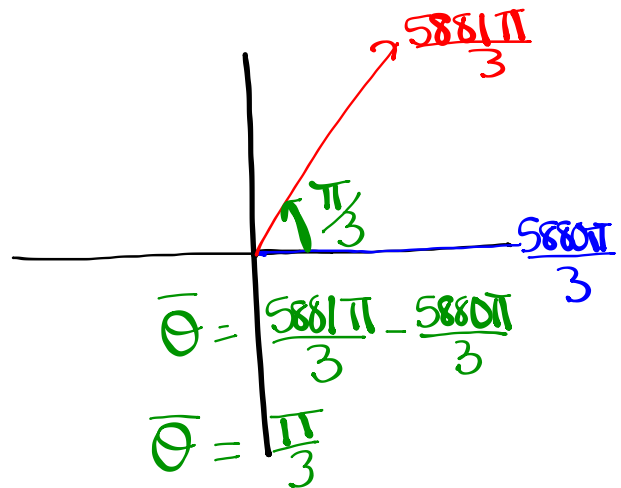


Sketch the following and determine a negative angle co-terminal with:

$$(i) \frac{5881\pi}{3}$$

$$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$$

$$1960\pi$$



Negative coterminal angle:

$$\frac{5881\pi}{3} - \frac{1962\pi}{1}$$

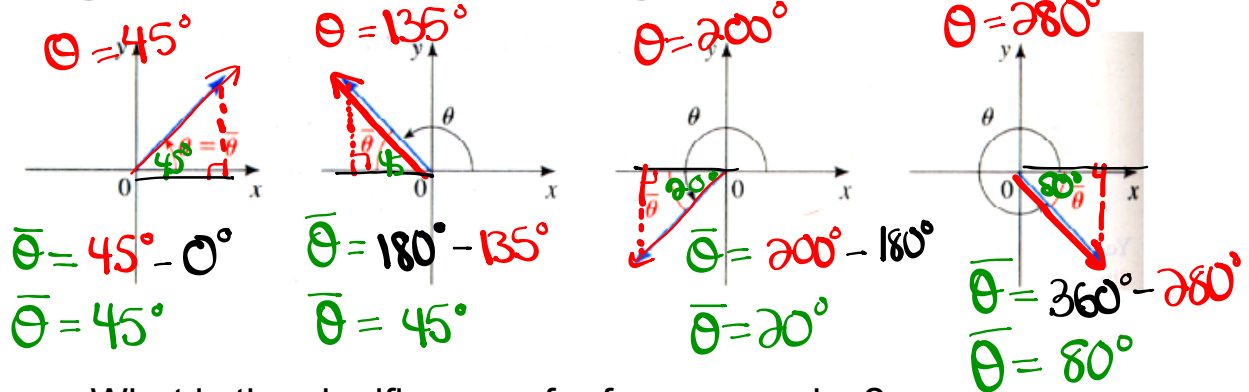
$$\frac{5881\pi}{3} - \frac{5886\pi}{3}$$

$$\frac{-5\pi}{3}$$

Reference Triangles:

Definition 17 The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.

The picture below illustrates this concept.



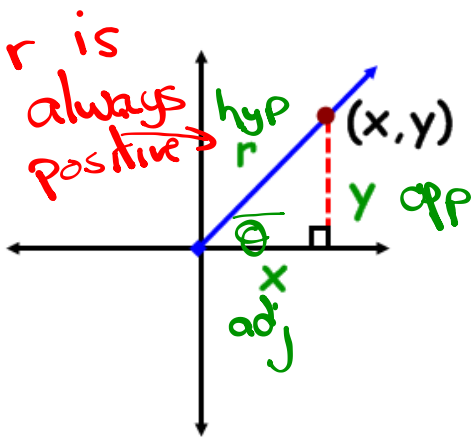
What is the significance of reference angles?

Angles on the Cartesian Plane

$< 90^\circ$ or $< \pi/2$ or $< 1.57 \text{ rads}$

- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin \theta = \frac{y}{r} = \frac{o}{h} \quad \csc \theta = \frac{r}{y} = \frac{h}{o}$$

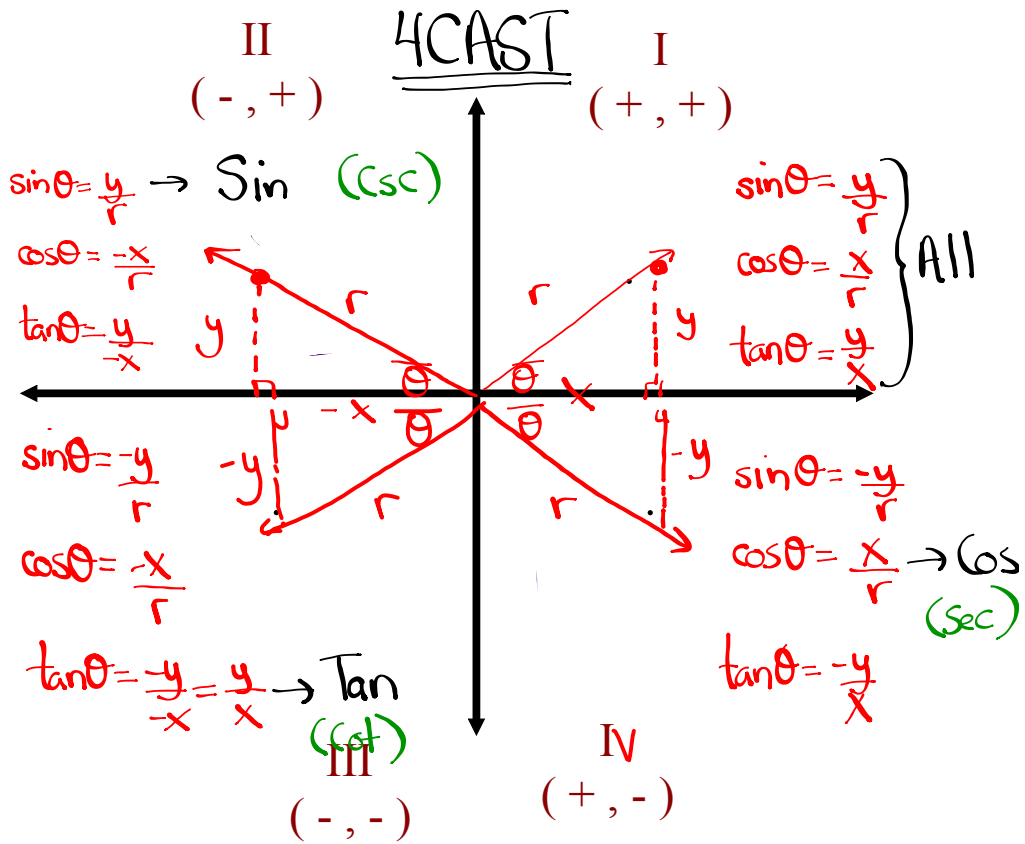
$$\cos \theta = \frac{x}{r} = \frac{a}{h} \quad \sec \theta = \frac{r}{x} = \frac{h}{a}$$

$$\tan \theta = \frac{y}{x} = \frac{o}{a} \quad \cot \theta = \frac{x}{y} = \frac{a}{o}$$

"Primary" **"Reciprocal"**

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are **POSITIVE** in...



Where is θ if...

$\csc\theta < 0$

$\sin\theta < 0$ & $\tan\theta < 0$

$\csc\theta > 0$ & $\cot\theta < 0$

Homework

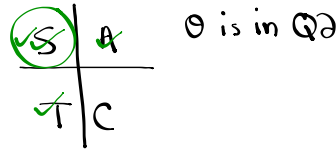
If $\sec\theta = -\sqrt{10}$ and $\sin\theta > 0$, determine the value of $\csc\theta$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$r = \sqrt{10} \text{ (always +)}$$

$$x = -1$$

① Determine what quadrant:
 $\sec\theta < 0$ + $\sin\theta > 0$
 or $\cos\theta < 0$ + $\sin\theta > 0$

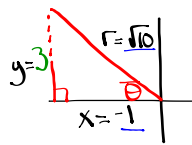


④ Find $\csc\theta$:

$$\csc\theta = \frac{r}{y}$$

$$\csc\theta = \frac{\sqrt{10}}{3}$$

② Draw a diagram



$$\textcircled{3} \quad x^2 + y^2 = r^2$$

$$(-1)^2 + y^2 = (\sqrt{10})^2$$

$$\textcircled{1} + y^2 = 10$$

$$y^2 = 9$$

$$y = \pm 3$$

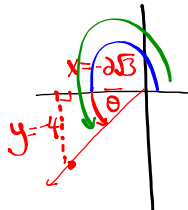
$$y = 3 \text{ (Q2)}$$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$$x = -2\sqrt{3}$$

$$y = -4$$

① Draw a diagram



② Find $\bar{\theta}$

$$\tan\bar{\theta} = \frac{y}{x}$$

$$\tan\bar{\theta} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\tan\bar{\theta} = 1.1547$$

$$\bar{\theta} = \tan^{-1}(1.1547)$$

convert calculator to radians

$$\bar{\theta} = 0.86 \text{ rads}$$

③ Find θ

$$\theta = \pi + \bar{\theta}$$

$$\theta = 3.14 + 0.86$$

$$\theta = 4 \text{ rads}$$

$$\theta = \pi - \bar{\theta}$$

$$\theta = 180^\circ - \bar{\theta}$$

$$\theta = 180^\circ + \bar{\theta}$$

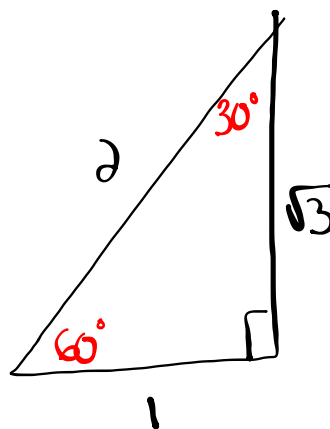
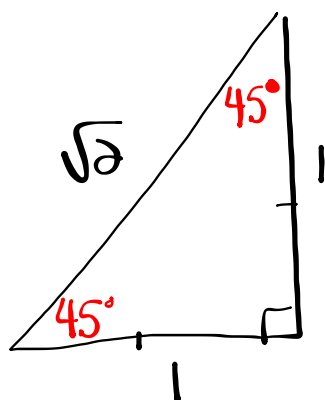
$$\theta = \pi + \bar{\theta}$$

$$\theta = \bar{\theta}$$

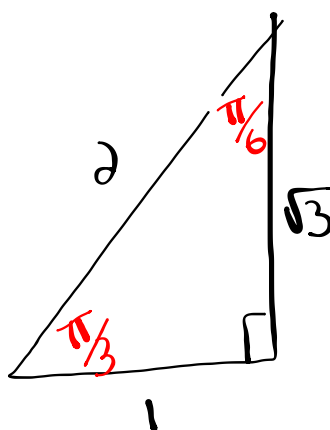
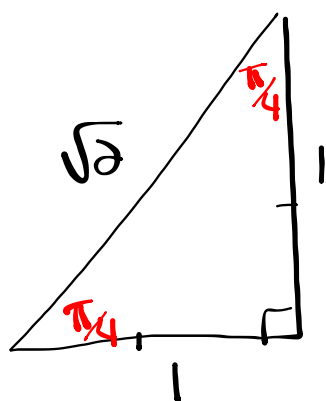
$$\theta = 360^\circ - \bar{\theta}$$

$$\theta = 2\pi - \bar{\theta}$$

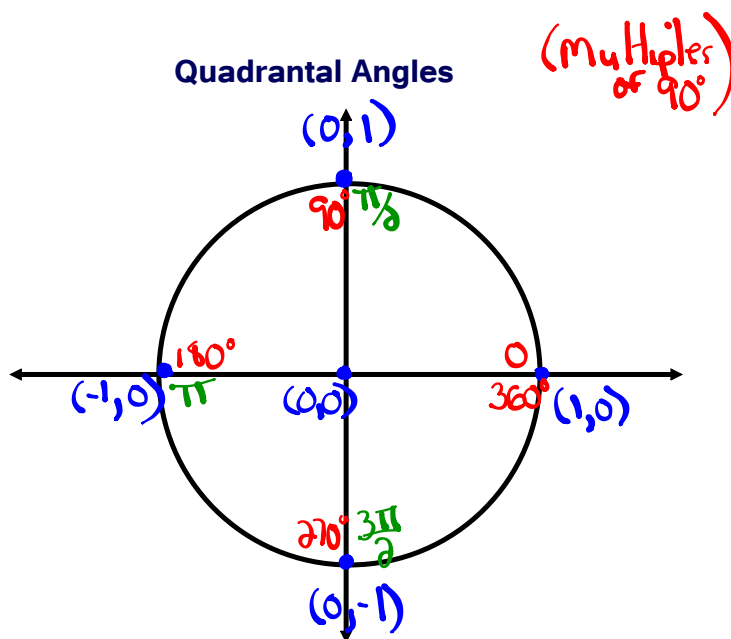
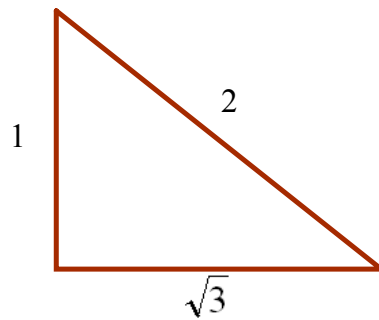
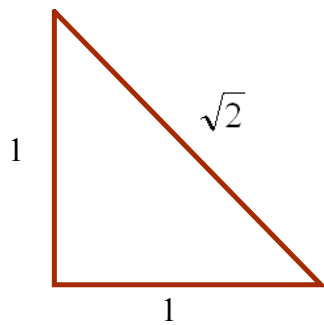
In Degrees



In Radians



Special Angles (in radians)



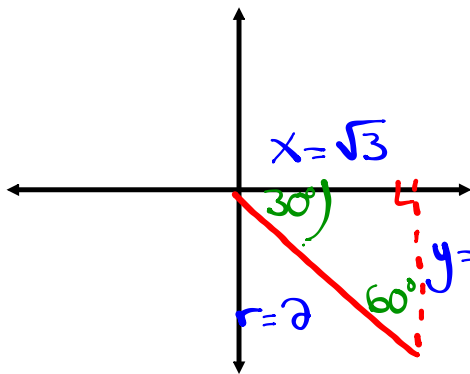
- The Unit Circle
- Center is @ $(0,0)$
 - radius is 1 unit

Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690^\circ$

Optional

(i) Find principal angle:
 $\theta = 690^\circ - 360^\circ = 330^\circ$



(ii) Sketch (Q4)

(iii) Find $\bar{\theta}$

$$\bar{\theta} = 360^\circ - 330^\circ = 30^\circ$$

(iv) Label

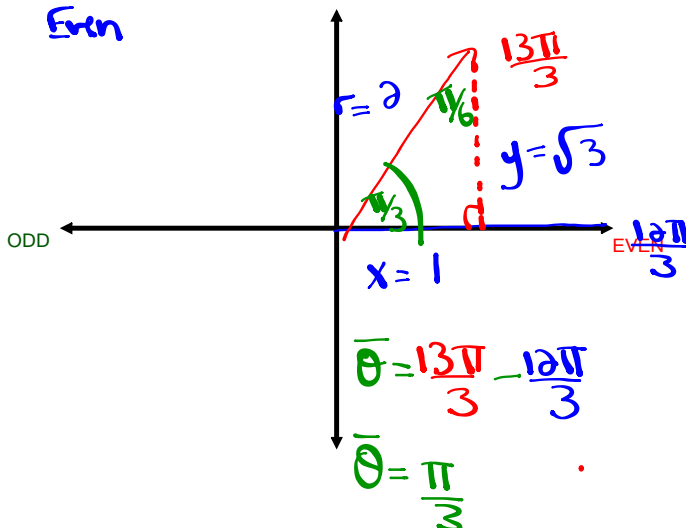
(v) Evaluate

$$\sin 690^\circ = -\frac{1}{2}$$

Ex. $\cos \frac{13\pi}{3} = \frac{2}{5} = \frac{x}{r} = \frac{1}{2}$

$\frac{12\pi}{3}$ $\frac{13\pi}{3}$ $\frac{14\pi}{3}$

4π
Even

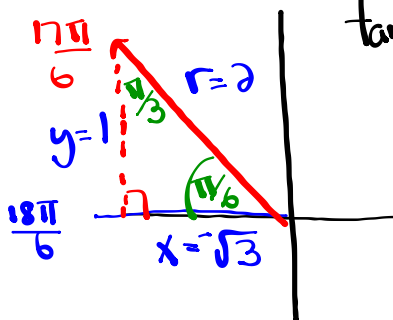


Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan \frac{17\pi}{6} = -\frac{1}{\sqrt{3}}$ 2. $\sin \frac{15\pi}{4} = -\frac{1}{\sqrt{2}}$ 3. $\cos\left(-\frac{21\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

① $\frac{16\pi}{6}$ $\frac{17\pi}{6}$ $\frac{18\pi}{6}$
 3π
 (odd)

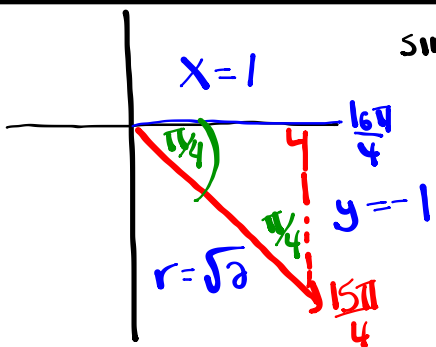


$$\tan \frac{17\pi}{6} = \frac{0}{a} = \frac{y}{x} = -\frac{1}{\sqrt{3}}$$

$$= -\frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

$\tan \frac{17\pi}{6} = -\frac{\sqrt{3}}{3}$

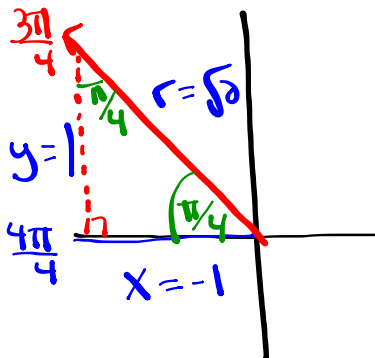
② $\frac{14\pi}{4}$ $\frac{15\pi}{4}$ $\frac{16\pi}{4}$
 4π
 (Even)



$$\sin \frac{15\pi}{4} = \frac{0}{h} = \frac{y}{r} = -\frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$\sin \frac{15\pi}{4} = -\frac{\sqrt{2}}{2}$

③ $-\frac{21\pi}{4} + \frac{6\pi}{1}$
 $-\frac{21\pi}{4} + \frac{24\pi}{4}$
 $\frac{3\pi}{4}$



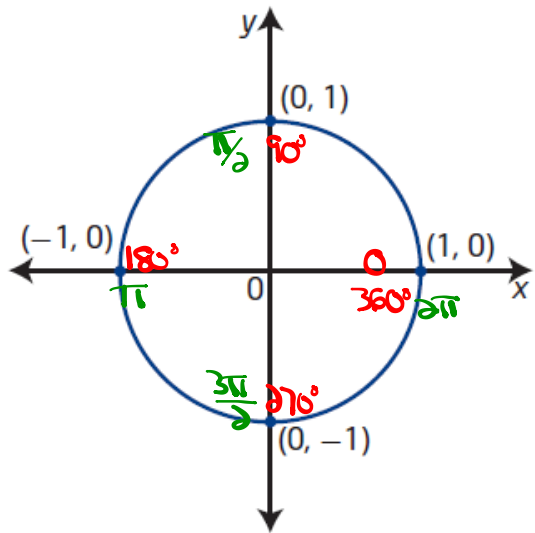
$$\cos\left(-\frac{21\pi}{4}\right) = \cos \frac{3\pi}{4} = \frac{a}{h} = \frac{x}{r}$$

$$= -\frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$\frac{2\pi}{4}$ $\frac{3\pi}{4}$ $\frac{4\pi}{4}$
 π
 (odd)

$\cos\left(-\frac{21\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Unit Circle



unit circle

- a circle with radius 1 unit ($r = 1$)
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y \quad \rightarrow \quad \text{Ex: } \sin 90^\circ = 1$$

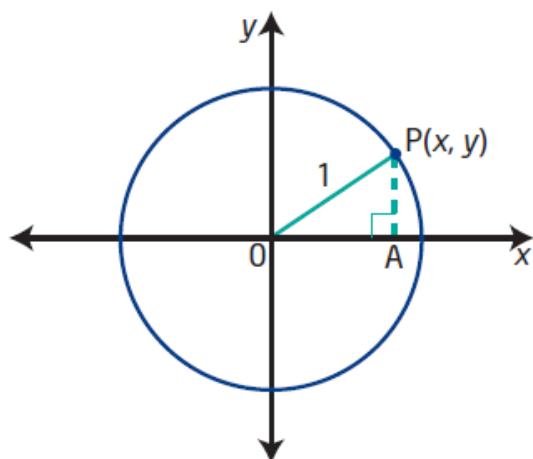
$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x \quad \rightarrow \quad \text{Ex: } \cos \pi = -1$$

$$\tan \theta = \frac{y}{x} \quad \rightarrow \quad \text{Ex: } \tan 270^\circ = \frac{-1}{0} = \text{undefined}$$

$$\csc \theta = \frac{1}{y} \quad \rightarrow \quad \text{Ex: } \csc 360^\circ = \frac{1}{0} = \text{undefined}$$

$$\sec \theta = \frac{1}{x} \quad \rightarrow \quad \text{Ex: } \sec 5\pi = \frac{1}{-1} = -1$$

$$\cot \theta = \frac{x}{y} \quad \rightarrow \quad \text{Ex: } \cot \frac{3\pi}{2} = \frac{0}{-1} = 0$$



$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = (1)^2$$

The equation of the unit circle is $x^2 + y^2 = 1$.

$$r=1$$

Determine the equation of a circle with centre at the origin and radius 6.

$$\underline{r=6}$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = (6)^2$$

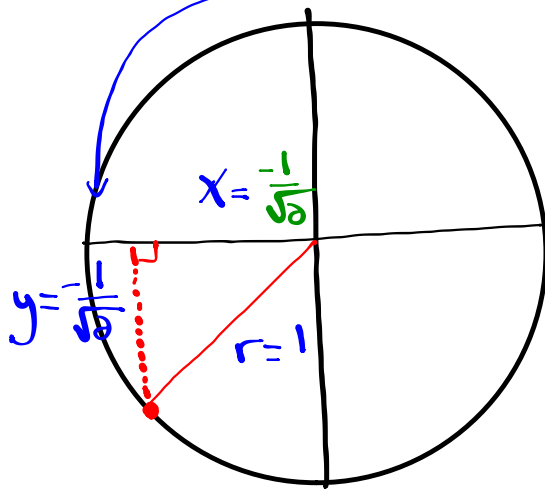
$$\boxed{x^2 + y^2 = 36}$$

Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

Determine the coordinates (x, y) for all points on the unit circle $r=1$ that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III



Find x

$$x^2 + y^2 = r^2$$

$$x^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = (1)^2$$

$$x^2 + \frac{1}{2} = 1$$

$$x^2 = 1 - \frac{1}{2}$$

$$x^2 = \frac{2}{2} - \frac{1}{2}$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{\sqrt{1}}{\sqrt{2}}$$

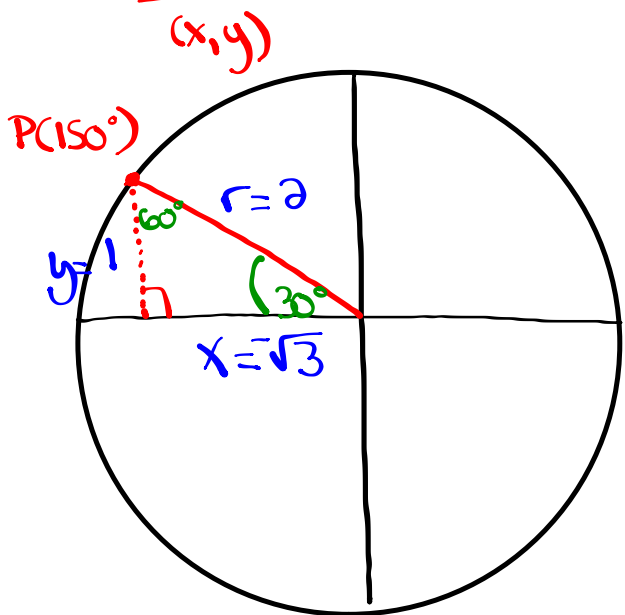
$$x = \pm \frac{1}{\sqrt{2}}$$

$$x = -\frac{1}{\sqrt{2}} \quad (\text{Q3})$$

coordinates are $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

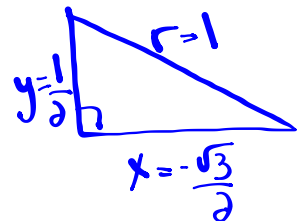
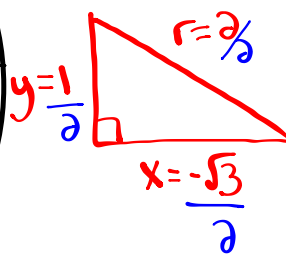
Problems Involving the Unit Circle:

If $P(150^\circ)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...



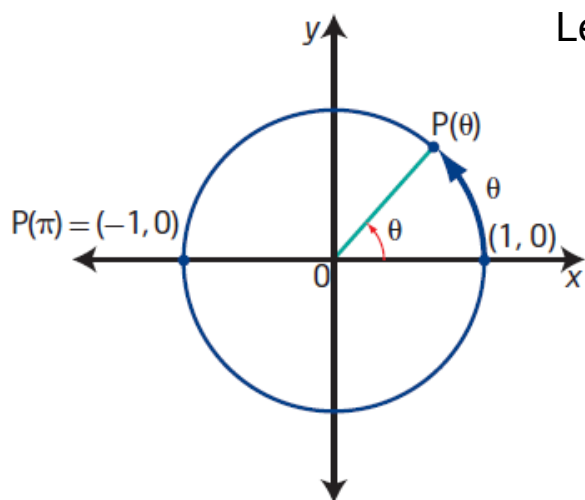
$$r=1$$

Scale the diagram so that $r=1$ (unit circle)



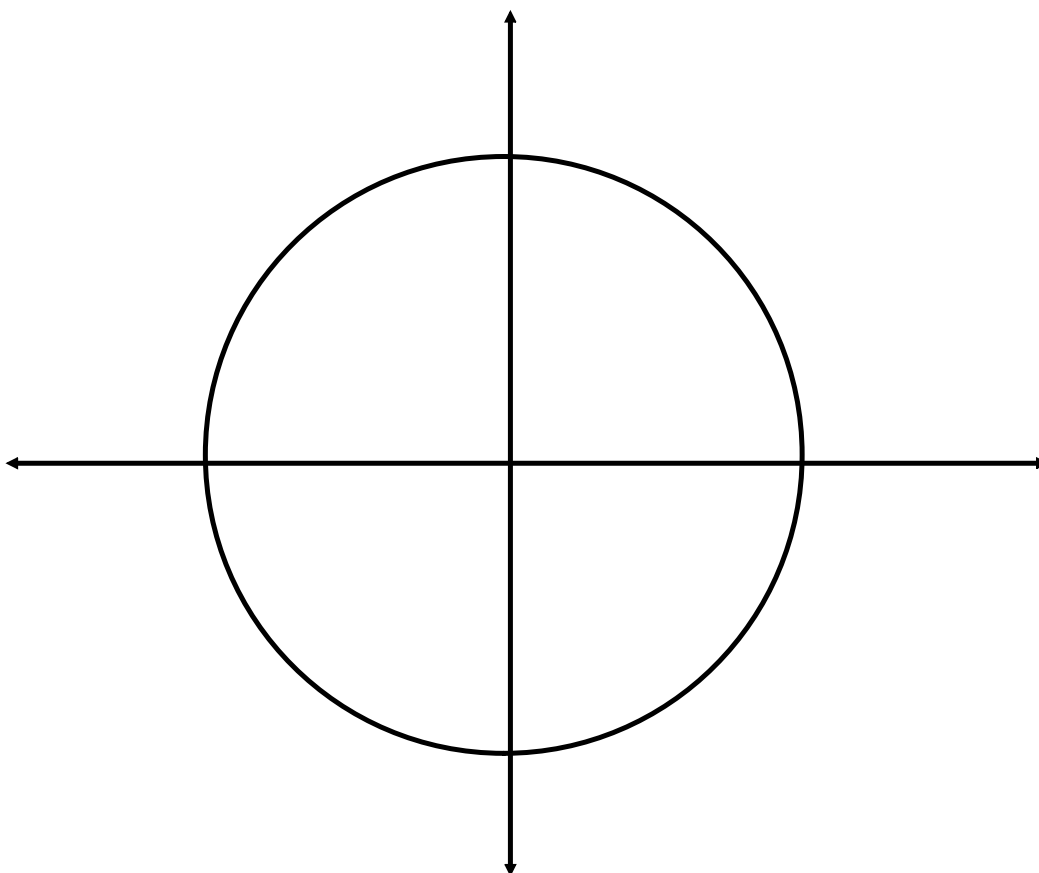
coordinates are $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

Special Angles on the Unit Circle:

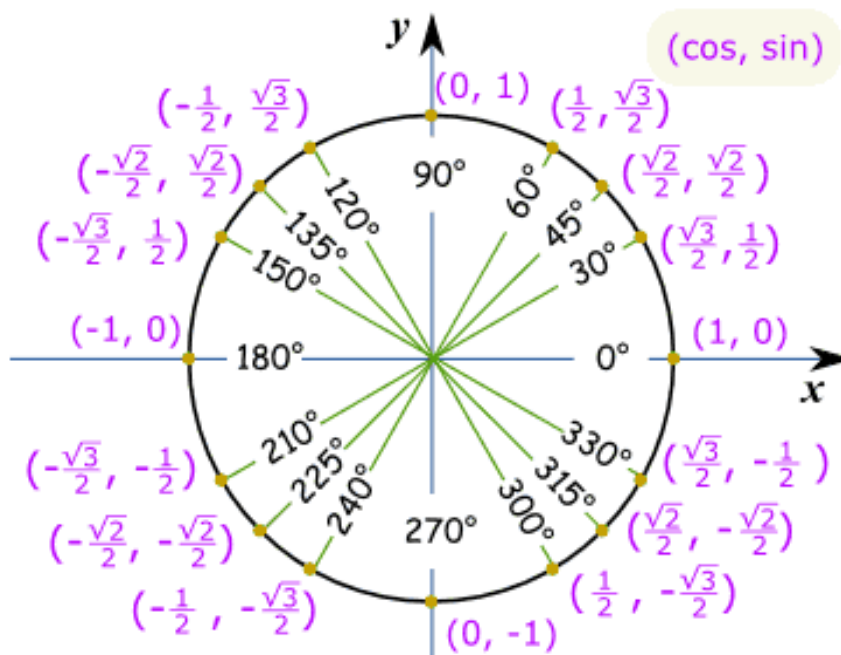


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

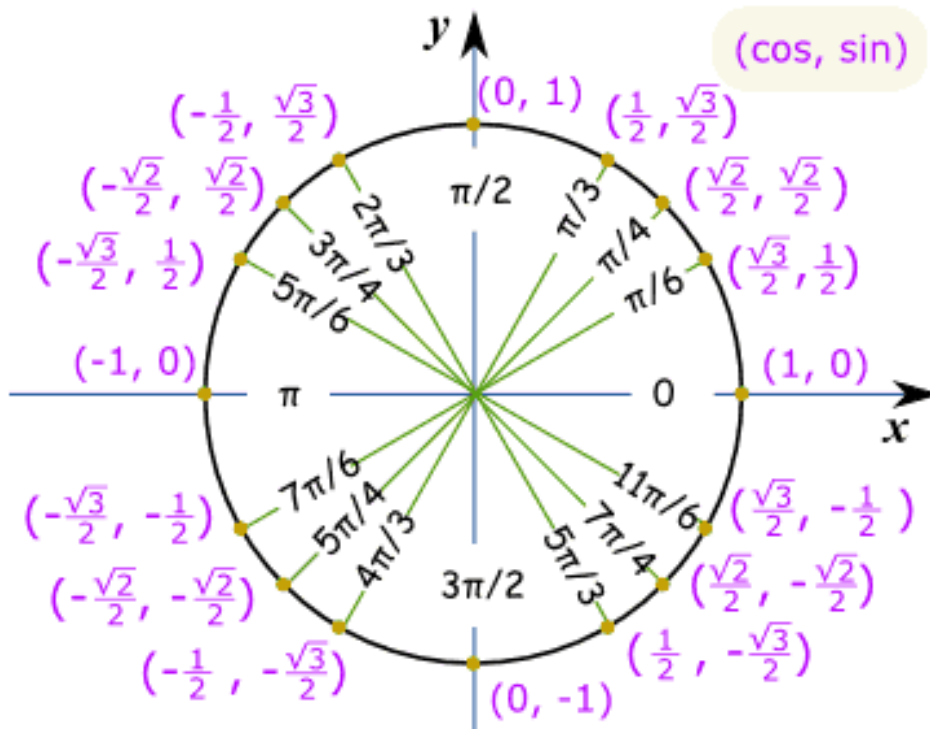


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Finish worksheet

Attachments

Worksheet - Sketching Angles in Radians.doc