

Notice... $g(5) =$

1. $\lim_{x \rightarrow 2^-} g(x) =$

"as x approaches 2 from the left"

2. $\lim_{x \rightarrow 2^+} g(x) =$

"as x approaches 2 from the right"

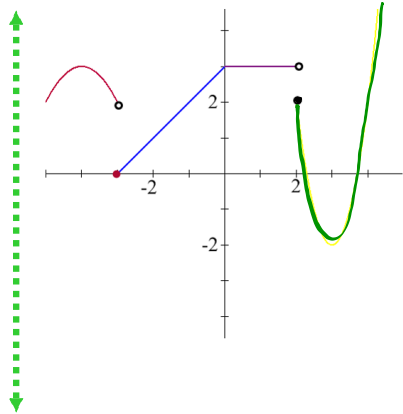
3. $\lim_{x \rightarrow 2} g(x) =$

4. $\lim_{x \rightarrow 5^-} g(x) =$

5. $\lim_{x \rightarrow 5^+} g(x) =$

6. $\lim_{x \rightarrow 5} g(x) =$

Questions from Homework



a) $\lim_{x \rightarrow 3^+} f(x) = 0$

b) $\lim_{x \rightarrow 3^-} f(x) = 2$

* $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

c) $f(3) = 0$ (closed dot)

① c) $\lim_{x \rightarrow 1} \frac{(x+2)^3 - 27}{x-1}$ ← diff of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$\lim_{x \rightarrow 1} \frac{[(x+2) - 3][(x+2)^2 + 3(x+2) + 9]}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)[(x+2)^2 + 3(x+2) + 9]}{(x-1)} = 9 + 9 + 9 = 27$$

① c) $(x+2)(x+2)(x+2)$
 $(x+2)(x^2 + 4x + 4)$
 $x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$
 $x^3 + 6x^2 + 12x + 8$

$$\lim_{x \rightarrow 1} \frac{(x+2)^3 - 27}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{x^3 + 6x^2 + 12x + 8 - 27}{x-1}$$

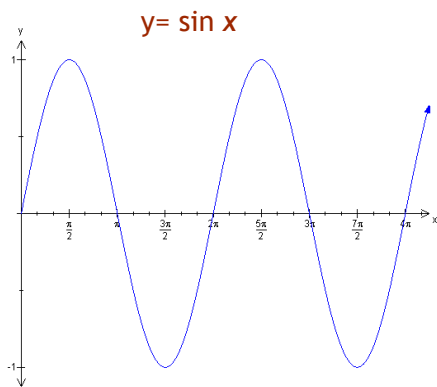
$$\lim_{x \rightarrow 1} \frac{x^3 + 6x^2 + 12x - 19}{x-1}$$

← synthetic sub

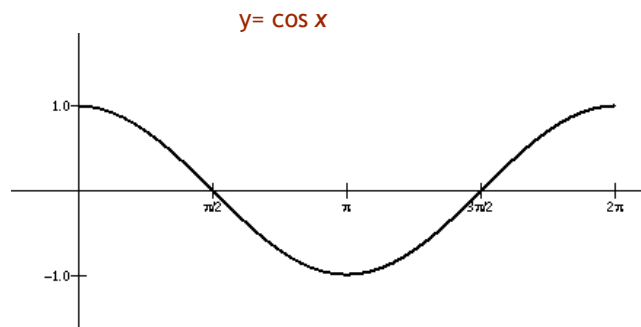
1)	1	6	12	-19
		1	7	19
	1	7	19	

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^2 + 7x + 19)}{(x-1)} = 27$$

Limits of Trigonometric Functions



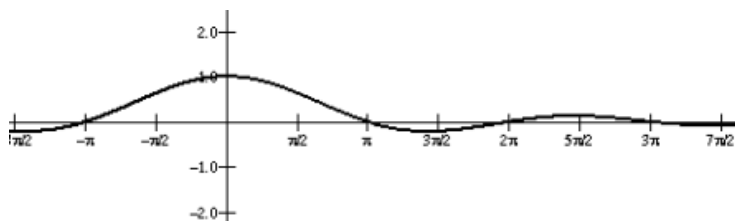
$$\lim_{x \rightarrow 0} \sin x = 0$$



$$\lim_{x \rightarrow 0} \cos x = 1$$

Here is the graph of

$$y = \frac{\sin x}{x}$$



X	Y1
3	.04704
2	.45465
-1	.84147
0	ERROR
1	.84147
2	.45465
3	.04704

X = -3

Examine the following limit...

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

Identity

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

Examples:

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot 5$$

$$= 5 \lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$$

$$= 5(1)$$

$$= (5)$$

$$\lim_{x \rightarrow 0} \frac{8x}{\sin 5x}$$

$$= \lim_{x \rightarrow 0} \frac{8x}{5x} \left(\frac{5x}{\sin 5x} \right)$$

$$= \frac{8}{5} \lim_{x \rightarrow 0} \frac{5x}{\sin 5x}$$

$$= \frac{8}{5} (1) = \left(\frac{8}{5} \right)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{4x}{\sin x}$$

Direct substitution

$$= \frac{4\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)}$$

$$= \pi \div \frac{1}{\sqrt{2}}$$

$$= \pi \cdot \sqrt{2}$$

$$= \pi\sqrt{2}$$

$$\lim_{x \rightarrow 0} \frac{6x}{\cos 3x}$$

$$= \frac{6(0)}{\cos(3 \cdot 0)}$$

$$= \frac{0}{\cos(0)}$$

$$= \frac{0}{1}$$

$$= 0$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan 7x}$$

Identity

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\frac{\sin 7x}{\cos 7x}}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 2x \cos 7x}{\sin 7x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \cdot \frac{\cos 7x}{1} \cdot \frac{7x}{\sin 7x} \cdot \frac{2x}{7x}$$

$$= \frac{2}{7} \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \lim_{x \rightarrow 0} \frac{\cos 7x}{1} \lim_{x \rightarrow 0} \frac{7x}{\sin 7x}$$

$$= \frac{2}{7} (1)(1)(1)$$

$$= \left(\frac{2}{7} \right)$$

$$\lim_{x \rightarrow 0} \frac{\sin^3 2x}{5x^3 + 10x^4}$$

$$\lim_{x \rightarrow 0} \frac{\sin^3 2x}{5x^3(1+2x)}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^3 2x}{x^3} \cdot \frac{1}{5(1+2x)}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^3 \cdot \frac{1}{5(1+2x)} \cdot 8$$

$$= 8 \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^3 \lim_{x \rightarrow 0} \frac{1}{5(1+2x)}$$

$$= 8(1)^3 \left(\frac{1}{5} \right)$$

$$= \left(\frac{8}{5} \right)$$

Homework

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#7, 9, 15, 16, 18, 20, 22, 23, 26, 27, 31, 37