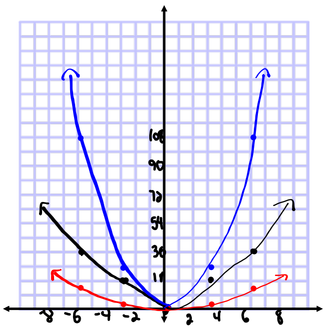
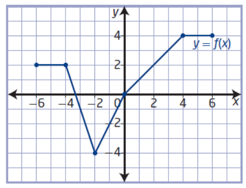
Questions from Homework

2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12



6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$



$(x,y) \to (x, 2y)$

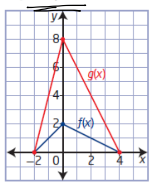
$f(x)$ $g(x)$

D: $[-6,6]$ D: $[-6,6]$

R: $[-4,4]$ R: $[-8,8]$

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



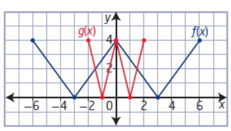
$(x,y) \to (x, 4y)$ A vertical stretch by a factor of 4

$f(x)$	$g(x)$
$(-2,0)$	$(-2,0)$
$(0,2)$	$(0,8)$
$(4,0)$	$(4,0)$

$a = 4$

$\boxed{y = 4f(x)}$

c)



$(x,y) \to (\frac{1}{3}x, y)$ A horizontal compression by a factor of $\frac{1}{3}$

$f(x)$	$g(x)$
$(-6,4)$	$(-2,4)$
$(-3,0)$	$(-1,0)$
$(0,4)$	$(0,4)$
$(3,0)$	$(1,0)$
$(6,4)$	$(2,4)$

$b = 3$

$\boxed{y = f(3x)}$

5 a) $y = 4f(x)$

$a = 4 \to$ A vertical stretch about the x-axis by a factor of 4

$(x,y) \to (x, 4y)$

b) $y = f(3x)$

$b = 3 \to$ A horizontal compression about the y-axis by a factor $\frac{1}{3}$

$(x,y) \to (\frac{1}{3}x, y)$

Warm-Up...

$y = af[b(x-h)]+k$

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

a = 3 → vertically stretched about the x-axis by a factor of 3

b = 1 → no horizontal stretch.

h = 0 → no horizontal trans

k = 0 → no vertical trans.

$(x,y) \to (x, 3y)$

$(-2,5) \to \boxed{(-2,15)}$

(2) $y = f\left(-\frac{1}{3}x\right)$

a = 1 → no vertical stretch

$b = -\frac{1}{3}$ → horizontally stretched about the y-axis by a factor 3 and a reflection in the y-axis

h = 0 → no horizontal trans.

k = 0 → no vertical trans.

$(x,y) \to (-3x, y)$

$(-2,5) \to \boxed{(6,5)}$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right]-3$

a = 4 → vertically stretched about the x-axis by a factor of 4

$b = \frac{1}{2}$ → horizontally stretched about the y-axis by a factor of 2.

h = -5 → horizontally translated 5 units left

k = -3 → vertically translated 3 units down

$(x,y) \to (2x-5, 4y-3)$

$(-2,5) \to \boxed{(-9,17)}$

(4) $y - 5 = -2f(-2x+6)$

factor

$y = -2f(-2x+6)+5$

$y = -2f[-2(x-3)]+5$

a = -2 → vertically stretched about the x-axis by a factor of 2 and reflected in the x-axis

b = -2 → horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and reflected in the y-axis

h = 3 → horizontally trans 3 units right

k = 5 → vertically trans. 5 units up

$(x,y) \to \left(-\frac{1}{2}x+3, -2y+5\right)$

$(-2,5) \to \boxed{(4,-5)}$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

factor

$g(x) = -3f(4x-16)-10$

$g(x) = -3f[4(x-4)]-10$

$a=-3 \quad b=4 \quad h=4 \quad k=-10$

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

Summary of Transformations...

Transformations of the graphs of functions

$f(x)+k$	shift $f(x)$ up k units	vertical trans.	$(x,y)\rightarrow(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units		$(x,y)\rightarrow(x,y-k)$
$f(x+h)$	shift $f(x)$ left h units	horizontal trans.	$(x,y)\rightarrow(x-h,y)$
$f(x-h)$	shift $f(x)$ right h units		$(x,y)\rightarrow(x+h,y)$
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref.	$(x,y)\rightarrow(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref	$(x,y)\rightarrow(x,-y)$
$af(x)$	When $0<a<1$ – vertical shrinking of $f(x)$ When $a>1$ – vertical stretching of $f(x)$ **Multiply the y values by a**		$(x,y)\rightarrow(x,ay)$
$f(bx)$	When $0<b<1$ – horizontal stretching of $f(x)$ When $b>1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**	or multiply by $\frac{1}{b}$	$(x,y)\rightarrow(\frac{1}{b}x,y)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right)+k$$

Mapping Rule:

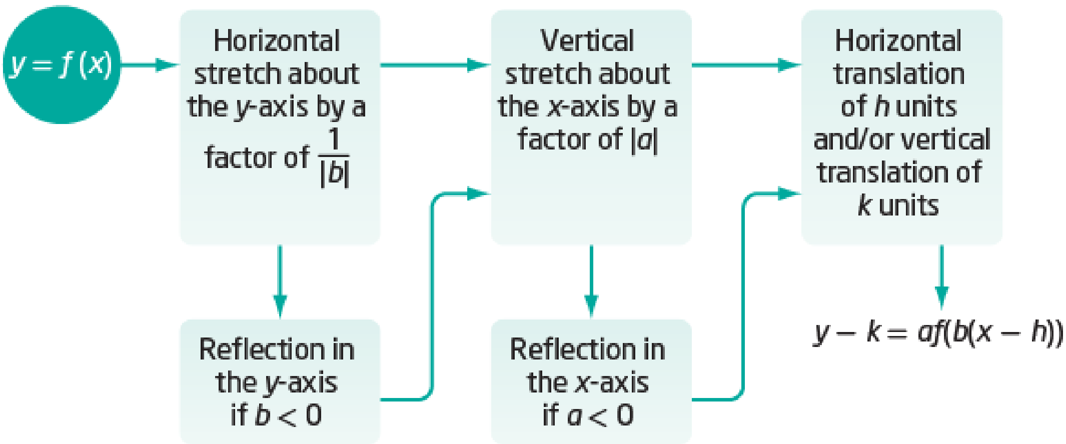
$$\boxed{(x,y) \to \left(\frac{1}{b}x + h, ay + k\right)}$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



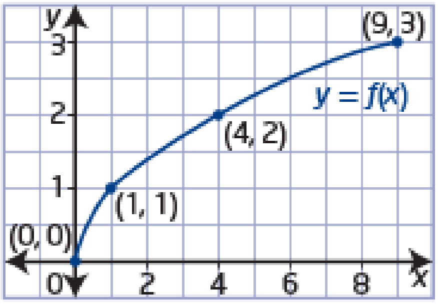
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

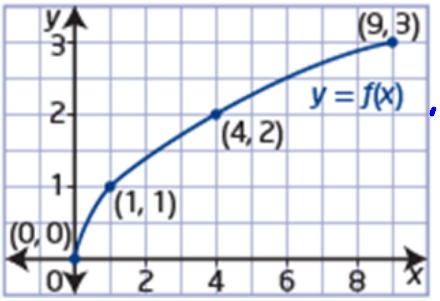
b) $y = f(3x + 6)$



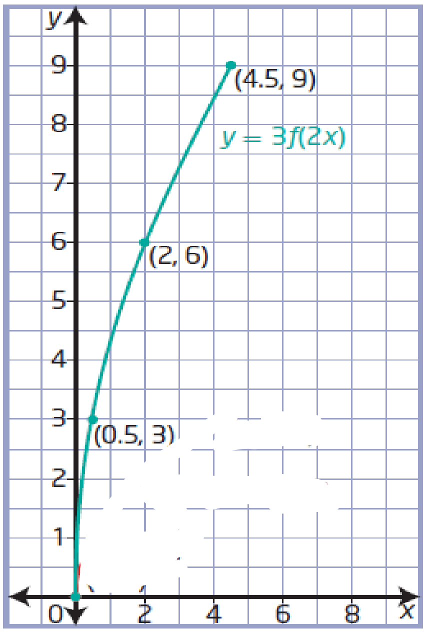
a) $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x, y) \rightarrow \left[\frac{1}{2}x, 3y\right]$



f(x)	g(x)
(0,0)	(0,0)
(1,1)	(½, 3)
(4,2)	(2,6)
(9,3)	(9/2, 9)

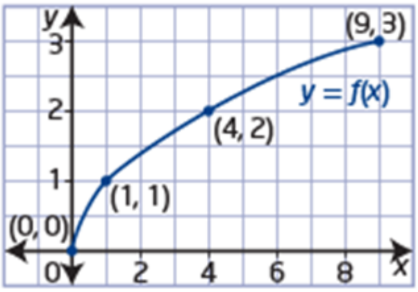


b) $y = f(3x + 6)$ $a=1$ $b=3$ $h=-2$ $k=0$

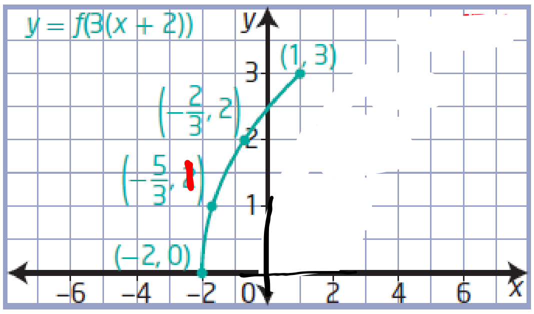
$y = f[3(x+2)]$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x,y) \rightarrow [\frac{1}{3}x - 2, y]$



f(x)	g(x)
(0,0)	(-2,0)
(1,1)	(-5/3,1)
(4,2)	(-2/3,2)
(9,3)	(1,3)



$\frac{1}{3}x - 2$ $\frac{1}{3}x - 2$

$\frac{1}{3}(1) - 2$ $\frac{1}{3}(4) - 2$

$\frac{1}{3} - \frac{2}{1}$ $\frac{4}{3} - \frac{2}{1}$

$\frac{1}{3} - \frac{6}{3}$ $\frac{4}{3} - \frac{6}{3}$

$-\frac{5}{3}$ $-\frac{2}{3}$

Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

	Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
(i)	$y - 4 = f(x - 5)$	—	—	—	4	5
(ii)	$y + 5 = 2f(3x)$	—	2	$\frac{1}{3}$	-5	-
(iii)	$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	—	$\frac{1}{2}$	2	—	4
(iv)	$y + 2 = -3f(2(x + 2))$	vertical reflection in x-axis	3	$\frac{1}{2}$	-2	-2

(i) $y = f(x-5) + 4$
$a=1 \quad b=1 \quad h=5 \quad k=4$

(ii) $y = 2f(3x) - 5$
$a=2 \quad b=3 \quad h=0 \quad k=-5$

(iii) $y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$
$a=\frac{1}{2} \quad b=\frac{1}{2} \quad h=4 \quad k=0$

(iv) $y = -3f(2(x+2)) - 2$
$a=-3 \quad b=2 \quad h=-2 \quad k=-2$

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

e) $y+3 = -\frac{1}{3}f[2(x+6)]$

$y = -\frac{1}{3}f[2(x+6)] - 3$

$a = -\frac{1}{3} \quad b = 2 \quad h = -6 \quad k = -3$

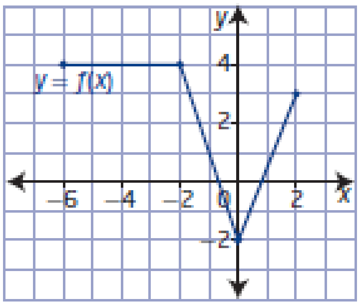
$(x,y) \rightarrow \left[\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right]$

$(-12,18) \rightarrow \boxed{[-12,-9]}$

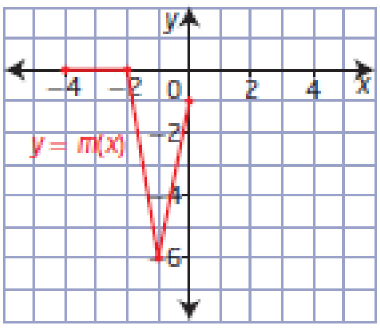
$\frac{1}{2}(-12) - 6$
$= -6 - 6$
$= -12$

$-\frac{1}{3}(18) - 3$
$= -6 - 3$
$= -9$

4. Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.



b)



$f(x)$	$m(x)$
$(-6,4)$	$(-4,0)$
$(-2,4)$	$(-2,0)$
$(0,-2)$	$(-1,-6)$
$(2,3)$	$(0,-1)$

$$(x,y) \longrightarrow \left(\tfrac{1}{2}x-1,\ y-4\right)$$

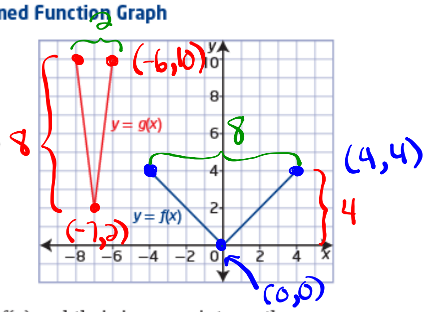
$a=1$ $b=2$ $h=-1$ $k=-4$

$$m(x) = 1f(2(x+1))-4$$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \to (-8, 10)$
$(0, 0) \to (-7, 2)$
$(4, 4) \to (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: none

② Vertical Stretch Factor: $VSF = \frac{8}{4} = 2$ $\quad a = 2$

Range $\frac{new}{old}$

③ Horizontal Stretch Factor: $HSF = \frac{2}{8} = \frac{1}{4}$ $\quad b = 4$

Domain $\frac{new}{old}$

④ Horizontal Translation: $(0,0) \to (-7,2)$ $\quad h = -7$ (left 7)

Pick a point on the original where $x=0$

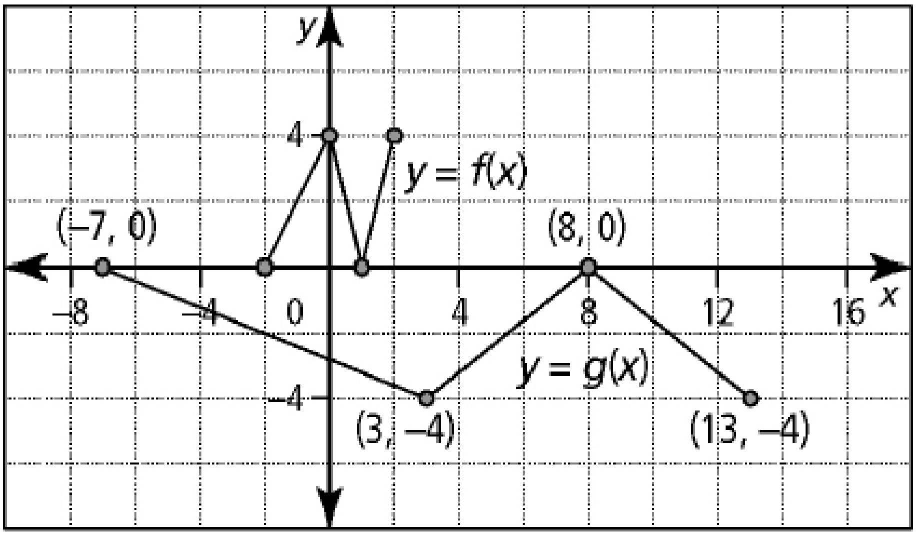
⑤ Vertical Translation: $(0,0) \to (-7,2)$ $\quad k = 2$ (up 2)

Pick a point on the original where $y=0$

⑥ Equation: $y = af[b(x-h)]+k$

$$\boxed{y = 2f[4(x+7)] + 2}$$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



Homework

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Plus 7, 8, 9 (a, c, e) and 10