60^\circ = 120^\circ \quad \theta = 180^\circ + 60^\circ = 240^\circ$$

$$\bar{\theta} = 60^\circ \quad 120^\circ \pm 360^\circ n, \text{n} \in \mathbb{N} \quad 240^\circ \pm 360^\circ n, \text{n} \in \mathbb{N}$$

③ $\tan \theta = -\frac{\sqrt{3}}{3}$ where is $\tan \theta < 0$ 

$$\tan \theta = -\frac{1}{\sqrt{3}}$$

$$\theta = 180^\circ - 30^\circ = 150^\circ \quad \theta = 360^\circ - 30^\circ = 330^\circ$$

$$\bar{\theta} = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) \quad 150^\circ \pm 360^\circ n, \text{n} \in \mathbb{N} \quad 330^\circ \pm 360^\circ n, \text{n} \in \mathbb{N}$$

$$\bar{\theta} = 30^\circ$$

④ $\tan \theta = \frac{\sqrt{3}}{1}$ where is $\tan \theta > 0$ 

$$\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$\theta = 60^\circ \quad \theta = 360^\circ - 60^\circ = 300^\circ$$

$$\bar{\theta} = 60^\circ \quad 60^\circ \pm 360^\circ n, \text{n} \in \mathbb{N} \quad 300^\circ \pm 360^\circ n, \text{n} \in \mathbb{N}$$

⑥ $\sin \theta = -1$

* Unit Circle

$$\theta = 270^\circ \rightarrow 270^\circ \pm 360^\circ n, \text{n} \in \mathbb{N}$$

⑤ $5 \sin \theta - 4 = 0$ where is $\sin \theta > 0$ 

$$\sin \theta = \frac{4}{5}$$

$$\sin \theta = 0.8 \quad (\text{approx. value})$$

$$\bar{\theta} = \sin^{-1}(0.8)$$

$$\bar{\theta} = 53.1^\circ$$

$$\theta = 53.1^\circ \quad \theta = 180^\circ - 53.1^\circ = 126.9^\circ$$

$$53.1^\circ \pm 360^\circ n, \text{n} \in \mathbb{N} \quad 126.9^\circ \pm 360^\circ n, \text{n} \in \mathbb{N}$$

Worksheet Solutions

$$\textcircled{8} \quad 2\sin\theta + 1 = 0$$

$$\frac{\partial \sin\theta}{\partial} = -\frac{1}{2}$$

$$\sin\theta = -\frac{1}{2}$$

\textcircled{9} Find $\bar{\theta}$

$$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right) \quad (\text{Triangle } \#3)$$

$$\bar{\theta} = 30^\circ$$

\textcircled{10} Where is $\sin\theta < 0$

$$\begin{array}{c} S \\ | \\ A \\ \hline T \\ | \\ C \end{array}$$

$$\textcircled{11} \quad \begin{array}{c|c} Q3 & Q4 \\ \hline \end{array}$$

$$\theta = 180^\circ + \bar{\theta}$$

$$\theta = 180^\circ + 30^\circ = 210^\circ$$

$$\theta = 360^\circ - \bar{\theta}$$

$$\theta = 360^\circ - 30^\circ = 330^\circ$$

$$210^\circ \pm 360^\circ n, n \in \mathbb{N}$$

$$330^\circ \pm 360^\circ n, n \in \mathbb{N}$$

$$\textcircled{12} \quad \begin{array}{c|c} \sqrt{3} & -2\sin\theta = 0 \\ \hline -2\sin\theta & = -\sqrt{3} \\ \hline \end{array}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

\textcircled{13} Find $\bar{\theta}$

$$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad (\text{Triangle } \#3)$$

$$\bar{\theta} = 60^\circ$$

\textcircled{14} Where is $\sin\theta > 0$

$$\begin{array}{c} S \\ | \\ A \\ \hline T \\ | \\ C \end{array}$$

$$\textcircled{15} \quad \begin{array}{c|c} Q1 & Q2 \\ \hline \end{array}$$

$$\theta = \bar{\theta}$$

$$\theta = 60^\circ$$

$$\theta = 180^\circ - \bar{\theta}$$

$$\theta = 180^\circ - 60^\circ = 120^\circ$$

$$60^\circ \pm 360^\circ n, n \in \mathbb{N}$$

$$120^\circ \pm 360^\circ n, n \in \mathbb{N}$$

$$\textcircled{16} \quad \begin{array}{c} \sqrt{3} \\ | \\ \tan^2\theta = 3 \\ \hline \end{array}$$

$$\tan\theta = \pm\sqrt{3}$$

\textcircled{17} Find $\bar{\theta}$

$$\bar{\theta} = \tan^{-1}(\sqrt{3}) \quad (\text{Triangle } \#3)$$

$$\bar{\theta} = 60^\circ$$

\textcircled{18} Where is $\tan\theta$ positive/negative

$$\begin{array}{c} S \\ | \\ A \\ \hline T \\ | \\ C \end{array}$$

$$\textcircled{19} \quad \begin{array}{c|c|c|c} Q1 & Q2 & Q3 & Q4 \\ \hline \theta = \bar{\theta} & \theta = 180^\circ - \bar{\theta} & \theta = 180^\circ + \bar{\theta} & \theta = 360^\circ - \bar{\theta} \\ \hline \theta = 60^\circ & \theta = 120^\circ & \theta = 240^\circ & \theta = 300^\circ \\ \hline 60^\circ \pm 360^\circ n & 120^\circ \pm 360^\circ n & 240^\circ \pm 360^\circ n & 300^\circ \pm 360^\circ n \end{array}$$

$$\theta = \bar{\theta}$$

$$\theta = 60^\circ$$

$$\theta = 180^\circ - \bar{\theta}$$

$$\theta = 120^\circ$$

$$\theta = 360^\circ - \bar{\theta}$$

$$\theta = 300^\circ$$

$$\textcircled{20} \quad 5\sin\theta - 4 = 0$$

$$\frac{5\sin\theta - 4}{5} = 0$$

$$\sin\theta = \frac{4}{5} = 0.8 \quad (\text{approx})$$

\textcircled{21} Find $\bar{\theta}$

$$\bar{\theta} = \sin^{-1}(0.8)$$

$$\bar{\theta} = 53.1^\circ$$

\textcircled{22} Where is $\sin\theta > 0$

$$\begin{array}{c} S \\ | \\ A \\ \hline T \\ | \\ C \end{array}$$

$$\textcircled{23} \quad \begin{array}{c|c} Q1 & Q2 \\ \hline \theta = \bar{\theta} & \theta = 180^\circ - \bar{\theta} \\ \hline \theta = 53.1^\circ & \theta = 106.9^\circ \\ \hline 53.1^\circ \pm 360^\circ n, n \in \mathbb{N} & 106.9^\circ \pm 360^\circ n, n \in \mathbb{N} \end{array}$$

$$\theta = \bar{\theta}$$

$$\theta = 53.1^\circ$$

$$\theta = 180^\circ - \bar{\theta}$$

$$\theta = 106.9^\circ$$

Worksheet Solutions

Back side of sheet $0 \leq \theta \leq 2\pi$

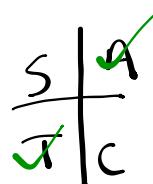
$$\textcircled{3} \quad \tan \theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

\textcircled{1} Find $\bar{\theta}$

$$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ (triangle #4)}$$

$$\bar{\theta} = \frac{\pi}{6}$$

\textcircled{2} Where is $\tan \theta > 0$



\textcircled{3} Find θ

$$\begin{array}{c} \text{Q1} \\ \hline \theta = \bar{\theta} \end{array}$$

$$\boxed{\theta = \frac{\pi}{6}}$$

$$\begin{array}{c} \text{Q3} \\ \hline \theta = \pi + \bar{\theta} \end{array}$$

$$\theta = \pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \boxed{\frac{7\pi}{6}}$$

Check-Up:

Solve:

$$\textcircled{1} \cot \theta = 0.7834, \quad -\frac{\pi}{2} < \theta < -\pi$$

$$\textcircled{2} 3\cos x + 5 = 6, \quad -360^\circ \leq x \leq 720^\circ$$

$$\textcircled{3} 2\csc x (1 - \csc x) = 0, \quad -4\pi < x < 4\pi$$

Factoring trinomials:

① Hard Trinomial

$$\underline{2x^2} + \underline{7x} + \underline{6}$$

$$\underline{3} \times \underline{4} = 12$$

$$(x\underline{+3}) (x\underline{+4})$$

$$\underline{3} + \underline{4} = 7$$

$$(2x+3)(x+2)$$

② Simple trinomial

$$\underline{x^2} + \underline{7x} + \underline{6}$$

$$\underline{6} \times \underline{1} = 6$$

$$(x+1)(x+6)$$

$$\underline{6} + \underline{1} = 7$$

③ Difference of Squares

$$x^2 - 16$$

$$(x-4)(x+4)$$

④ Common Factor

$$3x^3 - 9x^2 + 21x$$

$$3x(x-3x^2 + 7)$$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations? (Factor)
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2\sin^2 x + \sin x - 1 = 0$, $0 \leq x \leq 4\pi$ (Radians)

$$\begin{aligned} 2\sin^2 x + \sin x - 1 &= 0 \\ (\sin x - 1)(\sin x + 1) &= 0 \\ (\cancel{\sin x} - 1)(\sin x + 1) &= 0 \end{aligned}$$

Hard Trinomial
 $\underline{-1} + \underline{1} = \underline{0}$
 $\underline{-1} \times \underline{1} = -\underline{1} \quad (a \cdot b)$

$$2\sin x - 1 = 0$$

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

① Find \bar{x} :

$$\bar{x} = \sin^{-1}\left(\frac{1}{2}\right) \text{ (triangle #4)}$$

$$\bar{x} = \frac{\pi}{6}$$

② where is $\sin x > 0$



$$\sin x + 1 = 0$$

$$\sin x = -1 \text{ (Unit Circle)}$$

① Find x :

$$x = \boxed{\frac{3\pi}{2}}$$

$$x = \frac{3\pi}{2} + 2\pi$$

$$x = \frac{3\pi}{2} + \frac{4\pi}{6} = \boxed{\frac{11\pi}{6}}$$

③ Find x :

Q1

$$x = \bar{x}$$

$$x = \boxed{\frac{\pi}{6}}$$

$$x = \frac{\pi}{6} + 2\pi$$

$$x = \frac{\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{13\pi}{6}}$$

Q2

$$x = \pi - \bar{x}$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{6\pi}{6} - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$$

$$x = \frac{5\pi}{6} + 2\pi$$

$$x = \frac{5\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{17\pi}{6}}$$

Ex. $\cos^2 \theta - \frac{1}{2} \cos \theta = 0, -2\pi \leq \theta \leq 4\pi$
(Radians)

(Common Factor)

$$\cos \theta (\cos \theta - \frac{1}{2}) = 0$$

$\cos \theta = 0$ (Unit Circle)

① Find θ :

$$\theta = \frac{\pi}{2}$$

$$\theta = \frac{3\pi}{2}$$

$$\theta = \frac{\pi}{2} - 2\pi$$

$$\theta = \frac{3\pi}{2} - 2\pi$$

$$\theta = \frac{\pi}{2} - \frac{4\pi}{3} = -\frac{3\pi}{2}$$

$$\theta = \frac{3\pi}{2} - \frac{4\pi}{3} = -\frac{\pi}{6}$$

$$\theta = \frac{\pi}{2} + 2\pi$$

$$\theta = \frac{3\pi}{2} + 2\pi$$

$$\theta = \frac{\pi}{2} + \frac{4\pi}{3} = \frac{5\pi}{2}$$

$$\theta = \frac{3\pi}{2} + \frac{4\pi}{3} = \frac{7\pi}{2}$$

$$\cos \theta - \frac{1}{2} = 0$$

$$\cos \theta = \frac{1}{2}$$

② Find $\bar{\theta}$:

$$\bar{\theta} = \cos^{-1} \left(\frac{1}{2}\right)$$
(Triangle 4)

$$\bar{\theta} = \frac{\pi}{3}$$

③ Where is $\cos \theta > 0$ S A



④ Find θ :

Q1

$$\theta = \bar{\theta}$$

$$\theta = \frac{\pi}{3}$$

$$\theta = 2\pi - \bar{\theta}$$

$$\theta = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{3} - 2\pi$$

$$\theta = \frac{\pi}{3} - \frac{6\pi}{3} = -\frac{5\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2\pi$$

$$\theta = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$$

$$\theta = \frac{5\pi}{3} - 2\pi$$

$$\theta = \frac{5\pi}{3} - \frac{6\pi}{3} = -\frac{\pi}{3}$$

$$\theta = \frac{5\pi}{3} + 2\pi$$

$$\theta = \frac{5\pi}{3} + \frac{6\pi}{3} = \frac{11\pi}{3}$$

Ex. $6\sin^2 x - \sin x = 2, -2\pi \leq \theta \leq 4\pi$

Your Turn

Solve for θ .

$$\cos^2 \theta - \cos \theta - 2 = 0, 0^\circ \leq \theta < 360^\circ$$

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

General Solution of a Trigonometric Equation

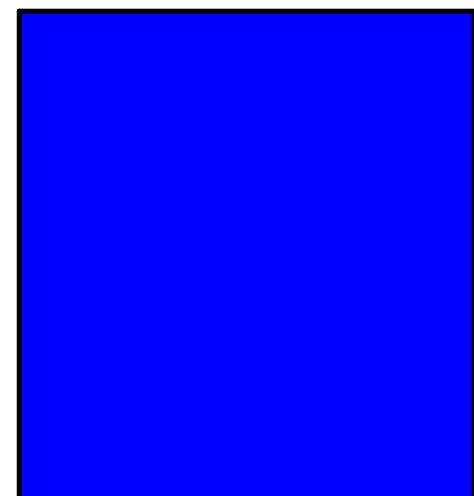
Solve: $3\cos^2 \theta - \cos \theta = 2; \theta \in \mathbb{R}$

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians.

Did You Know?

$2n$, where $n \in \mathbb{I}$, represents all even integers.

$2n + 1$, where $n \in \mathbb{I}$, is an expression for all odd integers.



Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Practice Problems:

Pages 212 - 214
#11 - 23

Check-Up problem...

Solve:

$$\sin x \sec x + 2 \sin x = 0 , x \in R \quad (x \text{ is measured in radians})$$

Unit Review...

What topics have we covered??

Review...

- C4 a)** Determine all solutions for the equation
 $2 \sin^2 \theta = 1 - \sin \theta$ in the domain
 $0^\circ \leq \theta < 360^\circ$.
- b)** Are your solutions exact or approximate? Why?
- c)** Show how you can check one of your solutions to verify its correctness.

A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

Determine the angular velocity of the minute hand on a clock.

Solve: $6\sin^2 \theta - 3\sin \theta = 0$, $0^\circ \leq \theta \leq 360^\circ$

- [A] $0^\circ, 30^\circ, 180^\circ, 330^\circ, 360^\circ$ [B] $0^\circ, 30^\circ, 180^\circ, 150^\circ, 360^\circ$
[C] $30^\circ, 90^\circ, 120^\circ, 270^\circ$ [D] $0^\circ, 180^\circ, 210^\circ, 330^\circ, 360^\circ$

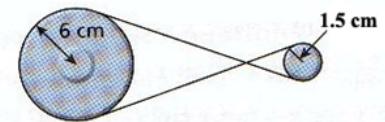
If $\csc \theta < 0$ and $\tan \theta > 0$, then which of the following could be a possible measure of angle θ ?

- [A] $\frac{11\pi}{6}$ [B] $\frac{4\pi}{3}$ [C] $\frac{3\pi}{4}$ [D] $\frac{\pi}{2}$

What is the principal angle of $-\frac{25\pi}{4}$?

- [A] $\frac{3\pi}{4}$ [B] $\frac{\pi}{4}$ [C] $-\frac{\pi}{4}$ [D] $\frac{7\pi}{4}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



- [A] $\frac{\pi}{9}$ radians [B] 20°
[C] 20 radians [D] 5°

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?
[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Solve: $2(1 - \sin \theta)^2 + \sin \theta = 2(3 - 4 \sin^2 \theta)$, $-360^\circ \leq \theta \leq 720^\circ$

| Little Johnny has a rock tied to the end of a piece of rope 1.5 m long and he is swinging it around his head in a circular pattern. Mrs. Centripetal, his physics teacher, is watching Johnny out the window of her physics lab and notes that the rock is making 12 revolutions every 48 seconds.

- (a) Determine the angular velocity with which little Johnny is twirling the rope above his head. [2]

- (b) The rock comes flying from the rope 3 minutes after Mrs. Centripetal started to time little Johnny. How far did the rock travel during the 3 minutes? [2]

Attachments

Worksheet - Sketching Angles in Radians.doc