

Understanding Logarithms

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

For the exponential function $y = c^x$, the inverse is $x = c^y$. This inverse is also a function and is called a **logarithmic function**. It is written as $y = \log_c x$, where c is a positive number other than 1.

Logarithmic Form Exponential Form



Since our number system is based on powers of 10, **logarithms** with base 10 are widely used and are called **common logarithms**. When you write a common logarithm, you do not need to write the base. For example, $\log 3$ means $\log_{10} 3$.

$$\text{or } \log_{10} 150 = \log 150$$

logarithmic function

- a function of the form $y = \log_c x$, where $c > 0$ and $c \neq 1$, that is the inverse of the exponential function $y = c^x$

logarithm

- an exponent
- in $x = c^y$, y is called the logarithm to base c of x

common logarithm

- a logarithm with base 10

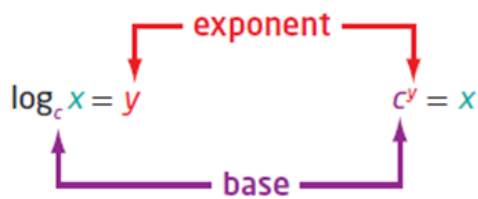
Write each of the following in logarithmic form

a) $32 = 2^5$
 (Handwritten: "ans" points to 5, "exp" points to 2, "Base" points to 2)
 $\log_2(32) = 5$

b) $2^{-5} = \frac{1}{32}$
 $\log_2\left(\frac{1}{32}\right) = -5$

c) $x = 10^y$
 $\log_{10}(x) = y$
 $\log x = y$

Logarithmic Form Exponential Form



Write each of the following in exponential form

a) $\log_4 16 = 2$
 (Handwritten: "ans" points to 2, "exp" points to 16, "Base" points to 4)
 $4^2 = 16$

b) $\log_2\left(\frac{1}{32}\right) = -5$
 $2^{-5} = \frac{1}{32}$

c) $\log 65 = 1.8129$
 $10^{1.8129} = 65$

Example 1

Evaluating a Logarithm

Evaluate.

a) $\log_7 49 = ?$

$$\frac{\log 49}{\log 7} = ?$$

$$x = \log_7 49$$

$$7^x = 49$$

~~$$7^x = (7)^2$$~~

$$x = 2$$

$$\boxed{\log_7 49 = 2}$$

b) $\log_6 1 = ?$

$$\frac{\log 1}{\log 6} = ?$$

$$x = \log_6 1$$

$$6^x = 1$$

~~$$6^x = (6)^0$$~~

$$x = 0$$

$$\boxed{\log_6 1 = 0}$$

c) $\log 0.001 = -3$

$$\frac{\log 0.001}{\log 10} = -3$$

$$x = \log 0.001$$

$$10^x = 0.001$$

~~$$10^x = (10)^{-3}$$~~

$$x = -3$$

$$\boxed{\log 0.001 = -3}$$

d) $\log_2 \sqrt{8} = 1.5$

$$\frac{\log \sqrt{8}}{\log 2} = 1.5$$

$$x = \log_2 \sqrt{8}$$

$$2^x = \sqrt{8}$$

$$2^x = (8)^{1/2}$$

$$2^x = (2^3)^{1/2}$$

~~$$2^x = 2^{3/2}$$~~

$$x = \frac{3}{2}$$

$$\boxed{\log_2 \sqrt{8} = \frac{3}{2}}$$

Example 2

Determine an Unknown in an Expression in Logarithmic Form

Determine the value of x . (convert to exponential form)

a) $\log_5 x = -3$

b) $\log_x 36 = 2$

c) $\log_{64} x = \frac{2}{3}$

a) $\log_5 x = -3$ (log. form)

$$5^{-3} = x \text{ (exp. form)}$$

$$\left(\frac{1}{5}\right)^3 = x$$

$$\boxed{\frac{1}{125} = x}$$

b) $\log_x 36 = 2$ (log. form)

$$x^2 = 36 \text{ (exp. form)}$$

$$x = \pm 6$$

$$\boxed{x = 6} \quad c > 0$$

c) $\log_{64} x = \frac{2}{3}$ (log. form)

$$64^{\frac{2}{3}} = x \text{ (exp. form)}$$

$$16 = x$$

$$2^4 = 16 \quad (\text{exponential form})$$

↑ Base ↑ ans.

← exp.

$$\log_2(16) = 4 \quad (\text{logarithmic form})$$

$$2^{-3} = \frac{1}{8} \quad (\text{exp.})$$

$$\log_2\left(\frac{1}{8}\right) = -3 \quad (\text{log.})$$

Exponential

$$y = 2^x$$

Find Inverse

$$\textcircled{1} \quad x = 2^y \quad (\text{exp.})$$

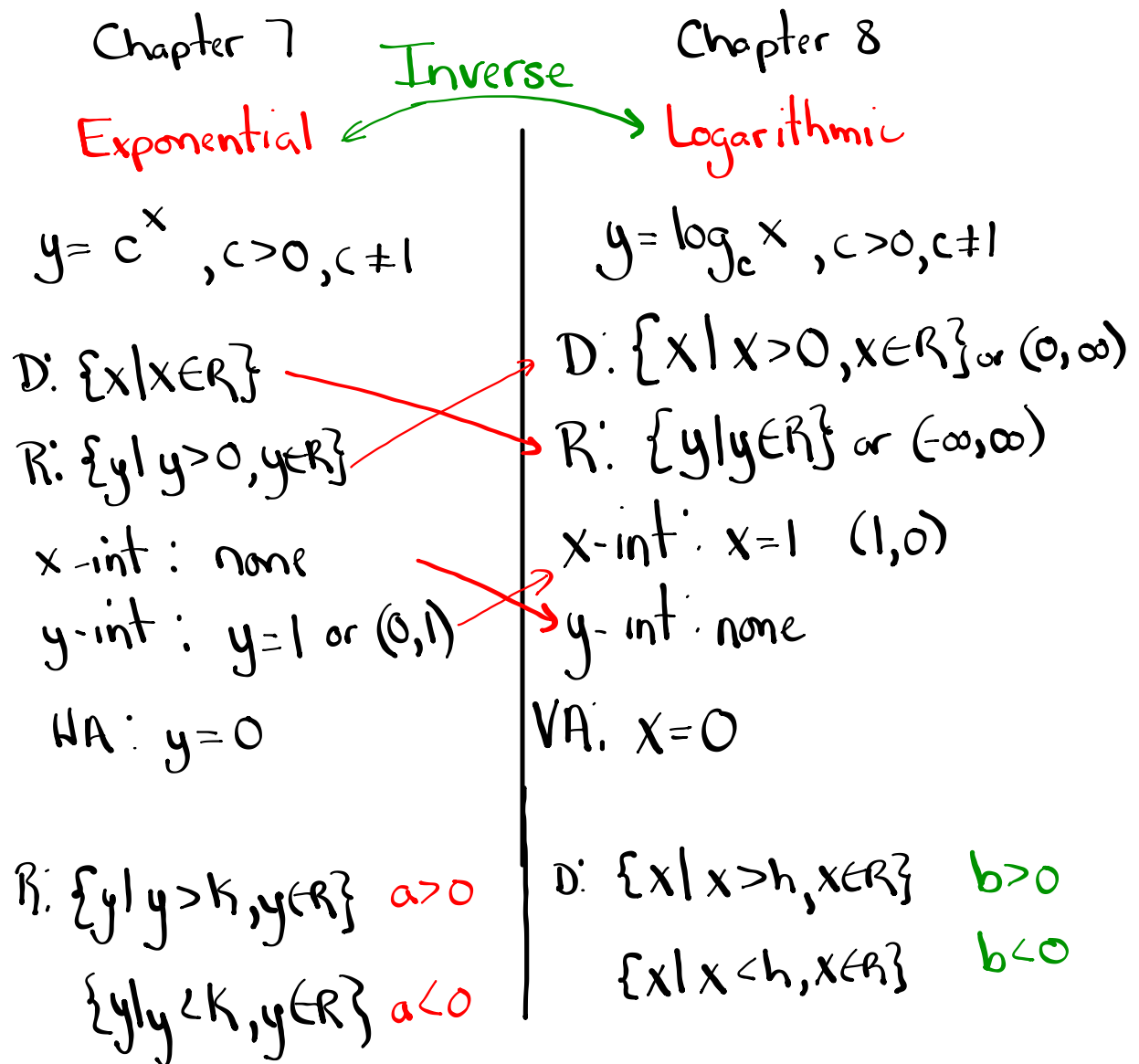
$$\log_2 x = y \quad (\text{log.})$$

$$y = \log_2 x$$

$$y = 2^x \rightarrow y = \log_2 x$$

$$y = 3^x \rightarrow y = \log_3 x$$

$$y = 5^x \rightarrow y = \log_5 x$$



Example 3

Graph the Inverse of an Exponential Function

- a) State the inverse of $f(x) = 3^x$.
- b) Sketch the graph of the inverse. Identify the following characteristics of the inverse graph: $(x, y) \rightarrow (y, x)$
- the domain and range
 - the x-intercept, if it exists
 - the y-intercept, if it exists
 - the equations of any asymptotes

$$a) f(x) = 3^x$$

$$y = 3^x$$

$$x = 3^y \quad (\text{exp. form})$$

$$\log_3 x = y \quad (\text{log form})$$

$$y = \log_3 x$$

$$f^{-1}(x) = \log_3 x$$

$y = 3^x$ passes the HLT

$$b) y = 3^x \quad (x, y) \rightarrow (y, x)$$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

Inverse

$$y = \log_3 x$$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Solution

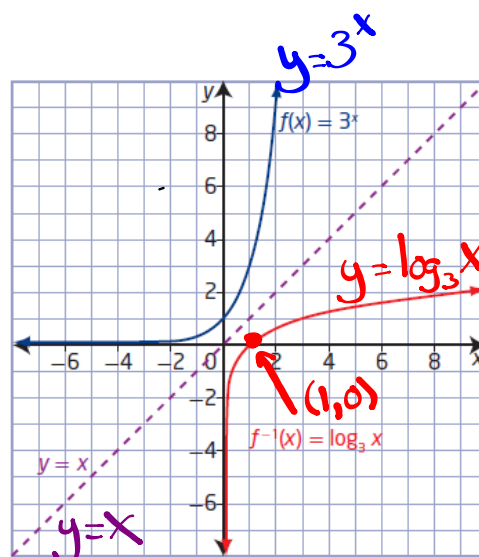
- a) The inverse of $y = f(x) = 3^x$ is $x = 3^y$ or, expressed in logarithmic form, $y = \log_3 x$. Since the inverse is a function, it can be written in function notation as

How do you know that $y = \log_3 x$ is a function?

- b) Set up tables of values for both the exponential function, $f(x)$, and its inverse, $f^{-1}(x)$. Plot the points and join them with a smooth curve.

$y = 3^x$ $f(x) = 3^x$	
x	y
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

$y = \log_3 x$ $f^{-1}(x) = \log_3 x$	
x	y
$\frac{1}{27}$	-3
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2
27	3



The graph of the inverse, $f^{-1}(x) = \log_3 x$, is a reflection of the graph

of $f(x) = 3^x$ about the line $y = x$. For $f^{-1}(x) = \log_3 x$,

- the domain is $\{x \mid x > 0, x \in \mathbb{R}\}$ and the range is $\{y \mid y \in \mathbb{R}\}$ or $(-\infty, \infty)$
- the x-intercept is 1 or $(1, 0)$
- there is no y-intercept
- the vertical asymptote, the y-axis, has equation $x = 0$; there is no horizontal asymptote

How do the characteristics of $f^{-1}(x) = \log_3 x$ compare to the characteristics of $f(x) = 3^x$?

Key Ideas

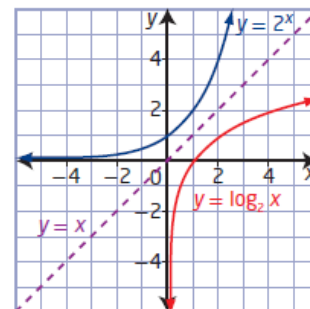
- A logarithm is an exponent.
- Equations in exponential form can be written in logarithmic form and vice versa.

Exponential Form **Logarithmic Form**

$$x = c^y \qquad y = \log_c x$$

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.
- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown.
- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,
 - the domain is $\{x \mid x > 0, x \in \mathbb{R}\}$
 - the range is $\{y \mid y \in \mathbb{R}\}$
 - the x-intercept is 1
 - the vertical asymptote is $x = 0$, or the y-axis
- A common logarithm has base 10. It is not necessary to write the base for common logarithms:

$$\log_{10} x = \log x$$



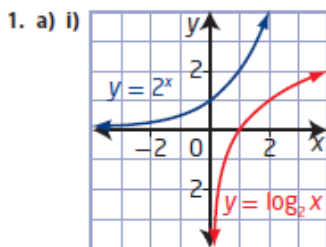
Homework

#1-5, 8, 10, 12, 13, 17 on page 380

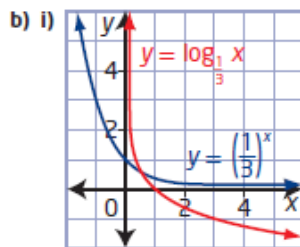
$$\begin{array}{l} y = 2^x \rightarrow y = \log_2 x \\ y = 3^x \rightarrow y = \log_3 x \\ y = 4^x \rightarrow y = \log_4 x \end{array}$$

Inverse

8.1 Understanding Logarithms, pages 380 to 382

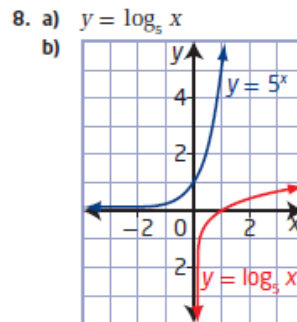


ii) $y = \log_2 x$
 iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1, no y-intercept,
 vertical asymptote $x = 0$



ii) $y = \log_3 x$
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2. a) $\log_{12} 144 = 2$ b) $\log_8 2 = \frac{1}{3}$
 c) $\log_{10} 0.000\ 01 = -5$ d) $\log_7 (y + 3) = 2x$
3. a) $5^2 = 25$ b) $8^{\frac{2}{3}} = 4$
 c) $10^6 = 1\ 000\ 000$ d) $11^y = x + 3$
4. a) 3 b) 0 c) $\frac{1}{3}$ d) -3
5. $a = 4; b = 5$



domain $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1,
 no y-intercept,
 vertical asymptote $x = 0$

10. They are reflections of each other in the line $y = x$.
11. a) They have the exact same shape.
 b) One of them is increasing and the other is decreasing.
12. a) 216 b) 81 c) 64 d) 8
13. a) 7 b) 6
14. a) 0 b) 1
15. -1
16. 16
17. a) $t = \log_{0.11} N$ b) 145 days
18. The larger asteroid had a relative risk that was 1479 times as dangerous.
19. 1000 times as great
20. 5
21. $m = 14, n = 13$
22. $4n$
23. $y = 3^{2^x}$

