

Warm Up

Review of laws of logarithms...

Express the following as a single logarithm:

$$\frac{2}{3} \ln b^6 - \frac{1}{2} [\ln b^4 + 8 \ln b + 6 \ln \sqrt[3]{b}]$$

$$\frac{2}{3} \ln b^6 - \frac{1}{2} \ln b^4 - 4 \ln b - 3 \ln b^{1/3}$$

$$\ln b^4 - \ln b^2 - \ln b^4 - \ln b$$

$$\ln \left(\frac{b^4}{b^2 b^4 b} \right)$$

$$\ln \left(\frac{1}{b^3} \right)$$

$$\ln b^{-3}$$

$$-3 \ln b$$

Questions from Homework

① b $y = \ln\left(\frac{x}{\sqrt{x^2+1}}\right)$ $u = \frac{x}{(x^2+1)^{1/2}}$ $du \Rightarrow$ quotient/chain

$$y' = \frac{(x^2+1)^{-1/2}}{x} \left[\frac{(x^2+1)^{-1/2}(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-3/2}(2x)}{x^2+1} \right]$$

$$y' = \frac{(x^2+1)^{-1/2}}{x} \left[\frac{(x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}}{x^2+1} \right]$$

$$y' = \frac{(x^2+1)^{-1/2}}{x} \left[\frac{(x^2+1)^{-1/2} [x^2+1 - x^2]}{x^2+1} \right]$$

$$y' = \frac{1}{x(x^2+1)}$$

② $y = \ln(\ln x)$ $u = \ln x$
 $du = \frac{1}{x}$

$$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

Questions from Homework

③ $g(x) = \frac{1 + \log_3 x}{x}$

$g'(x) = \frac{x \left(\frac{1}{x \ln 3} \cdot 1 \right) - (1 + \log_3 x)(1)}{x^2}$

$g'(x) = \frac{\frac{\ln 3}{\ln 3} - 1 - \log_3 x}{x^2 \ln 3}$

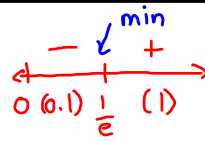
$g'(x) = \frac{1 - \ln 3 - \ln 3 (\log_3 x)}{x^2 \ln 3}$

Change of base formula: $\log_b M = \frac{\log_a M}{\log_a b} = \frac{\log M}{\log N} = \frac{\ln M}{\ln N}$

$g'(x) = \frac{1 - \ln 3 - \ln 3 \left(\frac{\ln x}{\ln 3} \right)}{x^2 \ln 3}$

$g'(x) = \frac{1 - \ln 3 - \ln x}{x^2 \ln 3}$

① $f(x) = x \ln x$



i) $f'(x) = x \left(\frac{1}{x} \right) + (1) \ln x$

$f'(x) = 1 + \ln x$

$f\left(\frac{1}{e}\right) = \left(\frac{1}{e}\right) \ln\left(\frac{1}{e}\right)$

$1 + \ln x = 0$

$\ln x = -1$

$e^{-1} = x$

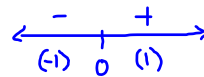
$= \frac{1}{e}(-1)$

$= -\frac{1}{e}$

CV: $\frac{1}{e} = x$

$\left(\frac{1}{e}, -\frac{1}{e}\right)$ Min

(ii) $f'(x) = 1 + \ln x$



$f''(x) = 0 + \frac{1}{x}$

$f''(x) = \frac{1}{x}$

CU on $(0, \infty)$

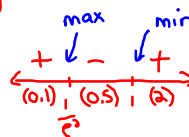
CD on $(-\infty, 0)$

CV: $x = 0$

② $f(x) = x (\ln x)^2$

$f'(x) = x \left(2 \ln x \right) \left(\frac{1}{x} \right) + (1) (\ln x)^2$

$f'(x) = 2 \ln x + (\ln x)^2$



$f'(x) = \ln x [2 + \ln x]$

$0 = \ln x [2 + \ln x]$

CV: $\ln x = 0$ | $2 + \ln x = 0$
 $e^0 = x$ | $\ln x = -2$
 $1 = x$ | $e^{-2} = x$
 $\frac{1}{e^2} = x$

$f\left(\frac{1}{e^2}\right) = \frac{4}{e^3}$ max
 $f(1) = 0$ min

Questions from Homework

$$\textcircled{6} \ln(x+y) = y^{-1}$$
$$\frac{1}{x+y} \cdot (1+y') = y'$$

$$\frac{1+y'}{x+y} \rightarrow y'$$

$$1+y' = xy' + yy'$$
$$1 = xy' + yy' - y'$$
$$1 = y'(x+y-1)$$

$$\boxed{\frac{1}{x+y-1} = y'}$$

Logarithmic Differentiation

A differentiation process that requires taking the logarithm of both sides before differentiating. (ln)

$$\frac{d}{dx} \ln u = \frac{du}{u}$$

This process will be used in TWO circumstances:

I. Simplifying messy products and quotients

What would it involve to differentiate the following?

$$y = \frac{(x^2 - 1)^5 \sqrt{2x + 9} (5x^3 + 2)^8}{(10x - 1) \sqrt{5 - x^7}}$$

- Quotient rule, multiple product rules and chain rules...

This would be possible but it would be easier to differentiate a group of terms added and subtracted rather than multiplied and divided

Laws of logarithms will do exactly that...turn this mess into a addition and subtraction of terms.

$$y = \frac{(x^2 - 1)^5 \sqrt{2x + 9} (5x^3 + 2)^8}{(10x - 1) \sqrt{5 - x^7}}$$

$$\ln y = \ln \left[\frac{(x^2 - 1)^5 (2x + 9)^{\frac{1}{2}} (5x^3 + 2)^8}{(10x - 1) (5 - x^7)^{\frac{1}{2}}} \right]$$

$$\ln y = \ln(x^2 - 1)^5 + \ln(2x + 9)^{\frac{1}{2}} + \ln(5x^3 + 2)^8 - \ln(10x - 1) - \ln(5 - x^7)^{\frac{1}{2}}$$

$$\ln y = 5 \ln(x^2 - 1) + \frac{1}{2} \ln(2x + 9) + 8 \ln(5x^3 + 2) - \ln(10x - 1) - \frac{1}{2} \ln(5 - x^7)$$

$$\frac{y'}{y} = 5 \left(\frac{2x}{x^2 - 1} \right) + \frac{1}{2} \left(\frac{2}{2x + 9} \right) + 8 \left(\frac{15x^2}{5x^3 + 2} \right) - \left(\frac{10}{10x - 1} \right) - \frac{1}{2} \left(\frac{-7x^6}{5 - x^7} \right)$$

$$\frac{y'}{y} = \left(\frac{10x}{x^2 - 1} + \frac{1}{2x + 9} + \frac{120x^2}{5x^3 + 2} - \frac{10}{10x - 1} + \frac{7x^6}{10 - 2x^7} \right) \cdot y$$

$$y' = \left[\frac{10x}{x^2 - 1} + \frac{1}{2x + 9} + \frac{120x^2}{5x^3 + 2} - \frac{10}{10x - 1} + \frac{7x^6}{10 - 2x^7} \right] \left[\frac{(x^2 - 1)^5 \sqrt{2x + 9} (5x^3 + 2)^8}{(10x - 1) \sqrt{5 - x^7}} \right]$$

Steps in Logarithmic Differentiation

1. Take logarithms of both sides of an equation. (ln)
2. Differentiate implicitly with respect to x .
3. Solve the resulting equation for y'

Use Logarithmic Differentiation to Differentiate the following:

$$y = \frac{e^x \sqrt{x^2 + 1}}{(x^2 + 2)^3}$$

$$\ln y = \ln \left[\frac{e^x (x^2 + 1)^{\frac{1}{2}}}{(x^2 + 2)^3} \right]$$

$$\ln y = \ln e^x + \ln (x^2 + 1)^{\frac{1}{2}} - \ln (x^2 + 2)^3 \rightarrow \ln e = \underline{1}$$

$$\ln y = x \ln e + \frac{1}{2} \ln (x^2 + 1) - 3 \ln (x^2 + 2)$$

$$\ln y = x + \frac{1}{2} \ln (x^2 + 1) - 3 \ln (x^2 + 2)$$

$$\frac{y'}{y} = 1 + \frac{1}{2} \left(\frac{2x}{x^2 + 1} \right) - 3 \left(\frac{2x}{x^2 + 2} \right)$$

$$\frac{y'}{y} = \left(1 + \frac{x}{x^2 + 1} - \frac{6x}{x^2 + 2} \right) \cdot y$$

$$y' = \left[1 + \frac{x}{x^2 + 1} - \frac{6x}{x^2 + 2} \right] \left[\frac{e^x \sqrt{x^2 + 1}}{(x^2 + 2)^3} \right]$$

II. Base and exponent both variables

Have a look at this example:

$$y = x^{x^5}$$

- Does not fit either the power rule or the rules for an exponential function

...What can be done to help this crazy situation??

Of Course...take the logarithm of both sides!!

$$\begin{aligned}
 y &= x^{x^5} \\
 \ln y &= \ln x^{x^5} \\
 \ln y &= x^5 \ln x \quad \leftarrow \text{product rule} \\
 \frac{y'}{y} &= 5x^4 \ln x + x^5 \left(\frac{1}{x}\right) \\
 \cancel{y} \cdot \frac{y'}{\cancel{y}} &= 5x^4 \ln x + x^4 \cdot y \\
 y' &= (5x^4 \ln x + x^4)(x^{x^5})
 \end{aligned}$$

Example:

Differentiate: $y = (\ln x^5)^{\cos x}$

Practice Questions...

Page 395–396

#1 b, d, e

#2 b, c, e

#3