② f) $f(t) = (at+b)(ct^2-d)$

$f'(t) = (a)(ct^2-d) + (at+b)(2ct)$

$f'(t) = \underline{act^2} - ad + \underline{2act^2} + 2bct$

$f'(t) = 3act^2 + 2bct - ad$

③a) $y = (1-2x)(3x-4)$, $x=2$

① Find y':

$y' = -2(3x-4) + (1-2x)(3)$

$y' = -6x+8+3-6x$

$y' = -12x+11$

② sub in $x=2$

$y' = -12x+11$

$y' = -12(2)+11$

$y' = -13$

$m = -13$

⑤ $y = (2-\sqrt{x})(1+\sqrt{x}+3x)$ @ $(1,5)$

$y = (2-x^{1/2})(1+x^{1/2}+3x)$

① find y':

$y' = \left(-\frac{1}{2}x^{-1/2}\right)(1+x^{1/2}+3x) + (2-x^{1/2})\left(\frac{1}{2}x^{-1/2}+3\right)$

$y' = \left(\frac{-1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$

② sub in $x=1$

$y' = \left(\frac{-1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1)) + (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right)$

$y' = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{7}{2}\right)$

$y' = \frac{-5}{2} + \frac{7}{2} = \frac{2}{2} = 1$

$m = 1$

③ $y - y_1 = m(x-x_1)$

$y - 5 = 1(x-1)$

$y-5 = x-1$

$y = x+4$

or $x-y+4=0$

② h) $g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$

$$g(v) = \underbrace{(v - v^{1/2})}_{f(x)}\underbrace{(v^2 + v^{1/2})}_{g(x)}$$

$1 + (-\frac{1}{2})$

$\frac{2}{2} + (-\frac{1}{2})$

$-\frac{1}{2} + \frac{2}{1}$

$-\frac{1}{2} + \frac{4}{2}$

$$g'(v) = (1 - \tfrac{1}{2}v^{-1/2})(v^2 + v^{1/2}) + (v - v^{1/2})(2v + \tfrac{1}{2}v^{-1/2})$$

$$g'(v) = v^2 + v^{1/2} - \tfrac{1}{2}v^{3/2} - \tfrac{1}{2}\cancel{v^0} + 2v^2 + \tfrac{1}{2}v^{1/2} - 2v^{3/2} - \tfrac{1}{2}\cancel{v^0}$$

$$g'(v) = \frac{3v^2}{1} - \frac{5}{2}v^{3/2} + \frac{3}{2}v^{1/2} - \frac{1}{1}$$

$$g'(v) = \frac{6v^2}{2} - \frac{5v^{3/2}}{2} + \frac{3v^{1/2}}{2} - \frac{2}{2}$$

$$g'(v) = \frac{6v^2 - 5v^{3/2} + 3v^{1/2} - 2}{2}$$

⑤ $y=(2-\sqrt{x})(1+\sqrt{x}+3x)$ @ $(1,5)$

x_1 y_1

① Find derivative

$$y=\underbrace{(2-x^{1/2})}_{f(x)}\underbrace{(1+x^{1/2}+3x)}_{g(x)}$$

$$y'=\underbrace{\left(-\frac{1}{2}x^{-1/2}\right)}_{f'(x)}\underbrace{(1+\sqrt{x}+3x)}_{g(x)}+\underbrace{(2-\sqrt{x})}_{f(x)}\underbrace{\left(\frac{1}{2}x^{-1/2}+3\right)}_{g'(x)}$$

$$y'=\left(\frac{-1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x)+(2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$$

② Solve for slope of tangent (m)

$$m=y'(1)=\left(\frac{1}{2}\right)(5)+(1)\left(\frac{1}{2}+3\right)$$

$$m=y'(1)=-\frac{5}{2}+1\left(\frac{1}{2}+\frac{6}{2}\right)$$

$$m=y'(1)=-\frac{5}{2}+\frac{7}{2}=\frac{2}{2}=1$$

m=1

③ Find the equation

$y-y_1=m(x-x_1)$

$y-5=1(x-1)$

$y-5=x-1$

$y=x+4$

$\boxed{0=x-y+4}$

multiply (Product

⑥ If:

$f(2)=3$

$f'(2)=5$

$g(2)=-1$

$g'(2)=-4$

Find $(fg)'(2)$

$$(fg)'(x)=f'(x)g(x)+f(x)g'(x)$$

$$(fg)'(2)=f'(2)g(2)+f(2)g'(2)$$

$$=(5)(-1)+(3)(-4)$$

$$=-5+-12$$

$$=-17$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$ = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the derivative of the first function times the second function, plus the first function times the derivative of the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$(fg)'x = f'(x)g(x) + f(x)g'(x)$

Differentiate the following function and simplify your answer:

$$h(t)=\underbrace{\left(t^3-5t\right)}_{f(x)}\underbrace{\left(6\sqrt{t}-t^{-5}\right)}_{g(x)}$$

$$h'(t) = (3t^2-5)(6t^{1/2}-t^{-5}) + (t^3-5t)(3t^{-1/2}+5t^{-6})$$

$$h'(t) = 18t^{5/2} - 3t^{-3} - 30t^{1/2} + 5t^{-5} + 3t^{5/2} + 5t^{-3} - 15t^{1/2} - 25t^{-5}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + 2t^{-3} - 20t^{-5}$$

$$\boxed{h'(t) = 21t^{5/2} - 45t^{1/2} + \frac{2}{t^3} - \frac{20}{t^5}}$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{(\text{First})}{(\text{Second})}$$

In words, ***the Quotient Rule*** says that the ***derivative of a quotient is:***
the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Examples:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Differentiate the following functions and simplify your answers:

$$F(x)=\frac{x^2+2x-3}{x^3+1} \qquad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$F'(x) = \frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$$

$$F'(x) = \frac{2x^4+2x+2x^3+2-3x^4-6x^3+9x^2}{(x^3+1)^2}$$

$$F'(x) = \frac{-x^4-4x^3+9x^2+2x+2}{(x^3+1)^2}$$

$$F(x)=\frac{\sqrt{x}}{1+2x} \qquad \begin{matrix} f(x) \\ g(x) \end{matrix} \qquad = \frac{x^{1/2}}{1+2x}$$

$$F'(x) = \frac{\frac{1}{2}x^{-1/2}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1}{2\sqrt{x}}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \cancel{2\sqrt{x}}\,\frac{\frac{1+2x}{\cancel{2\sqrt{x}}} - 2\sqrt{x}\cdot 2\sqrt{x}}{2\sqrt{x}(1+2x)^2}$$

CD: $2\sqrt{x}$

$$F'(x) = \frac{1+2x-4x}{2\sqrt{x}(1+2x)^2} = \frac{1-2x}{2\sqrt{x}(1+2x)^2}$$

Differentiate the following functions, do not simplify your answers:

$$f(x)=\frac{8-9x^7}{3x-7}$$

$$f'(x)=\frac{-63x^6(3x-7)-3(8-9x^7)}{(3x-7)^2}$$

$$f(x)=\frac{x^3-7x^2+2}{x^8-4x^5}$$

$$f'(x)=\frac{(3x^2-14x)(x^8-4x^5)-(x^3-7x^2+2)(8x^7-20x^4)}{(x^8-4x^5)^2}$$

Homework

$$y = \frac{x}{x-2} \qquad (4, 2)$$

① $$y' = \frac{1(x-2) - x}{(x-2)^2} = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

② $$y' = \frac{-2}{(x-2)^2} = \frac{-2}{(4-2)^2} = \frac{-2}{(2)^2} = \frac{-2}{4} = -\frac{1}{2}$$

③ $$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{2}(x-4)$$

$$y - 2 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 4$$

Exercise 2.5

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

① 2)

$$g(x) = \frac{x^3-1}{x^2+x+1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$g'(x) = \frac{3x^2(x^2+x+1) - (x^3-1)(2x+1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{3x^4+3x^3+3x^2-(2x^4+x^3-2x-1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)^2}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)(x^2+x+1)}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{x^4+x^3+x^2+x^3+x^2+x+x^2+x+1}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{x^4+2x^3+3x^2+2x+1}$$

$$\boxed{g'(x) = 1}$$

③ d) $y = \dfrac{x^3 - 1}{1 + 2x^2}$, $(1, 0)$

x_1 , y_1

(i) Find derivative

$$y' = \frac{3x^2(1+2x^2) - 4x(x^3-1)}{(1+2x^2)^2}$$

$$y' = \frac{3x^2 + 6x^4 - 4x^4 + 4x}{(1+2x^2)^2}$$

$$y' = \frac{2x^4 + 3x^2 + 4x}{(1+2x^2)^2}$$

(ii) Find m (sub in x=1)

$$m = y'(1) = \frac{2(1)^4 + 3(1)^2 + 4(1)}{[1+2(1)^2]^2}$$

$$m = y'(1) = \frac{2+3+4}{9} = \frac{9}{9} = 1$$

m

(iii) $y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 1)$

$\boxed{y = x - 1}$

$\boxed{x - y - 1 = 0}$