# Warm Up

1. Simplify: $\dfrac{9x^2 \cdot \frac{1}{\cancel{x^2}} - \frac{1}{\cancel{9}} \cdot \cancel{9}x^2}{\frac{(x-3)}{1} 9x^2}$

CD: $9x^2$

$$\frac{\frac{9\cancel{x^2}}{\cancel{x^2}} - \frac{\cancel{9}x^2}{\cancel{9}}}{9x^2(x-3)}$$

$$\frac{9 - x^2}{9x^2(x-3)}$$

$$\frac{-1(-9+x^2)}{9x^2(x-3)}$$

$$\frac{-(x^2-9)}{9x^2(x-3)}$$

diff of squares

$$\frac{-(x+3)\cancel{(x-3)}}{9x^2\cancel{(x-3)}}$$

$$\boxed{\frac{-(x+3)}{9x^2}}$$

3. Rationalize the denominator: (think conjugates)

$$\frac{(x+2)}{(\sqrt{x-4}-\sqrt{x-6})} \cdot \frac{(\sqrt{x-4}+\sqrt{x-6})}{(\sqrt{x-4}+\sqrt{x-6})}$$

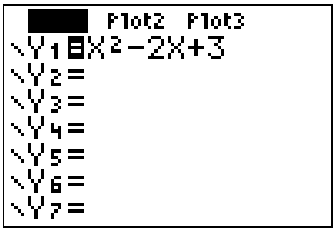
$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{x-4+\cancel{\sqrt{(x-4)(x-6)}}-\cancel{\sqrt{(x-4)(x-6)}}-(x-6)}$$

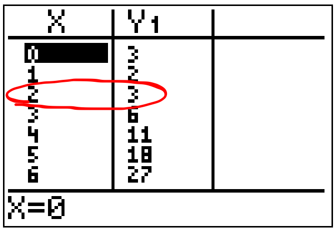
$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{x-4-x+6}$$

$$\boxed{\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{2}}$$

# Limit of a Function

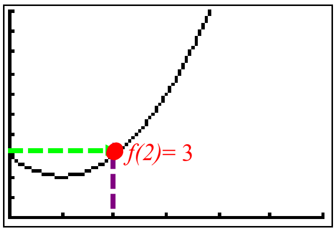
Let's examine the function $f(x) = x^2 - 2x + 3$ (parabola)



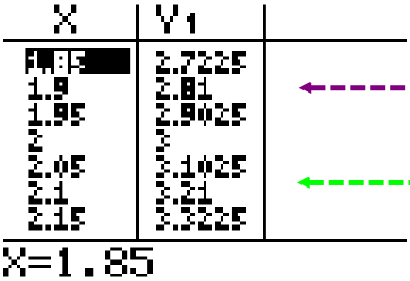



| X | Y1 |
|---|---|
| 0 | 3 |
| 1 | 2 |
| 2 | 3 |
| 3 | 6 |
| 4 | 11 |
| 5 | 18 |
| 6 | 27 |

X=0




We can see that $f(2) = 3$...let's check the behaviour of $f$ as we get closer and closer to $x = 2$.



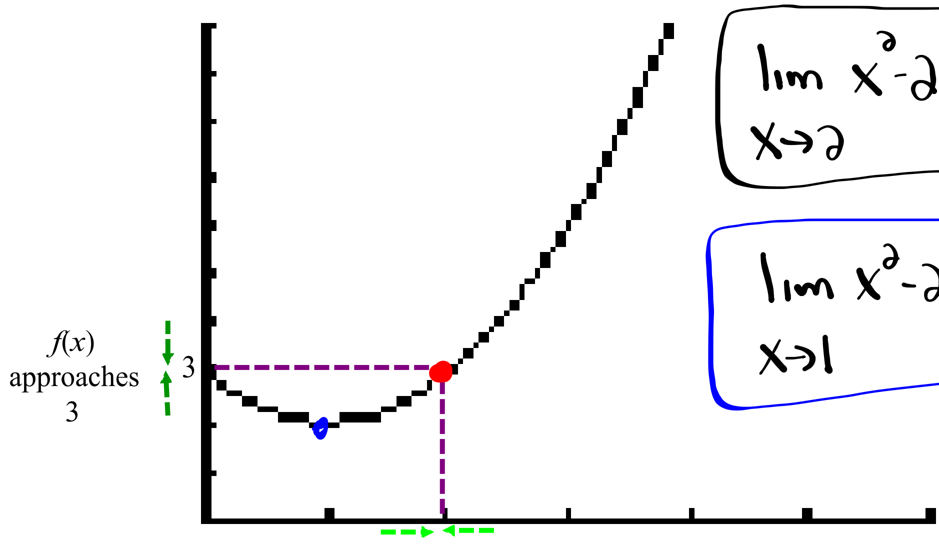
| X | Y1 |
|---|---|
| 1.85 | 2.7225 |
| 1.9 | 2.81 |
| 1.95 | 2.9025 |
| 2 | 3 |
| 2.05 | 3.1025 |
| 2.1 | 3.21 |
| 2.15 | 3.3225 |

X=1.85

As $x$ gets closer to 2 from the left $y$ is getting closer to 3.

As $x$ gets closer to 2 from the right $y$ is getting closer to 3.

From the above, the notion of the limit of a function arises...




As $x$ approaches 2

height

$$\boxed{\lim_{x \to 2} x^2 - 2x + 3 = 3}$$

height

$$\boxed{\lim_{x \to 1} x^2 - 2x + 3 = 2}$$

Notation: $\lim_{x \to 2} f(x) = 3$

"The limit of the function $f(x)$ as $x$ approaches 2 is equal to 3."

## The common sense definition of a limit...

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## A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to $L$

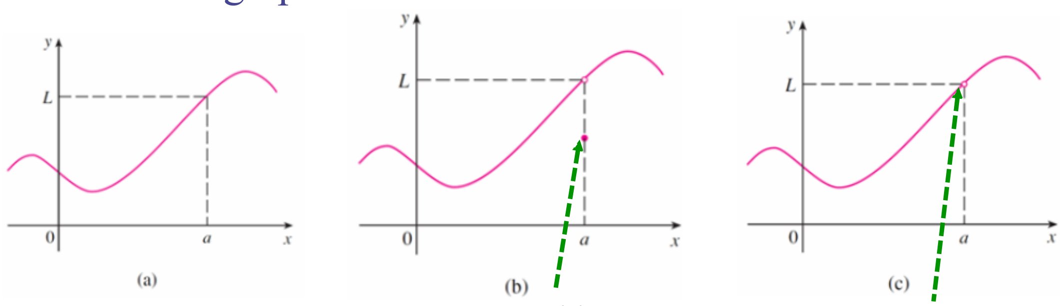
- (as close to $L$ as we like)

by taking $x$ to be sufficiently close to $a$

- (on either side of $a$)

but not equal to $a$.

Look at the graphs of these three functions...




Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at $a$, it is true that

$$\lim_{x \to a} f(x) = L$$

# Evaluating Limits

## I. Using a Graph:

- We looked at this in the previous two examples

## II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to 2}\frac{x^2-2x+1}{x+3}$$

$$\lim_{x\to -2}\frac{(-2)^2-2(-2)+1}{(-2)+3}=\frac{9}{1}$$

$$\lim_{x\to 3}(16-x^2)$$

$$\lim_{x\to 3}(16-(3)^2)=7$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

  $\Rightarrow$ Factor
  $\Rightarrow$ Rationalize
  $\Rightarrow$ Expand
  $\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x\to 4}\frac{x^2-16}{x-4}$$

$$\lim_{x\to 4}\frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$$

$$\lim_{x\to 4}(4+4)=8$$

$$\lim_{h\to 0}\frac{(\sqrt{4+h}-2)}{h}\frac{(\sqrt{4+h}+2)}{(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{1}{(\sqrt{4+0}+2)}=\frac{1}{4}$$

Try these...remember to use your algebra skills
to try and eliminate the **indeterminate form**.

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

$$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$$

# Homework