

Warm Up

Differentiate: $e^{xy^2} = 2x - 3xy + e^{\tan x}$

$$e^{xy^2} (1y^2 + 2xy \frac{dy}{dx}) = 2 - 3y - 3x \frac{dy}{dx} + e^{\tan x} (\sec^2 x)$$

$$(y^2 e^{xy^2}) + 2xy \frac{dy}{dx} e^{xy^2} = 2 - 3y - 3x \frac{dy}{dx} + \sec^2 x e^{\tan x}$$

$$2xy \frac{dy}{dx} e^{xy^2} + 3x \frac{dy}{dx} = 2 - 3y + \sec^2 x e^{\tan x} - y^2 e^{xy^2}$$

$$\frac{dy}{dx} (2xy e^{xy^2} + 3x) = 2 - 3y + \sec^2 x e^{\tan x} - y^2 e^{xy^2}$$

$$\frac{dy}{dx} = \frac{2 - 3y + \sec^2 x e^{\tan x} - y^2 e^{xy^2}}{2xy e^{xy^2} + 3x}$$

Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$. $x_1 = 0, y_1 = 1, m =$

(i) Find y :

$$y = 1 + xe^{2x}$$

$$y = 1 + (0)e^{2(0)}$$

$$y = 1 + 0$$

$$\underline{y = 1}$$

(ii) Differentiate:

$$y = 1 + xe^{2x}$$

$$y' = 1e^{2x} + xe^{2x} \cdot 2$$

$$y' = e^{2x} + 2xe^{2x}$$

$$y' = e^{2x}(1 + 2x)$$

(iii) Find m :

$$y' = e^{2(0)}(1 + 2(0))$$

$$y' = 1(1 + 0)$$

$$\underline{y' = 1} \quad m = 1$$

(iv) Find equation:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$\boxed{y = x + 1} \quad \text{or} \quad x - y + 1 = 0$$

Questions from Homework

⑥ $e^{xy} = 2x + y$

$$e^{xy} (y + x \frac{dy}{dx}) = 2 + \frac{dy}{dx}$$

$$ye^{xy} + e^{xy} x \frac{dy}{dx} = 2 + \frac{dy}{dx}$$

$$xe^{xy} \frac{dy}{dx} - \frac{dy}{dx} = 2 - ye^{xy}$$

$$\frac{dy}{dx} (xe^{xy} - 1) = 2 - ye^{xy}$$

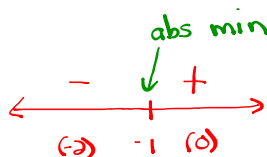
$$\boxed{\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}}$$

⑩ $f(x) = xe^x$

a) $f'(x) = e^x + xe^x$

$$f'(x) = e^x(1+x)$$

cv: $e^x \neq 0 \mid \begin{array}{l} 1+x=0 \\ x=-1 \end{array}$



abs min @ $x = -1$

$$f(x) = xe^x$$

$$f(-1) = (-1)e^{-1}$$

$$f(-1) = -\frac{1}{e}$$

$$\boxed{(-1, -\frac{1}{e}) \text{ abs. min}}$$

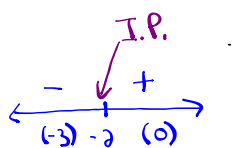
b) $f'(x) = e^x(x+1)$

$$f''(x) = e^x(x+1) + e^x$$

$$f''(x) = e^x[(x+1)+1]$$

$$f''(x) = e^x(x+2)$$

cv: $e^x \neq 0 \mid \begin{array}{l} x+2=0 \\ x=-2 \end{array}$



CD on $(-\infty, -2)$

CU on $(-2, \infty)$

c) Inflection Point @ $x = -2$

$$f(x) = xe^x$$

$$f(-2) = (-2)e^{-2}$$

$$f(-2) = -2\left(\frac{1}{e^2}\right) = -\frac{2}{e^2}$$

$$\boxed{(-2, -\frac{2}{e^2}) \text{ I.P.}}$$

Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate $y = e^x$ (exp. form)

What other function could this model?

$$y = e^x \quad (\text{exp. form})$$

$$\log_e(y) = x \quad (\text{log form})$$

$$\ln(y) = x$$

$$\ln y = x \quad (\text{log form})$$

$$\log_2(y) = x$$

$$e^x = y$$

Try to differentiate $\longrightarrow y = \ln x$. (log form)

$$e^y = x \quad (\text{exp. form})$$

$$e^y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \boxed{\frac{1}{x}}$$

Differentiate:

$$y = \ln x^3$$

$$e^y = \underline{x^3}$$

$$e^y \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{e^y}$$

$$\frac{dy}{dx} = \frac{3x^2}{x^3}$$

$$\frac{dy}{dx} = \frac{3}{x}$$

$$y = \ln x^3$$

$$u = x^3$$

$$du = 3x^2$$

$$\frac{dy}{dx} = \frac{1}{x^3} \cdot 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{x^3} = \boxed{\frac{3}{x}}$$

$$\text{Rule: } \frac{d(\ln u)}{du} = \frac{1}{u} du = \frac{du}{u}$$

$$\text{Ex: } y = \ln x^7 \quad u = x^7 \quad du = 7x^6$$

$$y' = \frac{1}{x^7} \cdot 7x^6 = \frac{7x^6}{x^7} = \boxed{\frac{7}{x}}$$

∴

Practice Problem:

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$$1 \div \frac{x+1}{x-1}$$

#1

$$1 \times \frac{x-1}{x+1}$$

$$j) \quad y = \ln \left(\frac{x+1}{x-1} \right)$$

$$y' = \left(\frac{x-1}{x+1} \right) \left[\frac{\overset{x-1}{1(x-1)} - \overset{-x-1}{1(x+1)}}{(x-1)^2} \right]$$

$$y' = \left[\frac{\cancel{x-1}}{x+1} \right] \left[\frac{-2}{(x-1)^2} \right] = \frac{-2}{(x+1)(x-1)} = \frac{-2}{x^2-1}$$

Laws of Logarithms

$$\log_b M + \log_b N = \log_b (MN)$$

$$\log_b M - \log_b N = \log_b \left(\frac{M}{N} \right)$$

$$\log_b (N^p) = p \log_b (N)$$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Differentiate:

$$y = \log_6 x^3$$

$$y = \log(5x^4)$$

Rule: $d(\log_b u) = \frac{1}{u \ln b} du$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

Try this one... $y = \pi^{x^5}$

Rule:

$$d(b^u) = b^u (\ln b) du, \text{ where } b \in R$$

Practice Problems:

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#1 #2 a #3 #4

#5 #6 #7 #8