① $\frac{dy}{dx}$ $y = \frac{(x+1)^3}{(x+2)^5(x+3)^7}$

$$\ln y = \ln \frac{(x+1)^3}{(x+2)^5(x+3)^7}$$

$$\ln y = \ln(x+1)^3 - \ln(x+2)^5 - \ln(x+3)^7$$

$$\ln y = 3\ln(x+1) - 5\ln(x+2) - 7\ln(x+3)$$

$$\frac{y'}{y} = 3\left(\frac{1}{x+1}\right) - 5\left(\frac{1}{x+2}\right) - 7\left(\frac{1}{x+3}\right)$$

$$\cancel{y} \cdot \frac{y'}{\cancel{y}} = \left[\frac{3}{x+1} - \frac{5}{x+2} - \frac{7}{x+3}\right] y$$

$$y' = \left[\frac{3}{x+1} - \frac{5}{x+2} - \frac{7}{x+3}\right]\left[\frac{(x+1)^3}{(x+2)^5(x+3)^7}\right]$$

(2) b) $y = x^{\sqrt{x}}$

$\ln y = \ln x^{\sqrt{x}}$

$\ln y = (\sqrt{x})(\ln x)$ product rule

$\frac{y'}{y} = \frac{1}{2}x^{-1/2}(\ln x) + x^{1/2}\left(\frac{1}{x}\right)$

$\frac{y'}{y} = \frac{1}{2}x^{-1/2}\ln x + x^{-1/2}$

$\cancel{y} \cdot \frac{y'}{\cancel{y}} = x^{-1/2}\left(\frac{1}{2}\ln x + 1\right) \cdot y$

$y' = x^{-1/2}\left(\frac{1}{2}\ln x + 1\right)x^{\sqrt{x}}$

$y' = x^{\sqrt{x} - \frac{1}{2}}\left(\frac{1}{2}\ln x + 1\right)$

③ $y = x^x$ @ $(2,4)$

① Find y'

$y = x^x$

$\ln y = \ln x^x$

$\ln y = (x)(\ln x)$ product

$\frac{y'}{y} = 1 \ln x + x\left(\frac{1}{x}\right)$

$y \cdot \quad \frac{y'}{\cancel{y}} = (\ln x + 1) y$

$y' = (\ln x + 1) x^x$

② sub in $x = 2$

$y' = (\ln 2 + 1) 2^2$

$y' = 4(\ln 2 + 1)$

$y' = 4\ln 2 + 4$

$m = 4\ln 2 + 4$

③ $y - y_1 = m(x - x_1)$

$y - 4 = (4\ln 2 + 4)(x - 2)$

$y - 4 = 4x\ln 2 - 8\ln 2 + 4x - 8$

$y = 4x\ln 2 + 4x - 8\ln 2 - 8 + 4$

$y = 4x(\ln 2 + 1) - 8\ln 2 - 4$

$y = 4x(\ln 2 + 1) - 4(2\ln 2 + 1)$

$0 = 4x(\ln 2 + 1) - y - 4(2\ln 2 + 1)$

Warm Up

Differentiate each of the following:

1. $f(x) = 6^{x^3} + \ln\left(\tan^{-1} 2x^4\right)$

$f'(x) = 6^{x^3}(\ln 6)(3x^2) + \left(\frac{1}{\tan^{-1}(2x^4)}\right)\left(\frac{1}{1+(2x^4)^2}\right)(8x^3)$

2. $y = (8x - 1)^{\sqrt{x}}$ (logarithmic differentiation)

$\ln y = \ln(8x-1)^{\sqrt{x}}$

$\ln y = (\sqrt{x})(\ln(8x-1))$ product rule

$y \cdot \frac{y'}{y} = \left[\left(\frac{1}{2\sqrt{x}}\right)(\ln(8x-1)) + \sqrt{x}\left(\frac{8}{8x-1}\right)\right] \cdot y$

$y' = \left[\frac{\ln(8x-1)}{2\sqrt{x}} + \frac{8\sqrt{x}}{8x-1}\right](8x-1)^{\sqrt{x}}$

Derivative Rules

Exponential Functions

$$d\left(b^u\right) = b^u \bullet (\ln b) \bullet du \text{ , where } b \in R$$

$$d\left(e^u\right) = e^u \bullet du \text{ , base is Euler's number}$$

Logarithmic Functions

$$d(\log_b u) = \frac{1}{u \ln b} \bullet du \text{ , where } b \in R$$

$$d(\ln u) = \frac{1}{u} \bullet du \text{ , base is Euler's number}$$

Inverse Trigonometric Functions

$$\frac{d\left(\sin^{-1} u\right)}{du} = \frac{1}{\sqrt{1-u^2}} du \qquad \frac{d\left(\csc^{-1} u\right)}{du} = \frac{-1}{u\sqrt{u^2-1}} du$$

$$\frac{d\left(\cos^{-1} u\right)}{du} = \frac{-1}{\sqrt{1-u^2}} du \qquad \frac{d\left(\sec^{-1} u\right)}{du} = \frac{1}{u\sqrt{u^2-1}} du$$

$$\frac{d\left(\tan^{-1} u\right)}{du} = \frac{1}{1+u^2} du \qquad \frac{d\left(\cot^{-1} u\right)}{du} = \frac{-1}{u^2+1} du$$

Quiz Monday: Derivatives of Transcendental Functions

- Inverse Trigonometric Functions
- Exponential Functions
- Logarithmic Functions

Practice Test

Solutions

① a) $f(x) = \log_5(x^3) + e^{\sin 5x}$

$f'(x) = \dfrac{3x^2}{x^3 \ln 5} + e^{\sin 5x}(\cos 5x)(5)$

b) $y = \cos^{-1}(2\sqrt{x}) - \ln(\ln x^2)$

$y' = \dfrac{-1}{\sqrt{1-(2\sqrt{x})^2}}\left(x^{-1/2}\right) - \dfrac{1}{\ln x^2}\left(\dfrac{1}{x^2}\right)(2x)$

c) $h(t) = \dfrac{5^{\tan t}}{\ln(3t^4+5)}$

$h'(t) = \dfrac{\ln(3t^4+5)\left[5^{\tan t}(\ln 5)(\sec^2 t)\right] - 5^{\tan t}\left[\dfrac{1}{3t^4+5} \cdot 12t^3\right]}{\left[\ln(3t^4+5)\right]^2}$

d) $y = (5-2x^2)^x$

$\ln y = \ln(5-2x^2)^x$

$\ln y = x\ln(5-2x^2)$

$\dfrac{1}{y} \cdot y' = x\left(\dfrac{-4x}{5-2x^2}\right) + \ln(5-2x^2)$

$y' = \left[x\left(\dfrac{-4x}{5-2x^2}\right) + \ln(5-2x^2)\right](5-2x^2)^x$

① e) $y = \tan^{-1}(\ln^3(x^5-1))$

$y = \tan^{-1}(\ln(x^5-1))^3$

$y' = \frac{1}{1+(\ln^3(x^5-1))^2}\left[3(\ln(x^5-1))^2\left(\frac{1}{x^5-1}\right)(5x^4)\right]$

f) $g(x) = 4^{5x}e^{\sin^{-1}\sqrt{x}}$

$g'(x) = 4^{5x}(e^{\sin^{-1}\sqrt{x}})\left(\frac{1}{\sqrt{1-x}}\right)\left(\frac{1}{2}x^{-1/2}\right) + 4^{5x}(\ln 4)(5)(e^{\sin^{-1}\sqrt{x}})$

② $y = \frac{(x^2-2x)^3(8x^5)}{\sqrt{(5-x^2)^5}\,(e^{x^5+2})}$

$\ln y = \ln\left[\frac{(x^2-2x)^3(8x^5)}{(5-x^2)^{5/2}(e^{x^5+2})}\right]$

$\ln y = 3\ln(x^2-2x) + \ln 8x^5 - \frac{5}{2}\ln(5-x^2) - (x^5+2)\underbrace{\ln e}_{(1)}$

$\ln y = 3\ln(x^2-2x) + \ln 8x^5 - \frac{5}{2}\ln(5-x^2) - (x^5+2)$

$\frac{1}{y}\cdot y' = \left[3\left(\frac{2x-2}{x^2-2x}\right) + \frac{40x^4}{8x^5} - \frac{5}{2}\left(\frac{-2x}{5-x^2}\right) - 5x^4\right]y$

$y' = \left[\frac{6x-6}{x^2-2x} + \frac{5}{x} + \frac{5x}{(5-x^2)} - 5x^4\right]\left[\frac{(x^2-2x)^3(8x^5)}{\sqrt{(5-x^2)^5}\,(e^{x^5+2})}\right]$

3. $e^{3x-y^5} = 5^{xy^3}$

$e^{3x-y^5}(3-5y^4y') = 5^{xy^3}(\ln 5)(3xy^2y' + y^3)$

$3e^{3x-y^5} - 5y^4y'e^{3x-y^5} = 5^{xy^3}\ln 5\, 3xy^2y' + 5^{xy^3}\ln 5\, y^3$

$3e^{3x-y^5} - 5^{xy^3}\ln 5\, y^3 = y'(5^{xy^3}\ln 5\, 3xy^2 + 5y^4e^{3x-y^5})$

$\dfrac{3e^{3x-y^5} - 5^{xy^3}\ln 5\, y^3}{5^{xy^3}\ln 5\, 3xy^2 + 5y^4e^{3x-y^5}} = y'$

4. $z = \sin^{-1}(y-3) - y^3 \qquad y = 3e^x + x^2$

$z = \sin^{-1}(3e^x + x^2 - 3) - (3e^x + x^2)^3$

$z' = \dfrac{1}{\sqrt{1-(3e^x+x^2-3)^2}}(3e^x+2x) - 3(3e^x+x^2)^2(3e^x+2x)$

$z' = \dfrac{3e^x+2x}{\sqrt{1-(3e^x+x^2-3)^2}} - 3(3e^x+x^2)^2(3e^x+2x)$

$z'(0) = \dfrac{3+0}{1} - 3(3+0)^2(3+0)$

$z'(0) = 3 - 81$

$z'(0) = -78$

⑤ $F(x) = x^2 e^{2x}$

$f'(x) = x^2(e^{2x})(2) + 2x e^{2x}$

$= 2x^2 e^{2x} + 2x e^{2x}$

$= 2x e^{2x}(x+1)$

max min

$+ \quad - \quad +$

$(-2) \quad -1 \quad (-0.5) \quad 0 \quad (1)$

CV: $x = 0, -1$

$F(-1) = (-1)^2 e^{2(-1)}$ $\quad \left(-1, \frac{1}{e^2}\right)$ Local max

$= 1 e^{-2}$

$= 1 \cdot \left(\frac{1}{e^2}\right)$

$= \frac{1}{e^2}$

$F(0) = (0)^2 e^{2(0)}$ $\quad (0, 0)$ Local min

$= 0$

4. Find $\frac{dz}{dx}$ at $x=0$ given that $z=\sin^{-1}(y-3)-y^3$ and $y=3e^x+x^2$

$z=\sin^{-1}(y-3)-y^3$

$\frac{dz}{dy}=\frac{1}{\sqrt{1-(y-3)^2}}-3y^2$

$y=3e^x+x^2$

$\frac{dy}{dx}=3e^x+2x$

When $x=0$

$y=3e^{(0)}+(0)^2$

$y=3(1)+0$

$y=3$

Since $\frac{dz}{dx}=\frac{dz}{dy}\cdot\frac{dy}{dx}$

$$\frac{dz}{dx}=\left[\frac{1}{\sqrt{1-(y-3)^2}}-3y^2\right]\left[3e^x+2x\right]$$

$$\frac{dz}{dx}=\left[\frac{1}{\sqrt{1-(3-3)^2}}-3(3)^2\right]\left[3e^0+2(0)\right]$$

$$\frac{dz}{dx}=(-26)(3)=\boxed{-78}$$

May 06

1 a) $f'(x) = \frac{1}{x^3 \ln 5}(3x^2) + e^{\sin 5x}(\cos 5x)$

b) $y' = \frac{-1}{\sqrt{1-(2/x)^2}}(x^{-1/2}) - \frac{1}{\ln x^2}\left(\frac{1}{x^2}\right)(2x)$

c) $h'(t) = \dfrac{5^{\tan t}\ln 5(\sec^2 t)\ln(3t^4+5) - 5^{\tan t}\left(\frac{1}{3t^4+5}(12t^3)\right)}{[\ln(3t^4+5)]^2}$

d) $\ln y = x\ln(5-2x^2)$

$\frac{1}{y}y' = \left[\ln(5-2x^2) + x\left(\frac{1}{5-2x^2}\right)(-4x)\right]y$

$y' = \left(\ln(5-2x^2) - \frac{4x^2}{5-2x^2}\right)(5-2x^2)^x$

e) $y' = \frac{1}{1+[\ln^3(x^5-1)]^2}\left[3[\ln(x^5-1)]^2\left(\frac{1}{x^5-1}\right)(5x^4)\right]$

f) $g'(x) = 4^{5x}\ln 4(5)e^{\sin^{-1}\sqrt{x}} + 4^{5x}e^{\sin^{-1}\sqrt{x}}\left(\frac{1}{\sqrt{1-x}}\frac{1}{2}x^{-1/2}\right)$

2. $\ln y = 3\ln(x^2-2x) + \ln 8x^5 - \frac{5}{2}\ln(5-x^2) - (x^5+2)$

$\frac{1}{y}y' = \left[3\left(\frac{2x-2}{x^2-2x}\right) + \frac{40x^4}{8x^5} - \frac{5}{2}\left(\frac{(-2x)}{5-x^2}\right) - 5x^4\right]y$

$y' = \left(\frac{3(2x-2)}{x^2-2x} + \frac{5}{x} + \frac{5x}{5-x^2} - 5x^4\right)\left[\frac{(x^2-2x)^3(8x^5)}{(5-x^2)^{5/2}(e^{x^5+2})}\right]$

3. $e^{3x-y^5}\left(3-5y^4\frac{dy}{dx}\right) = 5^{xy^3}\ln 5\left(y^3 + x\left(3y^2\frac{dy}{dx}\right)\right)$

$3e^{3x-y^5} - 5y^4e^{3x-y^5}\frac{dy}{dx} = 5^{xy^3}\ln 5\, y^3 + 3xy^2 5^{xy^3}\ln 5\frac{dy}{dx}$

$\frac{3e^{3x-y^5} - 5^{xy^3}\ln 5\, y^3}{3xy^2 5^{xy^3}\ln 5 + 5y^4e^{3x-y^5}} = \frac{dy}{dx}$

4. $\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}}(1) - 3y^2$ ~~when x = 0~~

$\frac{dy}{dx} = 3e^x + 2x$

$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx} = \frac{dz}{dx}$

at $x = 0 \ldots$ $y = 3+0$, $y = 3$

$\frac{dz}{dy} = \frac{1}{\sqrt{1-0^2}} - 27 = -26$ $\frac{dy}{dx} = 3e^0 + 2(0) = 3$

$\boxed{\frac{dz}{dx} = -78}$

5. $f'(x) = 2xe^{2x} + x^2e^{2x}(2)$

$f'(x) = 2xe^{2x}(1+x)$

Critical Values: $x = -1$, $x = 0$

	$2x$	e^{2x}	$1+x$	f'	f
$(-\infty,-1)$	−	+	−	+	Inc
$(-1, 0)$	−	+	+	−	Dec
$(0,\infty)$	+	+	+	+	Inc

LOCAL MAX: $(-1, \frac{1}{e^2})$

LOCAL MIN: $(0, 0)$

① c) $h(t) = \dfrac{5^{\tan t}}{\ln(3t^4+5)}$

$$h'(t) = \frac{\underbrace{5^{\tan t}(\ln 5)(\sec^2 t)}_{f'(x)}\underbrace{\left(\ln(3t^4+5)\right)}_{g(x)} - \underbrace{5^{\tan t}}_{f(x)}\underbrace{\left(\frac{12t^3}{3t^4+5}\right)}_{g'(x)}}{\left[\ln(3t^4+5)\right]^2}$$

$\underbrace{}_{g(x)}$

Attachments

Review of Transcendentals.doc

logs & arcfunctions test 2006.doc