

10.4

⑤ b)  $t_5 = 8$ ,  $t_{10} = \frac{1}{4}$        $t_3 = ?$  Find "a" + "r"

$t_n = ar^{n-1}$	$t_n = ar^{n-1}$
$t_5 = ar^{5-1}$	$t_{10} = ar^{10-1}$
$t_5 = ar^4$	$t_{10} = ar^9$
$8 = ar^4$	$\frac{1}{4} = ar^9$
$8 = a\left(\frac{1}{2}\right)^4$	

Elimination

$$\frac{\frac{1}{4} = ar^9}{8 = ar^4}$$

$$\frac{1}{4} \div 8 = r^{9-4}$$

$$\frac{1}{4} \times \frac{1}{8} = r^5$$

$$\frac{1}{32} = r^5$$

$$8 = a\left(\frac{1}{16}\right)$$

$$16 \cdot 8 = \frac{a}{16} \cdot 16$$

$$128 = a$$

$$\left(\frac{1}{32}\right)^{\frac{1}{5}} = \left(r^5\right)^{\frac{1}{5}} \quad \text{or} \quad \frac{\sqrt[5]{1}}{\sqrt[5]{32}} = \sqrt[5]{r^5}$$

$$\frac{1}{2} = r$$

$$\frac{1}{2} = r$$

$\frac{128}{4}, \frac{64}{4}, \frac{32}{4}$

$$t_3 = ar^{3-1}$$

$$t_3 = (128)\left(\frac{1}{2}\right)^2$$

$$t_3 = (128)\left(\frac{1}{4}\right)$$

$$t_3 = \frac{128}{4} = 32$$

10.9

② Find  $S_7$  for  $\underline{30}^{-5} + \frac{5}{6} - \dots$

$\swarrow$        $\swarrow$   
 $-\frac{1}{6}$     $-\frac{1}{6}$

Given:

$$a = 30$$

$$r = \frac{-5}{30} = -\frac{1}{6}$$

$$n = 7$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{30 \left[ \left(-\frac{1}{6}\right)^7 - 1 \right]}{-\frac{1}{6} - 1}$$

$$S_7 = \frac{30 \left[ \frac{-1}{279936} \left( \frac{-1}{1} \right) \right]}{-\frac{1}{6} - \left( \frac{1}{1} \right)}$$

$$S_7 = \frac{30 \left[ \frac{-1}{279936} \left( \frac{279936}{279936} \right) \right]}{-\frac{1}{6} - \left( \frac{6}{6} \right)}$$

$$S_7 = 30 \left( \frac{-279936}{279936} \right) \div \frac{-7}{6}$$

$$S_7 = \frac{5}{30} \left( \frac{39991}{-279936} \right) \left( \frac{-6}{7} \right)$$

$\frac{46656}{776}$

$$S_7 = 5 \left( \frac{-39991}{776} \right) \left( \frac{-1}{1} \right) = \boxed{\frac{199955}{776}}$$

or

$$S_7 = \frac{50388660}{1999552} = \boxed{\frac{199955}{776}}$$

10.9

$$\textcircled{a} \quad \underline{30} - 5 + \frac{5}{6}$$

$\swarrow$   $\searrow$   
 $-\frac{1}{6}$   $-\frac{1}{6}$

Given:

$a = 30$

$r = -\frac{1}{6}$

$$S_7 = \frac{(30) \left[ \left( -\frac{1}{6} \right)^7 - 1 \right]}{\left( -\frac{1}{6} \right) - 1}$$

$$S_7 = \frac{(30) \left[ \frac{-1}{279936} - \frac{1}{1} \right]}{-\frac{1}{6} - \frac{1}{1}}$$

$$S_7 = \frac{(30) \left[ \frac{-1}{279936} - \frac{279936}{279936} \right]}{\frac{-1}{6} - \frac{6}{6}}$$

$$S_7 = 30 \left( \frac{-279937}{279936} \right) \div \frac{-7}{6}$$

$$S_7 = 30 \left( \frac{-279937}{279936} \right) \left( \frac{6}{7} \right)$$

$$S_7 = \frac{50388600}{1959552} \div 6$$

$$S_7 = \frac{8398110}{326592} \div 2$$

$$S_7 = \frac{4199055}{163296} \div 21$$

$$S_7 = \frac{199955}{776} \text{ or } 25 \frac{555}{776}$$

10.9  
 ③ a)  $81 + 27 + 9 + \dots$   $S_6 = \frac{81 \left[ \left( \frac{1}{3} \right)^6 - 1 \right]}{\left( \frac{1}{3} \right) - 1}$   
 $S_6 = ?$   
 $a = 81$   
 $r = \frac{1}{3}$   
 $S_6 = \frac{81 \left[ \frac{1}{729} - 1 \right]}{\frac{1}{3} - 1}$   
 $S_6 = \frac{81 \left[ \frac{1 - 729}{729} \right]}{\frac{1}{3} - \frac{3}{3}}$   
 $S_6 = \frac{81 \left( \frac{-728}{729} \right) \div \frac{-2}{3}}$   
 $S_6 = 81 \left( \frac{-728}{729} \right) \left( \frac{-3}{2} \right)$   
 $S_6 = \frac{176904}{1458} = \frac{364}{3}$

③ b)  $1 + \frac{5}{9} + \frac{25}{9} + \dots + \frac{15625}{64}$   
 $a = 1$   
 $r = \frac{5}{3}$   
 $t_n = \frac{15625}{64}$   
 Find n:  
 $t_n = ar^{n-1}$   
 $\frac{15625}{64} = \left( \frac{5}{3} \right)^{n-1}$   
 $\frac{15625}{64} = \left( \frac{5}{3} \right)^{n-1}$   
 $\left( \frac{5}{3} \right)^6 = \left( \frac{5}{3} \right)^{n-1}$   
 $6 = n - 1 + 1$   
 $n = 7$   
 $\frac{\log \left( \frac{15625}{64} \right)}{\log \left( \frac{5}{3} \right)} = 6$

④ Find  $S_7$   
 $S_7 = \frac{1 \left[ \left( \frac{5}{3} \right)^7 - 1 \right]}{\left( \frac{5}{3} \right) - 1}$   
 $S_7 = \frac{1 \left[ \frac{78125}{108} - \frac{1}{1} \right]}{\frac{5}{3} - \frac{1}{3}}$   
 $S_7 = \frac{1 \left[ \frac{78125}{108} - \frac{128}{108} \right]}{\frac{5}{3} - \frac{1}{3}}$   
 $S_7 = \frac{1 \left( \frac{77997}{108} \right) \div \frac{4}{3}}$   
 $S_7 = \frac{1 \left( \frac{77997}{108} \right) \left( \frac{3}{4} \right)}$   
 $S_7 = \frac{155994}{384} = \frac{25999}{64}$

⑤  $S_7 = 1093$   $S_7 = \frac{a \left[ \left( \frac{1}{3} \right)^7 - 1 \right]}{\left( \frac{1}{3} \right) - 1}$   
 $r = \frac{1}{3}$   
 $a = ?$   
 $1093 = \frac{a \left[ \frac{1}{2187} - \frac{2187}{2187} \right]}{\frac{1}{3} - \frac{3}{3}}$   
 $1093 = \frac{a \left[ \frac{-2186}{2187} \right] \div \frac{-2}{3}}$   
 $1093 = a \left( \frac{-2186}{2187} \right) \left( \frac{-3}{2} \right)$   
 $4374 \cdot 1093 = \frac{6558a}{4374} \cdot 4374$   
 $4780782 = \frac{6558a}{658}$   
 $729 = a$

10.9

$$\textcircled{1} \text{ b) } \underline{2} + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$$

$$a = 2$$

$$\begin{aligned} r &= \frac{2}{3} \div \frac{2}{1} \\ &= \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} \\ &= \frac{1}{3} \end{aligned}$$

$$n = 4$$

$$S_4 = \frac{(2) \left[ \left( \frac{1}{3} \right)^4 - 1 \right]}{\left[ \left( \frac{1}{3} \right) - 1 \right]}$$

$$= \frac{2 \left[ \frac{1}{81} - \frac{81}{81} \right]}{\left[ \frac{1}{3} - \frac{3}{3} \right]}$$

$$= \frac{2 \left( -\frac{80}{81} \right)}{\left( -\frac{2}{3} \right)}$$

$$= \cancel{2} \left( -\frac{80}{\cancel{81} \cancel{2}} \right) \left( -\frac{\cancel{3}}{\cancel{2}} \right)$$

$$= \frac{480}{162}$$

$$= \frac{80}{27}$$

10.9

① Given:

$$t_7 = 192$$

$$a = 3$$

$$S_8 = ?$$

(1) Find  $r$ :

$$t_n = ar^{n-1}$$

$$192 = (3)r^{7-1}$$

$$\frac{192}{3} = \frac{3r^6}{3}$$

$$64 = r^6$$

$$\pm 2 = r$$

if  $r = 2$ 

$$S_8 = \frac{(3)[(2)^8 - 1]}{(2) - 1}$$

$$= \frac{3(256 - 1)}{1}$$

$$= 3(255)$$

$$= 765$$

if  $r = -2$ 

$$S_8 = \frac{(3)[(-2)^8 - 1]}{(-2) - 1}$$

$$= \frac{3(256 - 1)}{-3}$$

$$= \frac{\cancel{3}(255)}{\cancel{-3}}$$

$$= -255$$

Review

② b)  $16 + 8 + 4 + 2 + \dots + \frac{1}{32}$

$\underbrace{\quad}_{\frac{1}{2}} \quad \underbrace{\quad}_{\frac{1}{2}} \quad \underbrace{\quad}_{\frac{1}{2}}$

$$a = 16$$

$$r = \frac{1}{2}$$

$$t_n = \frac{1}{32}$$

(1) Find n:

$$t_n = ar^{n-1}$$

$$\frac{1}{32} = \overset{\div 16}{(16)} \left(\frac{1}{2}\right)^{n-1} \quad \div 16$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1} \quad \frac{\log \frac{1}{512}}{\log \frac{1}{2}} = 9$$

$$9 = n - 1$$

$$10 = n$$

Review

$$\textcircled{5} \text{ a) } S_6 = 1365$$

$$r = \frac{1}{4}$$

$$n = 6$$

$$a = ?$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1365 = \frac{a\left(\frac{1}{4}^6 - 1\right)}{\frac{1}{4} - 1}$$

$$1365 = \frac{a\left(\frac{1}{4096} - \frac{4096}{4096}\right)}{\frac{1}{4} - \frac{4}{4}}$$

$$1365 = a\left(\frac{\overset{1365}{-4095}}{\underset{1024}{4096}}\right)\left(\frac{\overset{1}{-4}}{\underset{1}{3}}\right) \quad |$$

$$1365 = \frac{1365a}{1024}$$

$$1024 = a$$



10.9

$$\textcircled{5} \text{ b) } 1 + \frac{5}{2} + \frac{25}{4} + \dots + \frac{15625}{64} \leftarrow \text{last term}$$

$$a = 1$$

$$r = \frac{5}{2}$$

$$t_n = \frac{15625}{64}$$

① Find  $n$ .

$$t_n = ar^{n-1}$$

$$\frac{15625}{64} = (1) \left(\frac{5}{2}\right)^{n-1}$$

$$\frac{15625}{64} = \left(\frac{5}{2}\right)^{n-1}$$

$$\left(\frac{5}{2}\right)^6 = \left(\frac{5}{2}\right)^{n-1}$$

$$6 = n - 1$$

$$7 = n$$

② Find  $S_7$ 

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{(1) \left(\left(\frac{5}{2}\right)^7 - 1\right)}{\left(\frac{5}{2}\right) - 1}$$

$$S_7 = \frac{1 \left(\frac{78125}{128} - \frac{128}{128}\right)}{\frac{5}{2} - \frac{2}{2}}$$

$$S_7 = \left(\frac{77997}{128}\right) \div \left(\frac{3}{2}\right)$$

$$S_7 = \left(\frac{77997}{128}\right) \left(\frac{2}{3}\right)$$

$$S_7 = \frac{155994}{384} = \frac{25999}{64}$$

$$\underline{10.9}$$

$$\textcircled{6} S_7 = 1093$$

$$r = \frac{1}{3}$$

$$n = 7$$

$$a = ?$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1093 = \frac{a\left(\left(\frac{1}{3}\right)^7 - 1\right)}{\frac{1}{3} - 1}$$

$$1093 = a \frac{\left(\frac{1}{2187} - \frac{2187}{2187}\right)}{\frac{1}{3} - \frac{3}{3}}$$

$$1093 = a \left(\frac{-2186}{2187}\right) \times -\frac{3}{2}$$

$$1093 = \frac{6558a}{4374}$$

$$6558a = 4780782$$

$$a = 729$$

$$b) t_4 = ?$$

$$a = 729$$

$$r = \frac{1}{3}$$

$$n = 4$$

$$t_4 = (729)\left(\frac{1}{3}\right)^{4-1}$$

$$t_4 = 729\left(\frac{1}{27}\right)$$

$$t_4 = \frac{729}{27}$$

$$t_4 = 27$$

10.9

$$\textcircled{1} \text{ b) } 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$$

$$a = 2$$

$$r = \frac{1}{3}$$

$$S_n = \frac{2\left(\frac{1}{3}^n - 1\right)}{\frac{1}{3} - 1}$$

$$= \frac{2\left(\frac{1}{3}^n - 1\right)}{-\frac{2}{3}}$$

$$= \cancel{2}\left(\frac{1}{3}^n - 1\right) \times \frac{3}{-\cancel{2}}$$

$$= -3\left(\frac{1}{3}^n - 1\right)$$

10.9

$$\textcircled{3} \text{ c) } 81 + 27 + 9 + \dots$$

$$a = 81$$

$$r = \frac{1}{3}$$

$$n = 6$$

$$S_6 = \frac{81 \left( \left( \frac{1}{3} \right)^6 - 1 \right)}{\frac{1}{3} - 1}$$

$$= \frac{81 \left( \frac{1}{729} - \frac{729}{729} \right)}{\frac{1}{3} - \frac{3}{3}}$$

$$= \frac{81 \left( \frac{-728}{729} \right)}{-\frac{2}{3}}$$

$$\frac{-2184}{-18}$$

$$= \frac{-728}{9} \times \frac{3}{-2}$$

$$= \frac{364}{3} = 121 \frac{1}{3}$$

Ex 10.9

$$\textcircled{5} \text{ b) } \textcircled{1} + \frac{5}{2} + \frac{25}{4} + \dots + \frac{15625}{64}$$

$$S_n = ?$$

$$a = 1$$

$$r = \frac{5}{2}$$

$$t_n = \frac{15625}{64}$$

Solve for n:

$$t_n = ar^{n-1}$$

$$\frac{15625}{64} = \left( \cancel{1} \right) \left( \frac{5}{2} \right)^{n-1}$$

$$\frac{15625}{64} = \left( \frac{5}{2} \right)^{n-1}$$

$$\left( \frac{5}{2} \right)^6 = \left( \frac{5}{2} \right)^{n-1}$$

$$6 = n - 1$$

$$\boxed{7 = n}$$

Find  $S_7$ :

$$S_7 = \frac{1 \left( \left( \frac{5}{2} \right)^7 - 1 \right)}{\frac{5}{2} - 1}$$

$$= \frac{1 \left( \frac{78125}{128} - \frac{128}{128} \right)}{\frac{5}{2} - \frac{2}{2}}$$

$$= \left( \frac{77997}{128} \right) \div \left( \frac{3}{2} \right)$$

$$= \frac{77997}{128} \times \frac{2}{3}$$

$$= \frac{155994}{384} = \boxed{\frac{25999}{64}}$$

Review

$$\begin{array}{l} \textcircled{2} a) \quad n=? \quad t_n = a + (n-1)d \\ \quad \quad \quad a=3 \quad \quad 39 = 3 + (n-1)4 \\ \quad \quad \quad d=4 \quad \quad 36 = 4n - 4 \\ \quad \quad \quad t_n = 39 \quad 40 = 4n \\ \quad \quad \quad \quad \quad 10 = n \end{array}$$

$$3, 7, 11, 15, 19, 23, 27, 31, 35, 39$$

$$\begin{array}{l} \textcircled{1} b) \quad t_9 = -6 \quad t_{12} = -12 \\ \quad \quad \quad t_9 = a + 8d \quad t_{12} = a + 11d \\ \quad \quad \quad \boxed{a + 8d = -6} \quad \boxed{a + 11d = -12} \end{array}$$

$$\begin{array}{l} a + 11d = -12 \\ \leftarrow \begin{array}{l} \text{(-)} \frac{a + 8d = -6}{3d = -6} \\ \quad \quad \quad \boxed{d = -2} \end{array} \quad \begin{array}{l} a + 8(-2) = -6 \\ a - 16 = -6 \\ \quad \quad \quad \boxed{a = 10} \end{array} \quad \begin{array}{l} t_n = a + (n-1)d \\ t_n = 10 + (n-1)(-2) \\ t_n = 10 - 2n + 2 \\ t_n = 12 - 2n \end{array} \end{array}$$

$$\textcircled{2} b) \quad t_5 = 8 \quad t_{10} = \frac{1}{4} \quad t_3 = ?$$

$$\begin{array}{l} t_5 = ar^4 \quad t_{10} = ar^9 \\ \boxed{ar^4 = 8} \quad \boxed{ar^9 = \frac{1}{4}} \end{array}$$

$$\begin{array}{l} \frac{ar^9 = \frac{1}{4}}{ar^4 = 8} \\ \quad \quad \quad r^5 = \frac{1}{32} \\ \quad \quad \quad \boxed{r = \frac{1}{2}} \end{array} \quad \begin{array}{l} a \left(\frac{1}{2}\right)^4 = 8 \\ a \left(\frac{1}{16}\right) = 8 \\ \quad \quad \quad \frac{a}{16} = 8 \\ \quad \quad \quad \boxed{a = 128} \end{array} \quad \begin{array}{l} t_3 = (128) \left(\frac{1}{2}\right)^3 \\ t_3 = 128 \left(\frac{1}{4}\right) \\ t_3 = 32 \end{array}$$

$$51a = a(-a)^{n-1}$$

$$a5b = (-a)^{n-1}$$

$$\cancel{(-a)}^8 = \cancel{(-a)}^{n-1}$$

$$8 = n - 1$$

$$9 = n$$

Review

⑩  $t_7 = 192$

$a = t_1 = 3$

$S_8 = ?$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$ar^6 = 192$

$3r^6 = 192$

$r^6 = 64$

$r = \pm 2$

$$S_8 = \frac{3(2^8 - 1)}{2 - 1}$$

$$= \frac{3(256 - 1)}{1}$$

$$= 3(255)$$

$$= 765$$

$$S_8 = \frac{3((-2)^8 - 1)}{(-2) - 1}$$

$$= \frac{3(256 - 1)}{-3}$$

$$= \frac{3(255)}{-3}$$

$$= -255$$



