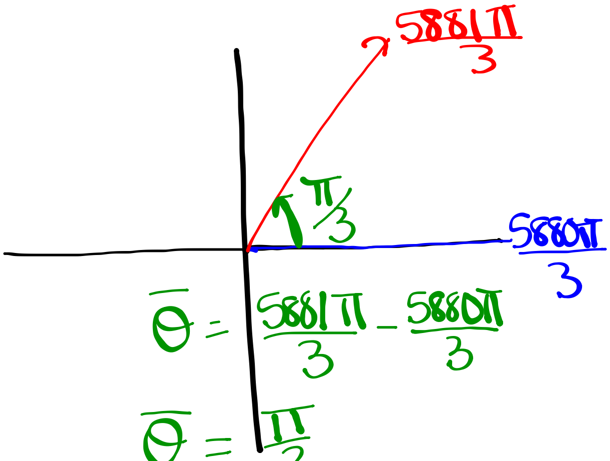
Sketch the following and determine a negative angle co-terminal with:

$(i)\ \dfrac{5881\pi}{3}$

$\dfrac{5880\pi}{3}, \dfrac{5881\pi}{3}, \dfrac{5882\pi}{3}$

1960π



$\bar{\theta} = \dfrac{5881\pi}{3} - \dfrac{5880\pi}{3}$

$\bar{\theta} = \dfrac{\pi}{3}$

Negative coterminal angle:

$\dfrac{5881\pi}{3} - \dfrac{1962\pi}{1}$

$\dfrac{5881\pi}{3} - \dfrac{5886\pi}{3}$

$-\dfrac{5\pi}{3}$

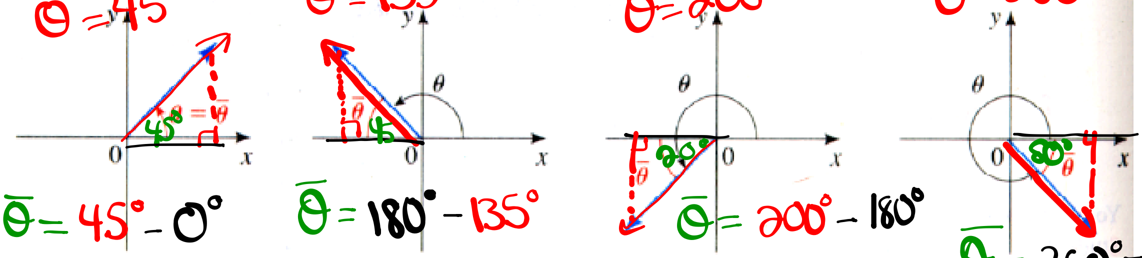
Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

arm

The picture below illustrates this concept.

$\theta = 45°$ $\theta = 135°$ $\theta = 200°$ $\theta = 280°$



$\bar{\theta} = 45° - 0°$ $\bar{\theta} = 180° - 135°$ $\bar{\theta} = 200° - 180°$ $\bar{\theta} = 360° - 280°$

$\bar{\theta} = 45°$ $\bar{\theta} = 45°$ $\bar{\theta} = 20°$ $\bar{\theta} = 80°$

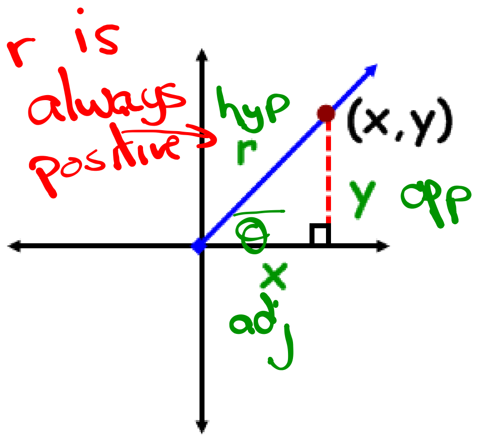
What is the significance of reference angles?

Angles on the Cartesian Plane

$< 90°$ or $< \pi/2$ or < 1.57 rads

- **Reference Angle** (θ) - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin\theta = \frac{y}{r} = \frac{o}{h} \qquad \csc\theta = \frac{r}{y} = \frac{h}{o}$$

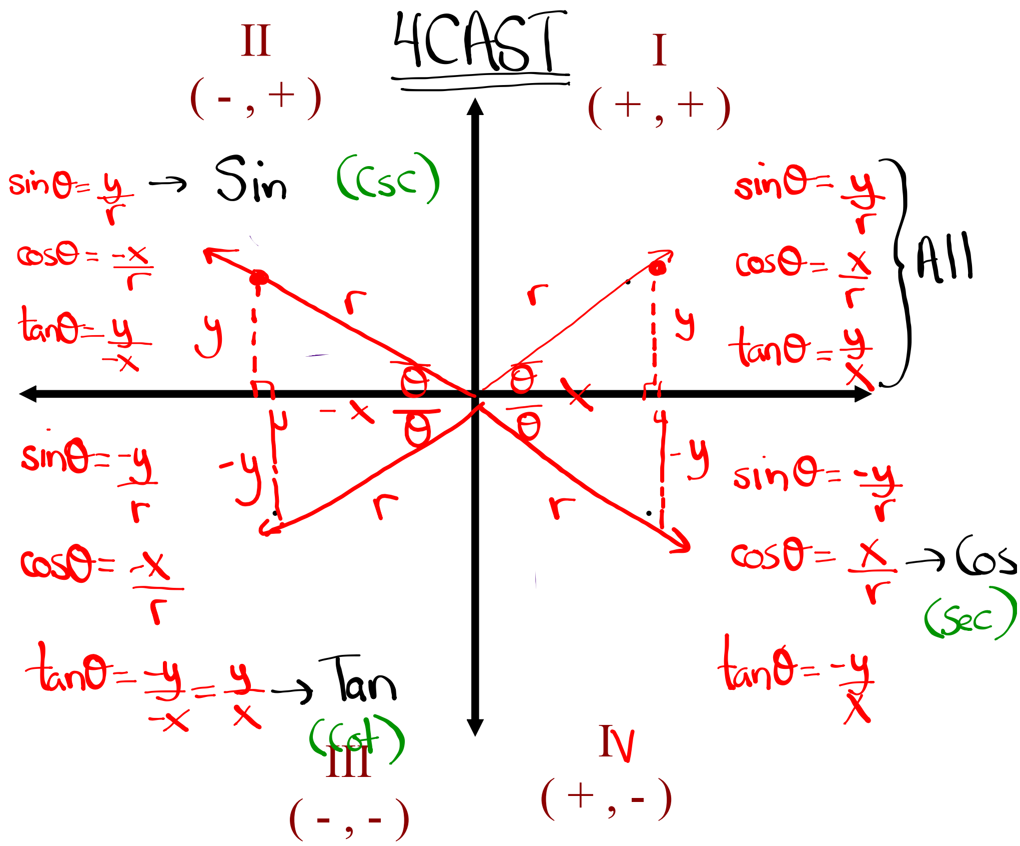
$$\cos\theta = \frac{x}{r} = \frac{a}{h} \qquad \sec\theta = \frac{r}{x} = \frac{h}{a}$$

$$* \tan\theta = \frac{y}{x} = \frac{o}{a} \qquad \cot\theta = \frac{x}{y} = \frac{a}{o}$$

"Primary" "Reciprocal"

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



Where is θ if...

cscθ<0

sinθ<0 & tanθ<0

cscθ>0 & cotθ<0

Homework

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta$

$\sec\theta = -\frac{\sqrt{10}}{1} = \frac{hyp}{adj} = \frac{r}{x}$

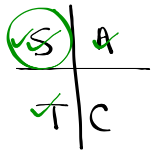
$r = \sqrt{10}$ (always +)

$x = -1$

① Determine what quadrant:

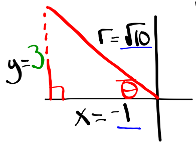
$\sec\theta < 0$ & $\sin\theta > 0$

or $\cos\theta < 0$ & $\sin\theta > 0$



θ is in Q2

② Draw a diagram



③ $x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

$\boxed{y = 3 \ (Q2)}$

④ Find $\csc\theta$:

$\csc\theta = \frac{r}{y}$

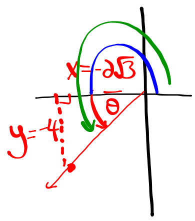
$\boxed{\csc\theta = \frac{\sqrt{10}}{3}}$

Determine the measure ~~(in radians)~~ of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$x = -2\sqrt{3}$

$y = -4$

① Draw a diagram



② Find $\bar{\theta}$

$\tan\bar{\theta} = \frac{y}{x}$

$\tan\bar{\theta} = \frac{-4^{2}}{-2\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

$\bar{\theta} = \tan^{-1}(1.1547)$

convert calculator to radians

$\boxed{\bar{\theta} = 0.86 \text{ rads}}$

③ Find θ

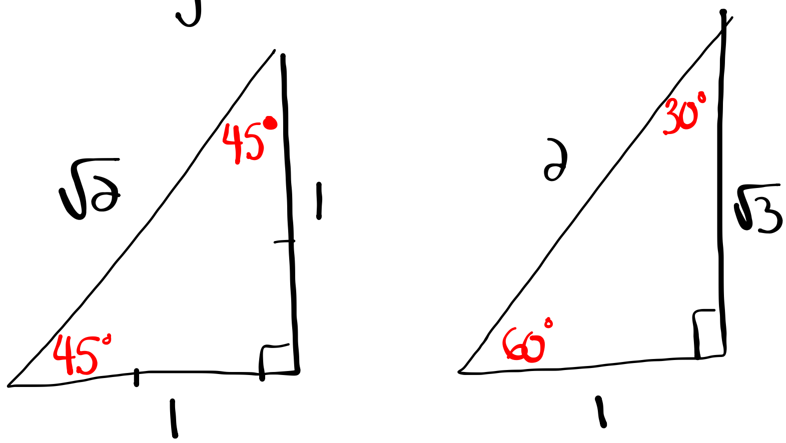
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

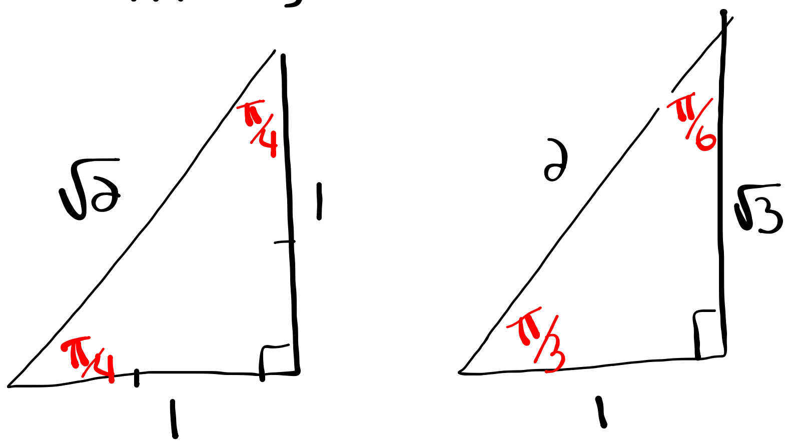
$\boxed{\theta = 4 \text{ rads}}$

$\theta = \pi - \bar{\theta}$ $\theta = 180^\circ - \bar{\theta}$	$\theta = \bar{\theta}$
$\theta = 180^\circ + \bar{\theta}$ $\theta = \pi + \bar{\theta}$	$\theta = 360^\circ - \bar{\theta}$ $\theta = 2\pi - \bar{\theta}$

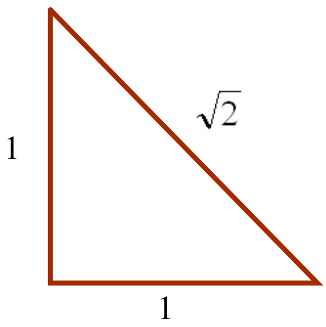
In Degrees

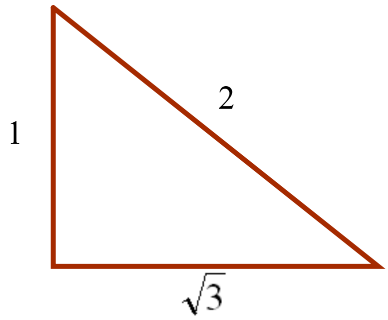


In Radians



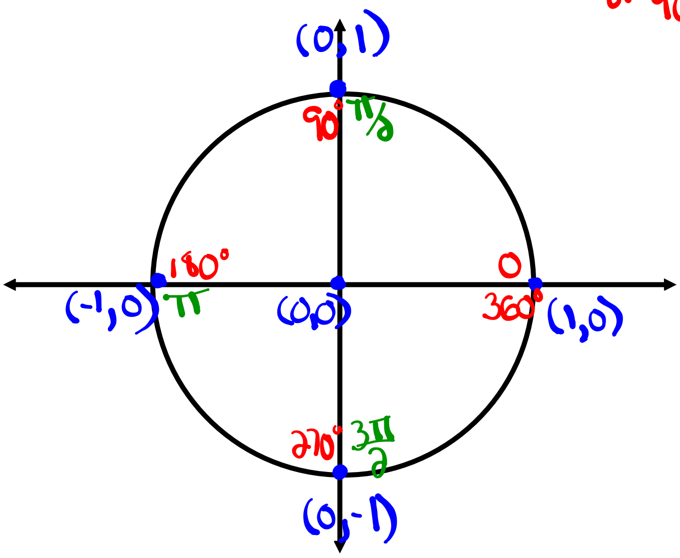
Special Angles (in radians)





Quadrantal Angles

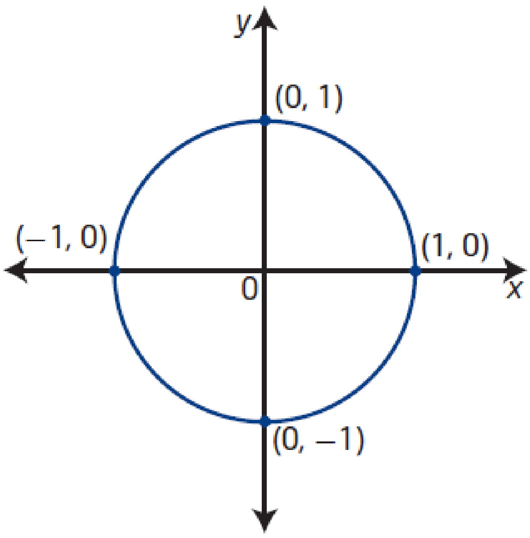
(Multiples of 90°)



The Unit Circle

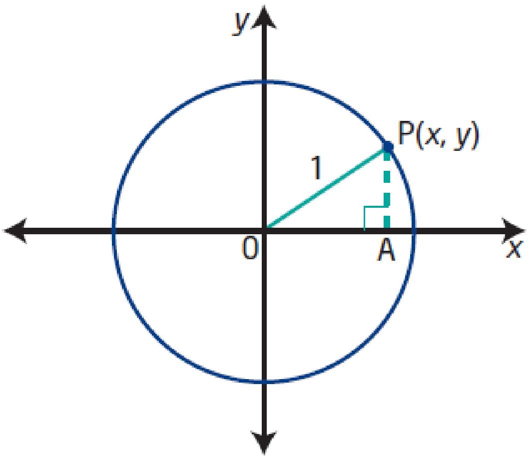
- Center is @ (0,0)
- radius is 1 unit

Unit Circle



unit circle

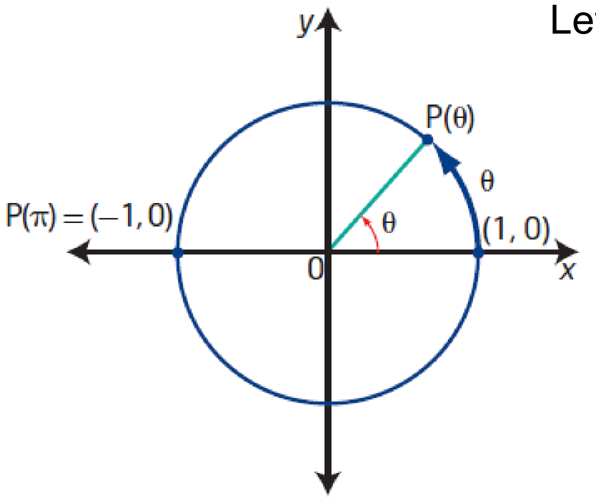
- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle



The equation of the unit circle is $x^2 + y^2 = 1$.

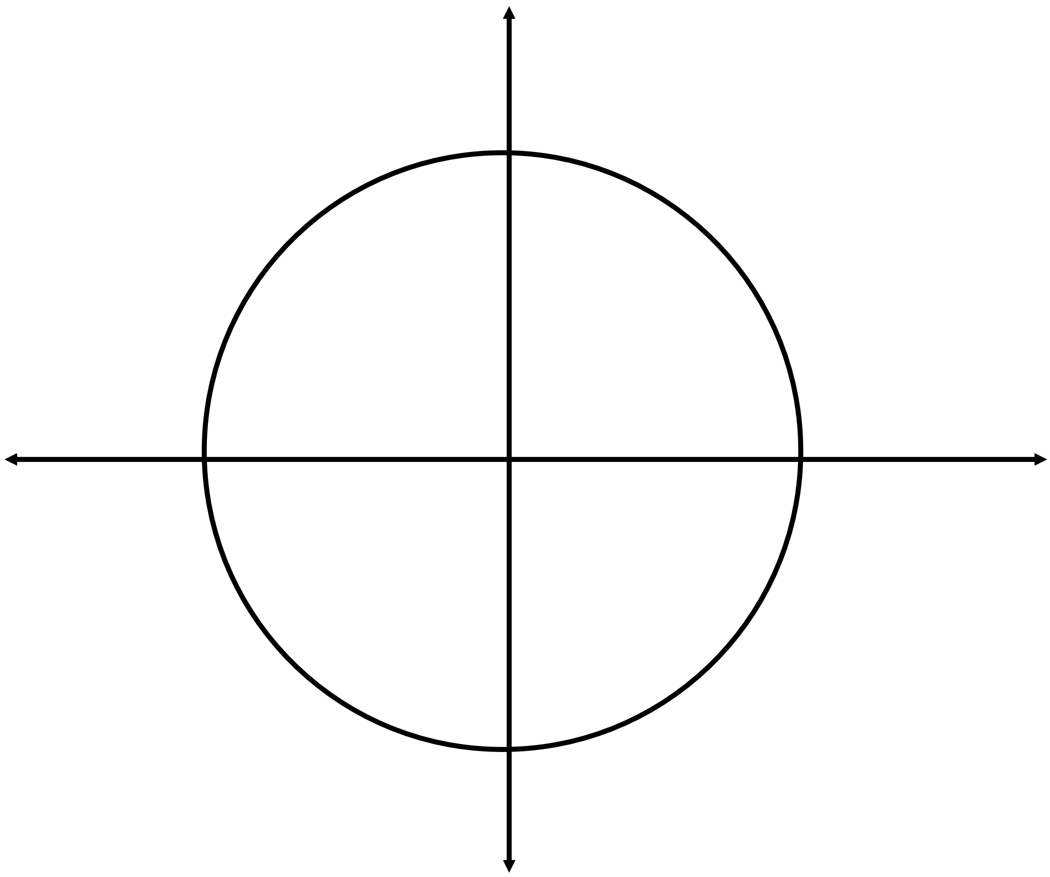
Determine the equation of a circle with centre at the origin and radius 6.

Special Angles on the Unit Circle:

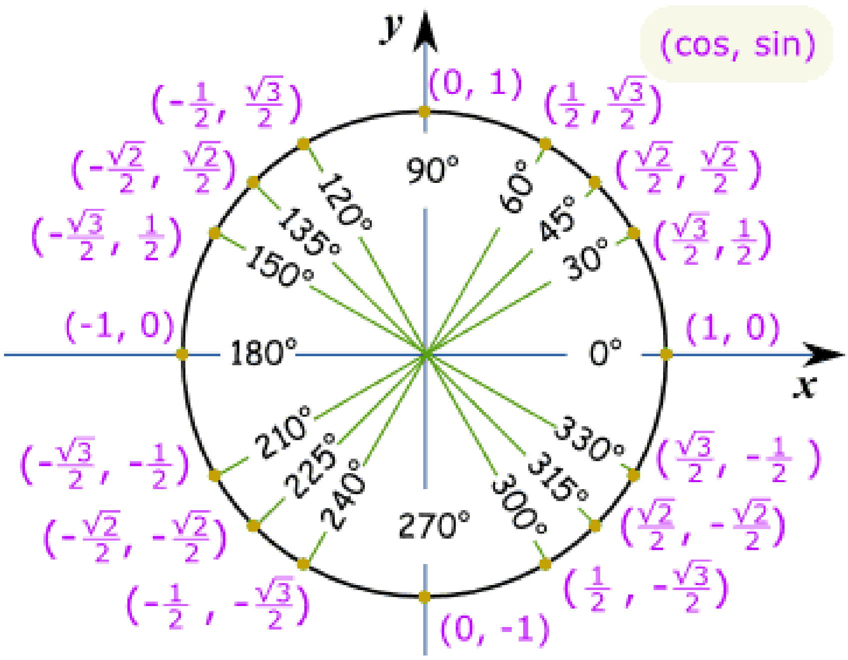


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

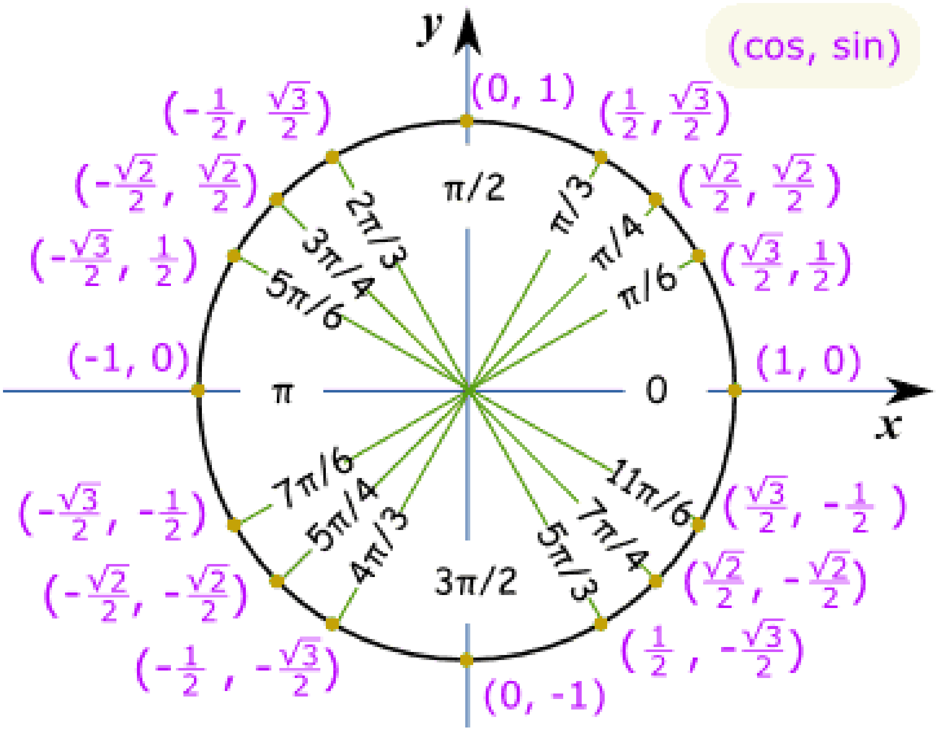


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



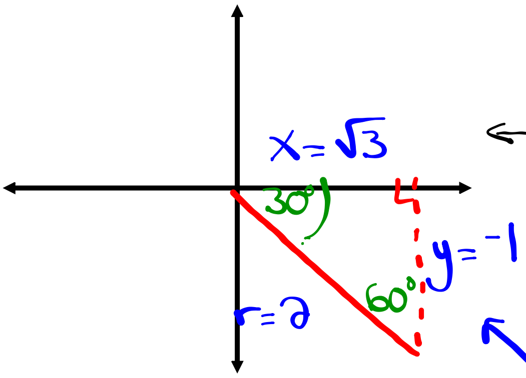
Solving ~~Trig Expressions~~ by Sketching Angles

Ex. Evaluate the $\sin 690°$

Optional

(i) Find principal angle:

$\theta = 690° - 360° = 330°$



(ii) Sketch (Q4)

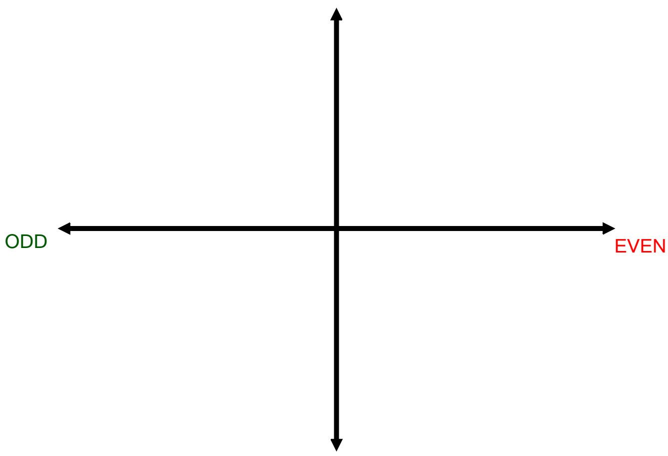
(iii) Find $\bar{\theta}$

$\bar{\theta} = 360° - 330° = \boxed{30°}$

(iv) Label

(v) Evaluate $\boxed{\sin 690° = \frac{-1}{2}}$

Ex. $\cos\dfrac{13\pi}{3}$



Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan\dfrac{17\pi}{6}$

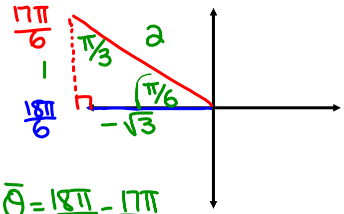
2. $\sin\dfrac{15\pi}{4}$

3. $\cos\left(-\dfrac{21\pi}{4}\right)$

Ex. $\tan\frac{17\pi}{6} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\frac{16\pi}{6}, \frac{17\pi}{6}, \frac{18\pi}{6}$

3π



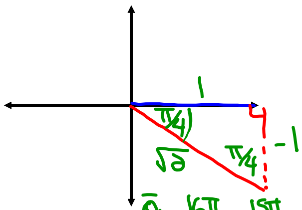
$\bar{\theta} = \frac{18\pi}{6} - \frac{17\pi}{6}$

$\bar{\theta} = \frac{\pi}{6}$

Ex. $\sin\frac{15\pi}{4} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$

4π



$\bar{\theta} = \frac{16\pi}{4} - \frac{15\pi}{4}$

$\bar{\theta} = \frac{\pi}{4}$

Ex. $\cos\left(-\frac{21\pi}{4}\right) = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

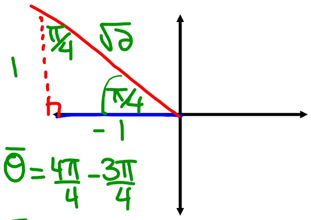
$\frac{-21\pi}{4} + \frac{6\pi}{1}$

$\frac{-21\pi}{4} + \frac{24\pi}{4} = \frac{3\pi}{4}$

$\cos\frac{3\pi}{4}$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

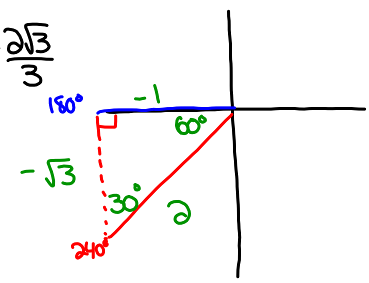
1π



$\bar{\theta} = \frac{4\pi}{4} - \frac{3\pi}{4}$

$\bar{\theta} = \frac{\pi}{4}$

$\csc 240^\circ = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$



$\bar{\theta} = 240^\circ - 180^\circ$

$\bar{\theta} = 60^\circ$

Attachments

Worksheet - Sketching Angles in Radians.doc