Questions from Homework

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(6) $f(x) = \dfrac{x+1}{4x-5}$ $\quad f(x+h) = \dfrac{(x+h)+1}{4(x+h)-5} = \dfrac{x+h+1}{4x+4h-5}$

CD: $(4x-5)(4x+4h-5)$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{x+h+1}{4x+4h-5} - \dfrac{x+1}{4x-5}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(4x-5)(x+h+1) - (x+1)(4x+4h-5)}{h(4x-5)(4x+4h-5)}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{4x^2} + \cancel{4xh} + \cancel{4x} - \cancel{5x} - 5h - \cancel{5} - (\cancel{4x^2} + \cancel{4xh} - \cancel{5x} + \cancel{4x} + 4h - \cancel{5})}{h(4x-5)(4x+4h-5)}$$

$$f'(x) = \lim_{h \to 0} \frac{-9\cancel{h}}{\cancel{h}(4x-5)(4x+4h-5)} = \frac{-9}{(4x-5)^2}$$

9) $f(x) = 2x^3 - 4x^2$

$$f(x+h) = 2(x+h)^3 - 4(x+h)^2$$

$$= 2(x^3+3x^2h+3xh^2+h^3) - 4(x^2+2xh+h^2)$$

$$= 2x^3+6x^2h+6xh^2+2h^3-4x^2-8xh-4h^2$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^3}+6x^2h+6xh^2+2h^3-\cancel{4x^2}-8xh-4h^2-\cancel{2x^3}+\cancel{4x^2}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{6x^2h+6xh^2+2h^3-8xh-4h^2}{h}$$ ← Common factor

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(6x^2+6xh+2h^2-8x-4h)}{\cancel{h}} = 6x^2-8x$$

$$\boxed{f'(x) = 6x^2-8x}$$

⑧ $f(x) = \dfrac{4x^2}{3x+2}$ $\qquad f(x+h) = \dfrac{4(x+h)^2}{3(x+h)+2} = \dfrac{4x^2+8xh+4h^2}{3x+3h+2}$

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{4x^2+8xh+4h^2}{3x+3h+2} - \dfrac{4x^2}{3x+2}}{h}$$

CD: $(3x+2)(3x+3h+2)$

$$f'(x) = \lim_{h\to 0} \frac{(3x+2)(4x^2+8xh+4h^2) - 4x^2(3x+3h+2)}{h(3x+2)(3x+3h+2)}$$

(expand)

$$f'(x) = \lim_{h\to 0} \frac{\cancel{12x^3} + \underline{24x^2h} + 12xh^2 \cancel{+8x^2} + 16xh + 8h^2 \cancel{-12x^3} \underline{-12x^2h} \cancel{-8x^2}}{h(3x+2)(3x+3h+2)}$$

$$f'(x) = \lim_{h\to 0} \frac{12x^2h + 12xh^2 + 16xh + 8h^2}{h(3x+2)(3x+3h+2)}$$

common factor

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(12x^2 + 12xh + 16x + 8h)}{\cancel{h}(3x+2)(3x+3h+2)} = \frac{12x^2+16x}{(3x+2)^2}$$

$$f'(x) = \frac{12x^2+16x}{(3x+2)^2}$$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If $f(x) = x^2 + 7x$, find $f'(3)$

Hint: find the derivative first then substitute 3 into that

$f(x) = x^2 + 7x$

$f(x+h) = (x+h)^2 + 7(x+h)$

$= x^2 + 2xh + h^2 + 7x + 7h$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+7x} + 7h - \cancel{(x^2 + 7x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + 7h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 7)}{\cancel{h}} = 2x + 7$$

$f'(x) = 2x + 7$

$f'(3) = 2(3) + 7$

$f'(3) = 6 + 7$

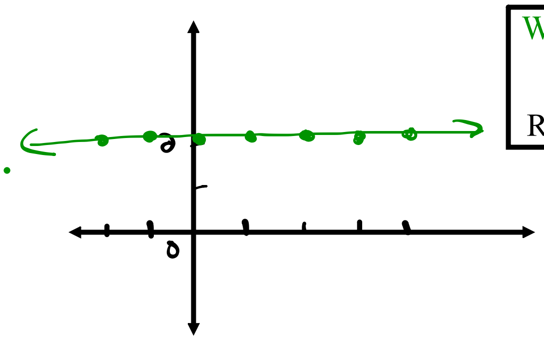
$f'(3) = 13$

Slope or an "m" value

Differentiation Rules

I. Constant Functions

- Sketch the function $y = 2$



What is the slope of the tangent to this graph?

Recall: slope of the tangent is the derivative

The derivative of a constant will always be equal to "0".

$f(x) = 3$ $\quad$ $y = 5$ $\quad$ $g(x) = \pi$

$f'(x) = 0$ $\quad$ $y' = 0$ $\quad$ $g'(x) = 0$

Formal Proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} 0 = 0$$

II. Power Functions

We want to come up with a rule to differentiate functions of the form $f(x) = x^n$, $x \in R$

Using the definition of a derivative to differentiate $f(x) = x^4$ $f(x+h) = (x+h)^4$
would lead to ...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - \cancel{x^4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x^3h + 6x^2h^2 + 4xh^3 + h^4)}{\cancel{h}}$$ (Factor out an h)

$$= \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = \boxed{4x^3}$$

Other examples we have looked at so far

$f(x) = x^2$	$f(x) = x^3$	$f(x) = x^4$
$\boxed{f'(x) = 2x}$	$\boxed{f'(x) = 3x^2}$	$\boxed{f'(x) = 4x^3}$

Do you see a pattern emerging?

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

derivative

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Let's practice using the power rule...

Differentiate each of the following functions:

1. $f(x) = x^{25}$

$f'(x) = 25x^{24}$

2. $f(x) = x^{-5}$

$f'(x) = -5x^{-6} = \frac{-5}{x^6}$

3. $f(x) = \frac{1}{x^{10}} = 1x^{-10}$

$f'(x) = -10x^{-11} = \frac{-10}{x^{11}}$

4. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}}$

$= \frac{1}{2\sqrt{x}}$

Constant Multiples

- The following formula says that <u>the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function:</u>

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

EXAMPLE 4

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}(x^4) = 3(4x^3) = 12x^3$

(b) $\frac{d}{dx}(-x) = \frac{d}{dx}[(-1)x] = (-1)\frac{d}{dx}(x) = -1(1) = -1$

Examples:

1. $f(x) = 4x^3$

$f'(x) = 12x^2$

2. $f(x) = \frac{8}{x^2} = 8x^{-2}$

$f'(x) = -16x^{-3} = \frac{-16}{x^3}$

3. $f(x) = 5x^{\frac{6}{5}}$

$f'(x) = 6x^{1/5}$

4. $f(x) = (3x^2)^2$

$f(x) = (3x^2)(3x^2)$

$f(x) = 9x^4$

$f'(x) = 36x^3$

Recall the derivative of a function is equal to the slope of a line that is tangent to the function.

Find the slope of the tangent line to the function at the given "x" coordinate!

$$f(x) = 3x^2 \quad \text{at x} = 4$$

① $f'(x) = 6x$ ② $f'(4) = 6(4)$

$f'(4) = 24$ slope of tangent $m = 24$

Homework

equation of tangent line

① slope of tangent line
② point (x_1, y_1)
③ $y - y_1 = m(x - x_1)$

④ a) $y = x^5$ $(2, 32)$

① $y' = 5x^4$

② $y'(2) = 5(2)^4$
$= 5(16)$
$= 80$

③ $y - 32 = 80(x - 2)$
$y - 32 = 80x - 160$
$y = 80x - 128$
or $\boxed{80x - y - 128 = 0}$