Questions from Homework

Apply

6. Consider the function $f(x) = \frac{1}{4}\sqrt{5x}$.

   a) Identify the transformations represented by $f(x)$ as compared to $y = \sqrt{x}$.

   b) Write two functions equivalent to $f(x)$: one of the form $y = a\sqrt{x}$ and the other of the form $y = \sqrt{bx}$.

   c) Identify the transformation(s) represented by each function you wrote in part b).

   d) Use transformations to graph all three functions. How do the graphs compare?

c) $a = \frac{1}{4} \rightarrow$ vertical stretch about the x-axis by a factor of $\frac{1}{4}$

$b = 5 \rightarrow$ horizontal stretch about the y-axis by a factor of $\frac{1}{5}$

b)(i) $y = \frac{1}{4}\sqrt{5x} = \frac{1}{4}\cdot\sqrt{5}\sqrt{x} = \frac{\sqrt{5}}{4}\sqrt{x}$ $(y = a\sqrt{x})$

$a = \frac{\sqrt{5}}{4}$

$VSF = \frac{\sqrt{5}}{4}$

(ii) $y = \frac{1}{4}\sqrt{5x} = \sqrt{\frac{1}{4}\cdot\frac{1}{4}\cdot 5x} = \sqrt{\frac{5}{16}x}$ $(y = \sqrt{bx})$

$b = \frac{5}{16}$

$HSF = \frac{16}{5}$

d) All three curves are the same.

Square Root of a Function

Focus on...

- sketching the graph of $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- explaining strategies for graphing $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- comparing the domains and ranges of the functions $y = f(x)$ and $y = \sqrt{f(x)}$, and explaining any differences

square root of a function

- the function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$
- $y = \sqrt{f(x)}$ is only defined for $f(x) \geq 0$

The function $y = \sqrt{2x + 1}$ represents the **square root of the function** $y = 2x + 1$.

x	$y = 2x + 1$	$y = \sqrt{2x + 1}$
0	1	$\sqrt{1} =$ 1
4	9	$\sqrt{9} =$ 3
12	25	$\sqrt{25} =$ 5
24	49	$\sqrt{49} =$ 7
⋮	⋮	⋮

when $x = 0$

$y = 2x + 1$

$y = 2(0) + 1$

$y = 0 + 1$

$y = 1$

Example 1

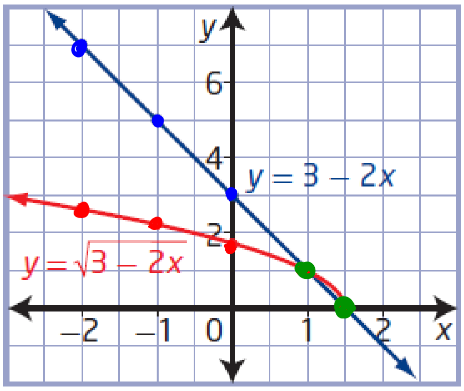
Compare Graphs of a Linear Function and the Square Root of the Function

linear

a) Given $f(x) = 3 - 2x$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
b) Compare the two functions.

Use a table of values to graph $y = 3 - 2x$ and $y = \sqrt{3 - 2x}$.

x	$y = 3 - 2x$	$y = \sqrt{3 - 2x}$
−2	7	$\sqrt{7} = 2.7$
−1	5	$\sqrt{5} = 2.2$
0	3	$\sqrt{3} = 1.7$
1	1	$\sqrt{1} = 1$
1.5	0	$\sqrt{0} = 0$



Invariant points are (1, 1) and (1.5, 0)

$y = 3 - 2x$:

D: $\{x | x \in R\}$ or $(-\infty, \infty)$

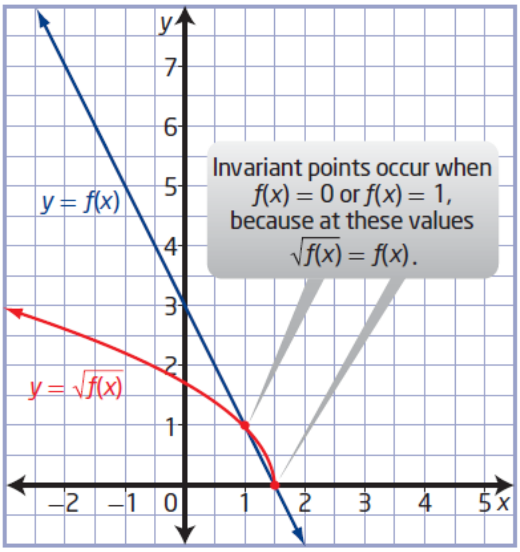
R: $\{y | y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{3 - 2x}$

D: $\{x | x \leq 1.5, x \in R\}$ or $(-\infty, 1.5]$

R: $\{y | y \geq 0, y \in R\}$ or $[0, \infty)$

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

For $y = f(x)$, the domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \in R\}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1.5, x \in R\}$ and the range is $\{y \mid y \geq 0, y \in R\}$.

Invariant points occur at (1, 1) and (1.5, 0).

How does the domain of the graph of $y = \sqrt{f(x)}$ relate to the restrictions on the variable in the radicand? How could you determine the domain algebraically?

Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$

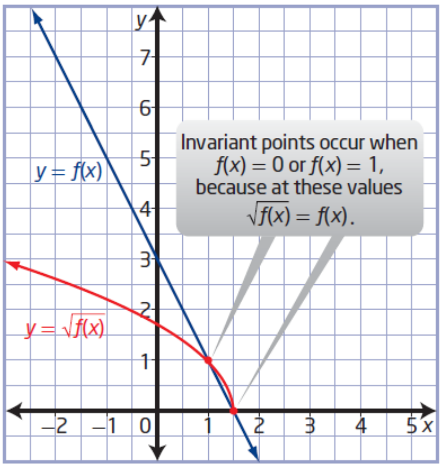
The domain of $y = \sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x) \geq 0$.

The range of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of $y = f(x)$ for which $\sqrt{f(x)}$ is defined.

The graph of $y = \sqrt{f(x)}$ exists only where $f(x) \geq 0$. You can predict the location of $y = \sqrt{f(x)}$ relative to $y = f(x)$ using the values of $f(x)$.

	undefined	I.P.	Above f(x)	I.P.	Below f(x)
Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative Location of Graph of $y = \sqrt{f(x)}$	The graph of $y = \sqrt{f(x)}$ is undefined.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ intersect on the x-axis.	The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ intersects the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ is below the graph of $y = f(x)$.

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

Your Turn

a) Given $g(x) = 3x + 6$, graph the functions $y = g(x)$ and $y = \sqrt{g(x)}$.

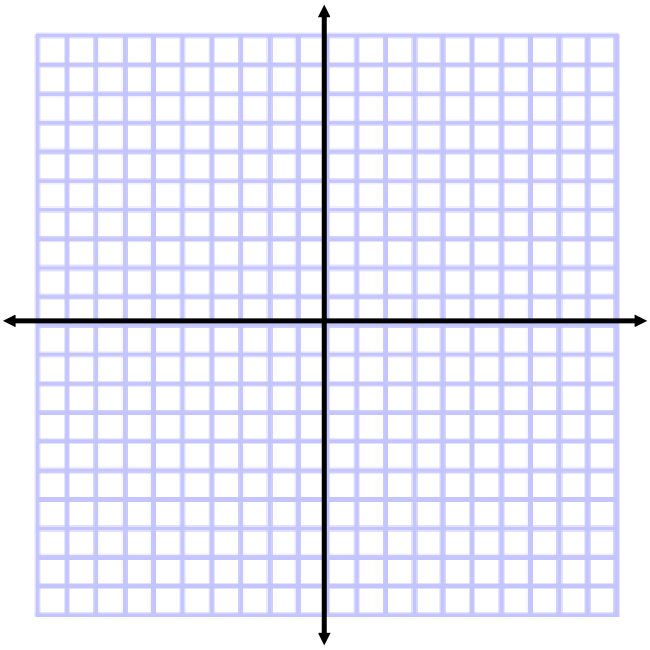
b) Identify the domain and range of each function and any invariant points.

Example 2

Compare the Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

Identify and compare the domains and ranges of the functions in each pair.

a) $y = 2 - 0.5x^2$ and $y = \sqrt{2 - 0.5x^2}$



Example 3

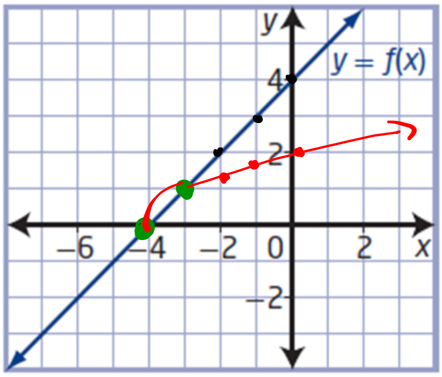
Graph the Square Root of a Function From the Graph of the Function

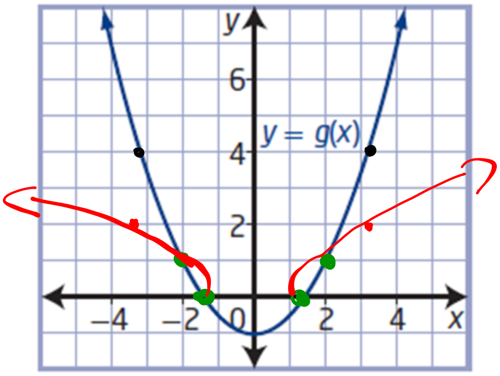
Step 1: Locate invariant points (Find points where $y=0$ and $y=1$) $\sqrt{f(x)}$

Step 2: Draw the portion of each graph between the invariant points (above $f(x)$)

Step 3: Locate other key points on $y = f(x)$ and $y = g(x)$ where the values are greater than 1. Transform these points to locate image points on the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$. ($\sqrt{f(x)}$ below $f(x)$)

Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.





$y = f(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \geq -4, x \in R\}$ or $[-4, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

$y = g(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -1, y \in R\}$ or $[-1, \infty)$

$y = \sqrt{g(x)}$

D: $\{x \mid x \leq -1.5, x \geq 1.5, x \in R\}$

or $(-\infty, -1.5] + [1.5, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

Key Ideas

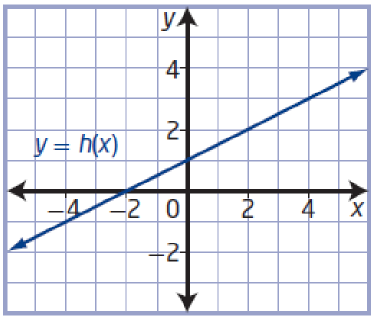
- You can use values of $f(x)$ to predict values of $\sqrt{f(x)}$ and to sketch the graph of $y = \sqrt{f(x)}$.
- The key values to consider are $f(x) = 0$ and $f(x) = 1$.
- The domain of $y = \sqrt{f(x)}$ consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$.
- The range of $y = \sqrt{f(x)}$ consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
- The y-coordinates of the points on the graph of $y = \sqrt{f(x)}$ are the square roots of the y-coordinates of the corresponding points on the original function $y = f(x)$.

What do you know about the graph of $y = \sqrt{f(x)}$ at $f(x) = 0$ and $f(x) = 1$? How do the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$ compare on either side of these locations?

Your Turn

1) Identify and compare the domains and ranges of the functions $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$. Verify your answers.

2) Using the graph of $y = h(x)$, sketch the graph of $y = \sqrt{h(x)}$.



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