Warm Up

Differentiate: $e^{xy^2} = 2x - 3xy + e^{\tan x}$

$e^{xy^2}\left(1y^2 + 2xy\frac{dy}{dx}\right) = 2 - 3y - 3x\frac{dy}{dx} + e^{\tan x}(\sec^2 x)$

$y^2e^{xy^2} + 2xy\frac{dy}{dx}e^{xy^2} = 2 - 3y - 3x\frac{dy}{dx} + \sec^2 x\, e^{\tan x}$

$2xy\frac{dy}{dx}e^{xy^2} + 3x\frac{dy}{dx} = 2 - 3y + \sec^2 x\, e^{\tan x} - y^2e^{xy^2}$

$\frac{dy}{dx}\left(2xye^{xy^2} + 3x\right) = 2 - 3y + \sec^2 x\, e^{\tan x} - y^2e^{xy^2}$

$$\boxed{\frac{dy}{dx} = \frac{2 - 3y + \sec^2 x\, e^{\tan x} - y^2e^{xy^2}}{2xye^{xy^2} + 3x}}$$

Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$. $x_1 = 0$, $y_1 = 1$ $m =$

(i) Find y:

$y = 1 + xe^{2x}$

$y = 1 + (0)e^{2(0)}$

$y = 1 + 0$

$y = 1$

(ii) Differentiate:

$y = 1 + xe^{2x}$

$y' = 1e^{2x} + xe^{2x} \cdot 2$

$y' = e^{2x} + 2xe^{2x}$

$y' = e^{2x}(1 + 2x)$

(iii) Find m:

$y' = e^{2(0)}(1 + 2(0))$

$y' = 1(1 + 0)$

$y' = 1$ $m = 1$

(iv) Find equation:

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y - 1 = x$

$\boxed{y = x + 1}$ or $x - y + 1 = 0$

Questions from Homework

⑥ $e^{xy} = 2x + y$

$e^{xy}\left(y + x\frac{dy}{dx}\right) = 2 + \frac{dy}{dx}$

$ye^{xy} + e^{xy}x\frac{dy}{dx} = 2 + \frac{dy}{dx}$

$xe^{xy}\frac{dy}{dx} - \frac{dy}{dx} = 2 - ye^{xy}$

$\frac{dy}{dx}\left(xe^{xy} - 1\right) = 2 - ye^{xy}$

$$\boxed{\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}}$$

⑩ $f(x) = xe^x$

a) $f'(x) = e^x + xe^x$

$f'(x) = e^x(1 + x)$

cv: $e^x \neq 0$ | $1 + x = 0$, $x = -1$

abs min

$-$ | $+$

(-2) -1 (0)

abs min @ $x = -1$

$f(x) = xe^x$

$f(-1) = (-1)e^{(-1)}$

$f(-1) = -\frac{1}{e}$

$\boxed{\left(-1, -\frac{1}{e}\right) \text{ abs. min}}$

b) $f'(x) = e^x(x+1)$

$f''(x) = e^x(x+1) + e^x$

$f''(x) = e^x[(x+1)+1]$

$f''(x) = e^x(x+2)$

cv: $e^x \neq 0$ | $x + 2 = 0$, $x = -2$

I.P.

$-$ | $+$

(-3) -2 (0)

CD on $(-\infty, -2)$

CU on $(-2, \infty)$

c) Inflection Point @ $x = -2$

$f(x) = xe^x$

$f(-2) = (-2)e^{(-2)}$

$f(-2) = -2\left(\frac{1}{e^2}\right) = \frac{-2}{e^2}$

$\boxed{\left(-2, \frac{-2}{e^2}\right) \text{ I.P.}}$

Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate $y = e^x$. (exp. form)

A B E

$\log_e(y) = x$ (log. form)

$\ln(y) = x$

What other function could this model?

$\ln y = x$ (log form)

$\log_e(y) = x$

$e^x = y$

Try to differentiate → y = ln x. (log form)

$e^y = x$ (exp. form)

$e^y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \boxed{\frac{1}{x}}$

Differentiate: $y = \ln x^3$

$e^y = x^3$

$e^y \frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{e^y}$

$\frac{dy}{dx} = \frac{3x^2}{x^3}$

$\frac{dy}{dx} = \frac{3}{x}$

$y = \ln x^3$ $\quad u = x^3$, $du = 3x^2$

$\frac{dy}{dx} = \frac{1}{x^3} \cdot 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{x^3} = \boxed{\frac{3}{x}}$

Rule: $\frac{d}{du}(\ln u) = \frac{1}{u}\,du = \frac{du}{u}$

Ex: $y = \ln x^7$ $\quad u = x^7 \quad du = 7x^6$

$y' = \frac{1}{x^7} \cdot 7x^6 = \frac{7x^6}{x^7} = \boxed{\frac{7}{x}}$

Practice Problem:

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$1 \div \frac{x+1}{x-1}$

#1

$1 \times \frac{x-1}{x+1}$

j) $y = \ln\left(\frac{x+1}{x-1}\right)$

$$y' = \left(\frac{x-1}{x+1}\right)\left[\frac{\overset{x-1}{1(x-1)} - \overset{-x-1}{1(x+1)}}{(x-1)^2}\right]$$

$$y' = \left[\frac{\cancel{x-1}}{x+1}\right]\left[\frac{-2}{(x-1)^{\cancel{2}}}\right] = \frac{-2}{(x+1)(x-1)} = \frac{-2}{x^2-1}$$

Laws of Logarithms

$$\log_b M + \log_b N = \log_b(MN)$$

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

$$\log_b(N^p) = p\log_b(N)$$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Differentiate:

$y = \log_6 x^3$

$y = \log(5x^4)$

Rule: $d\left(\log_b u\right) = \dfrac{1}{u \ln b}\,du$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

Try this one... $y = \pi^{x^5}$

Rule:

$$d(b^{u}) = b^{u}(\ln b)du \;, where \;\; b \in R$$

Practice Problems:

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#1 #2 a #3 #4

#5 #6 #7 #8