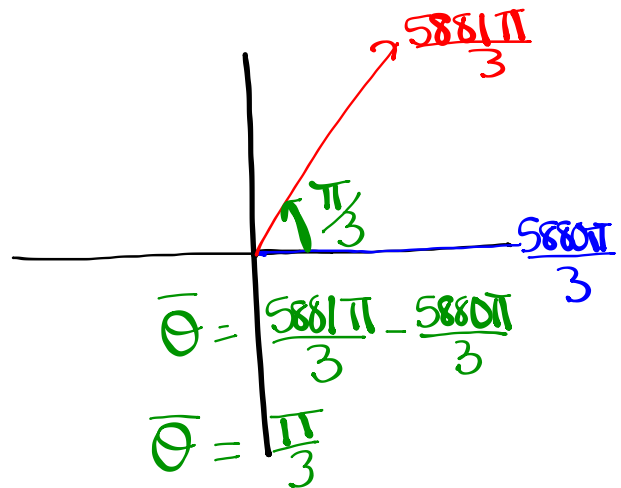


Sketch the following and determine a negative angle co-terminal with:

$$(i) \frac{5881\pi}{3}$$

$$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$$

$$1960\pi$$



Negative coterminal angle:

$$\frac{5881\pi}{3} - \frac{1962\pi}{1}$$

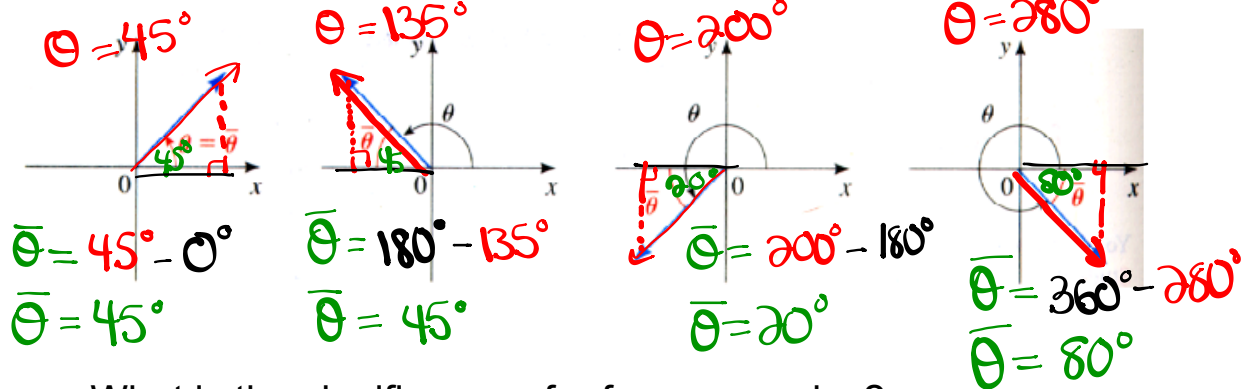
$$\frac{5881\pi}{3} - \frac{5886\pi}{3}$$

$$\frac{-5\pi}{3}$$

Reference Triangles:

Definition 17 The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.

The picture below illustrates this concept.



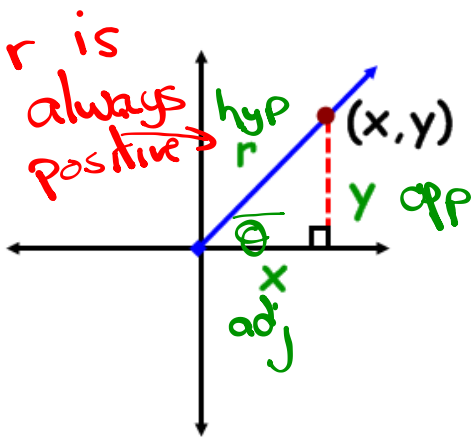
What is the significance of reference angles?

Angles on the Cartesian Plane

$< 90^\circ$ or $< \pi/2$ or $< 1.57 \text{ rads}$

- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.
 θ

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

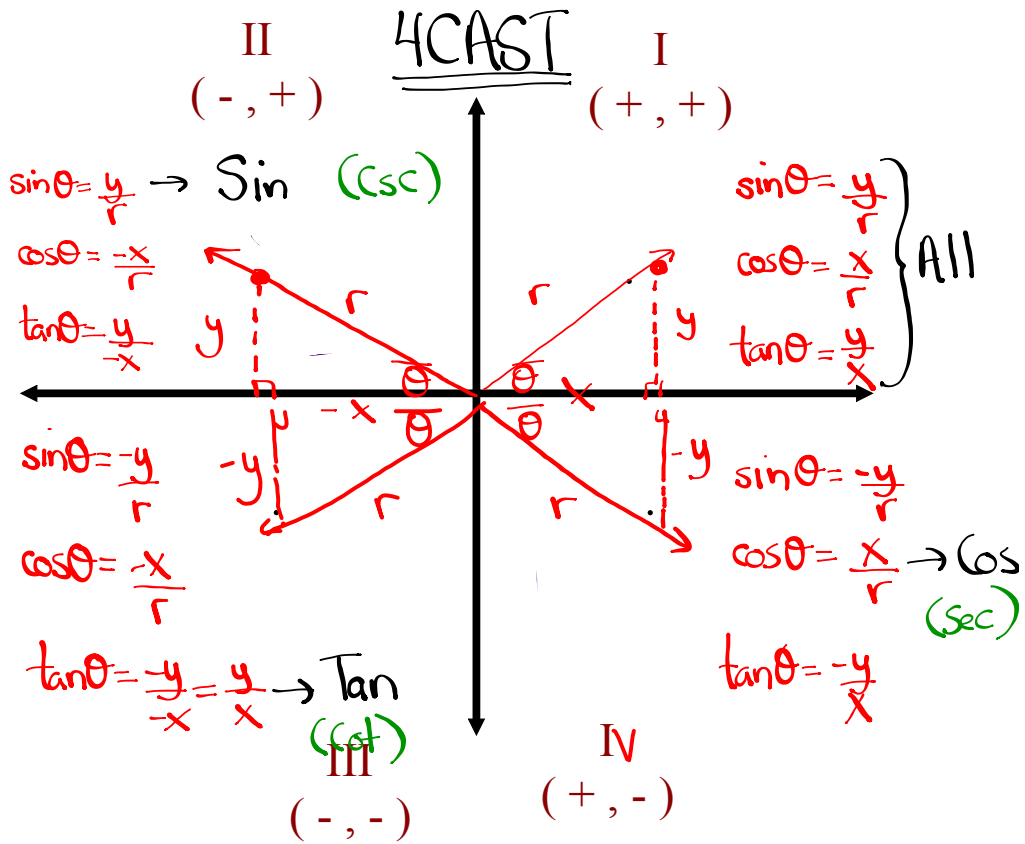
$\sin \theta = \frac{y}{r} = \frac{o}{h}$	$\csc \theta = \frac{r}{y} = \frac{h}{o}$
$\cos \theta = \frac{x}{r} = \frac{a}{h}$	$\sec \theta = \frac{r}{x} = \frac{h}{a}$
$\tan \theta = \frac{y}{x} = \frac{o}{a}$	$\cot \theta = \frac{x}{y} = \frac{a}{o}$

"Primary"

"Reciprocal"

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are **POSITIVE** in...



Where is θ if...

$\csc \theta < 0$

$\sin \theta < 0$ & $\tan \theta < 0$

$\csc \theta > 0$ & $\cot \theta < 0$

Homework

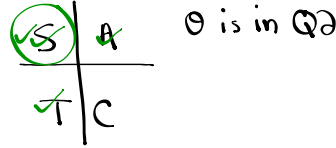
If $\sec\theta = -\sqrt{10}$ and $\sin\theta > 0$, determine the value of $\csc\theta$

$$\sec\theta = \frac{r}{x} = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$r = \sqrt{10} \text{ (always +)}$$

$$x = -1$$

① Determine what quadrant:
 $\sec\theta < 0$ + $\sin\theta > 0$
 or $\cos\theta < 0$ + $\sin\theta > 0$

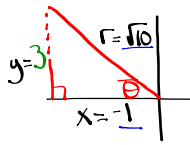


④ Find $\csc\theta$:

$$\csc\theta = \frac{r}{y}$$

$$\csc\theta = \frac{\sqrt{10}}{3}$$

② Draw a diagram



$$\textcircled{3} \quad x^2 + y^2 = r^2$$

$$(-1)^2 + y^2 = (\sqrt{10})^2$$

$$\textcircled{1} + y^2 = 10$$

$$y^2 = 9$$

$$y = \pm 3$$

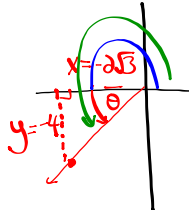
$$y = 3 \text{ (Q2)}$$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$$x = -2\sqrt{3}$$

$$y = -4$$

① Draw a diagram



② Find $\bar{\theta}$

$$\tan\bar{\theta} = \frac{y}{x}$$

$$\tan\bar{\theta} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\tan\bar{\theta} = 1.1547$$

$$\bar{\theta} = \tan^{-1}(1.1547)$$

convert calculator to radians

$$\bar{\theta} = 0.86 \text{ rads}$$

③ Find θ

$$\theta = \pi + \bar{\theta}$$

$$\theta = 3.14 + 0.86$$

$$\theta = 4 \text{ rads}$$

$$\theta = \pi - \bar{\theta}$$

$$\theta = 180^\circ - \bar{\theta}$$

$$\theta = 180^\circ + \bar{\theta}$$

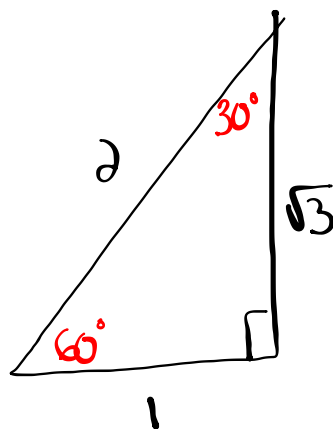
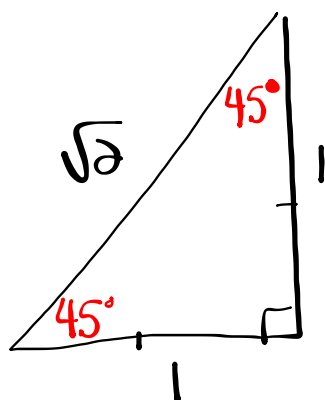
$$\theta = \pi + \bar{\theta}$$

$$\theta = \bar{\theta}$$

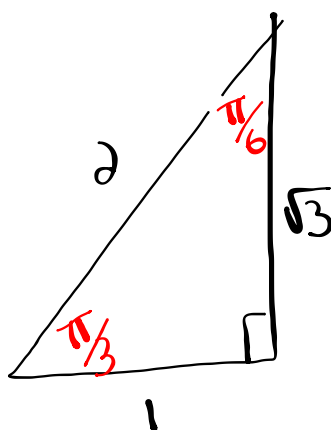
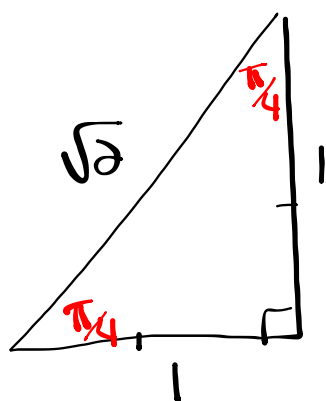
$$\theta = 360^\circ - \bar{\theta}$$

$$\theta = 2\pi - \bar{\theta}$$

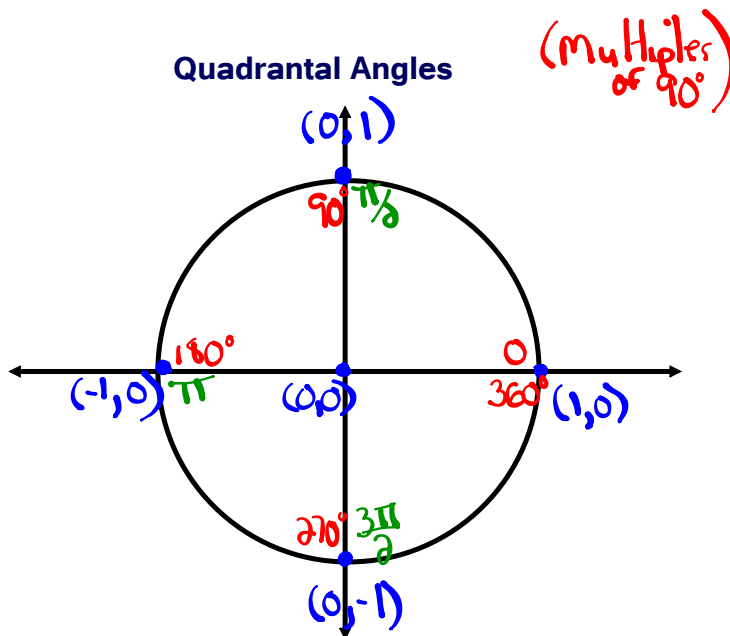
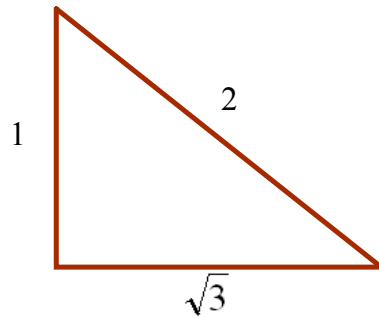
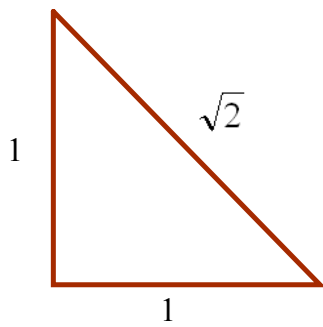
In Degrees



In Radians

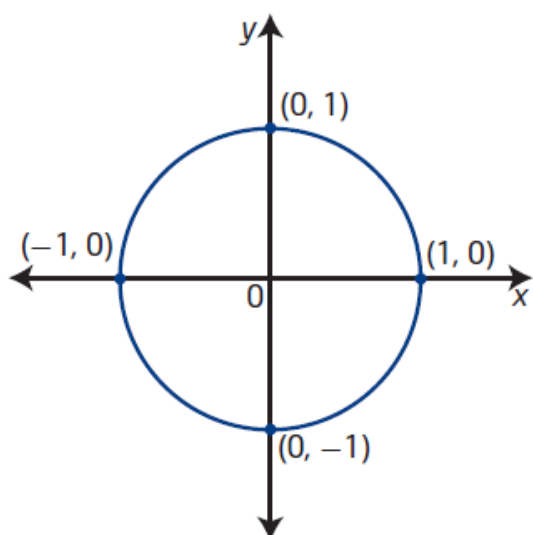


Special Angles (in radians)



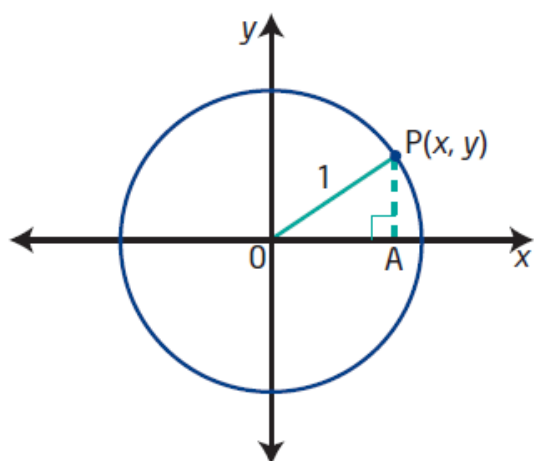
- The Unit Circle
- Center is @ $(0,0)$
 - radius is 1 unit

Unit Circle



unit circle

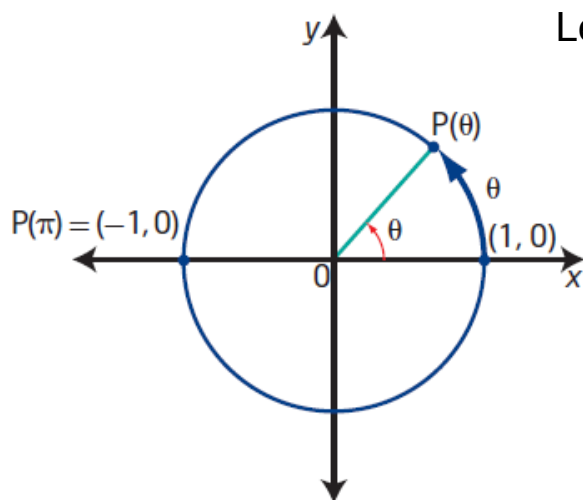
- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle



The equation of the unit circle is $x^2 + y^2 = 1$.

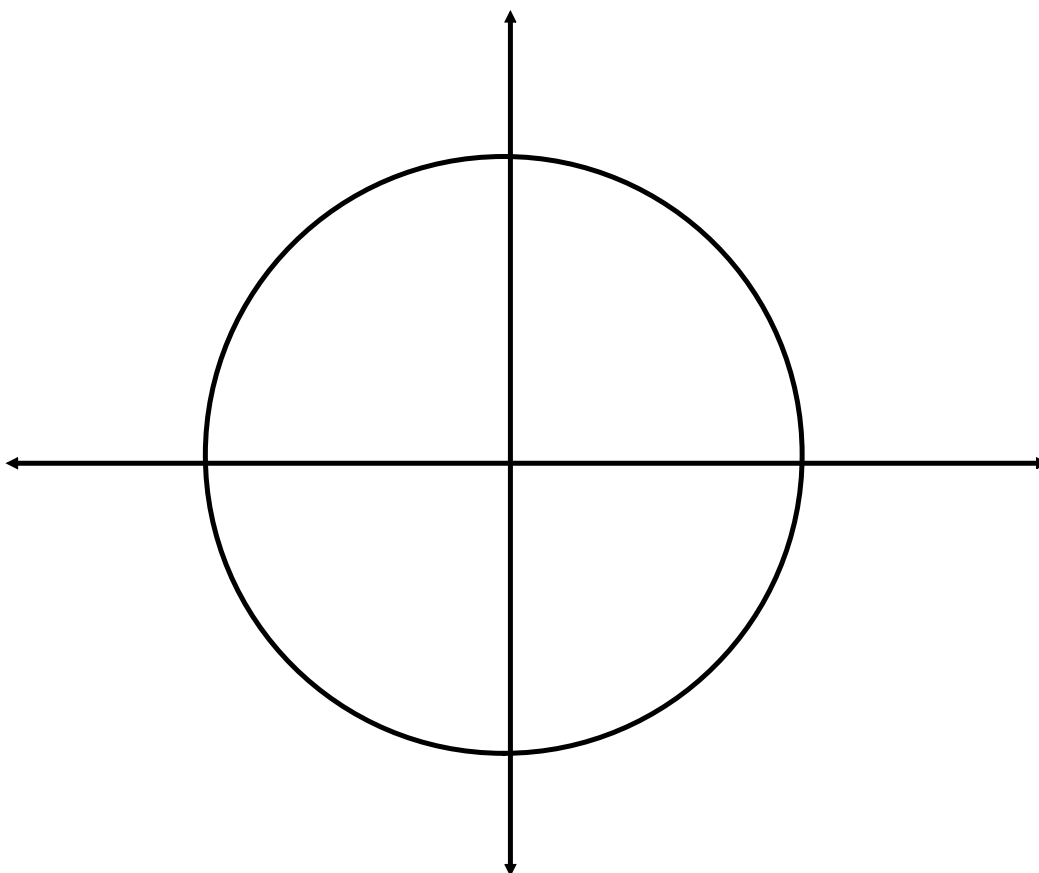
Determine the equation of a circle with centre at the origin and radius 6.

Special Angles on the Unit Circle:

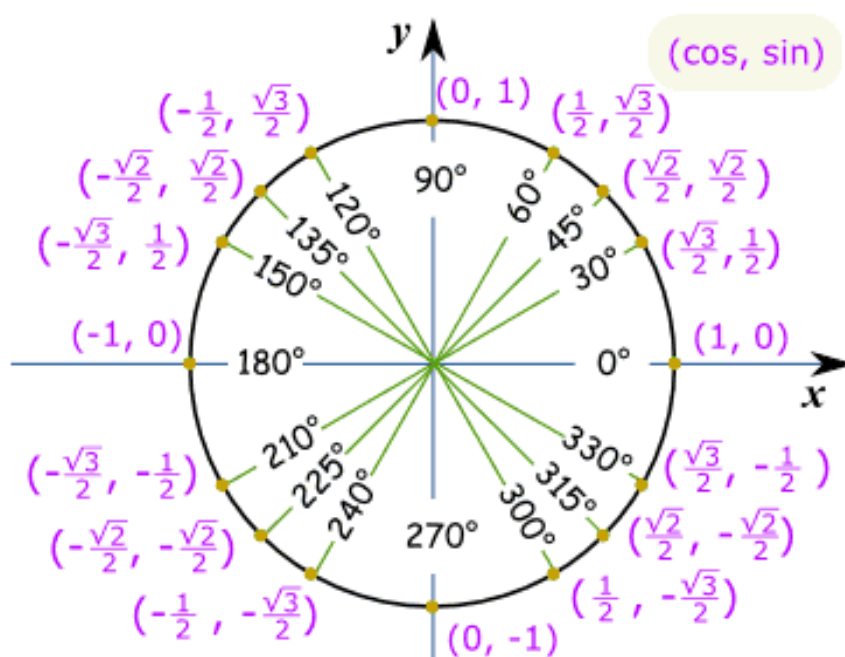


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

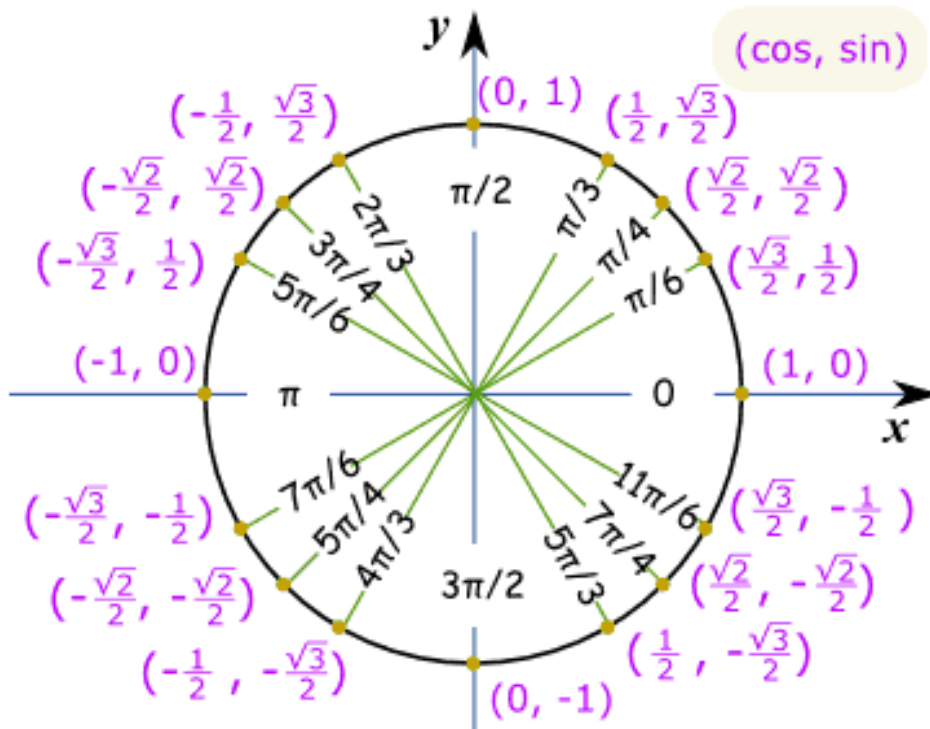


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians

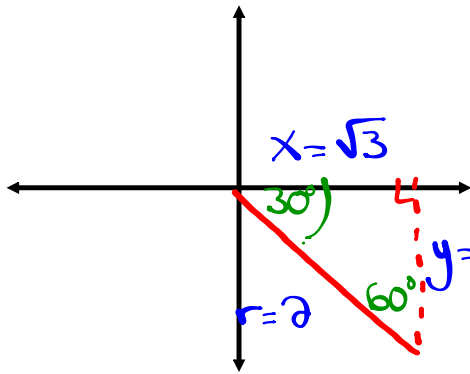


Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690^\circ$

Optional

(i) Find principal angle:
 $\theta = 690^\circ - 360^\circ = 330^\circ$



(ii) Sketch (Q4)

(iii) Find $\bar{\theta}$

$$\bar{\theta} = 360^\circ - 330^\circ = 30^\circ$$

(iv) Label

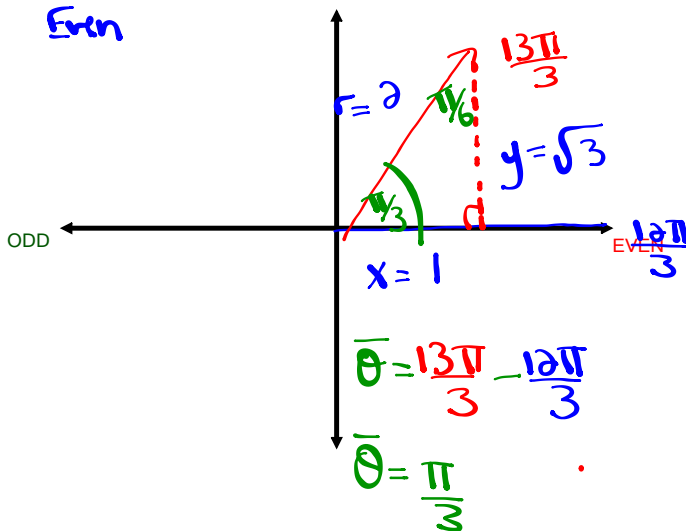
(v) Evaluate

$$\sin 690^\circ = -\frac{1}{2}$$

Ex. $\cos \frac{13\pi}{3} = \frac{2}{5} = \frac{x}{r} = \frac{1}{2}$

$\frac{12\pi}{3}$ $\frac{13\pi}{3}$ $\frac{14\pi}{3}$

4π
Even

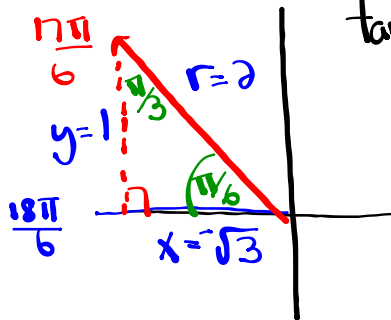


Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan \frac{17\pi}{6} = -\frac{1}{\sqrt{3}}$ 2. $\sin \frac{15\pi}{4} = -\frac{1}{\sqrt{2}}$ 3. $\cos \left(-\frac{21\pi}{4} \right) = \frac{-1}{\sqrt{2}}$

① $\frac{16\pi}{6}$ $\frac{17\pi}{6}$ $\frac{18\pi}{6}$
 3π
 (odd)

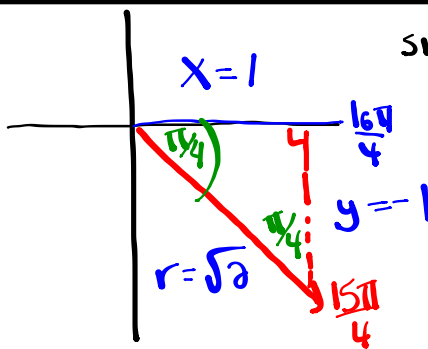


$$\tan \frac{17\pi}{6} = \frac{y}{x} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$= -\frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

$\tan \frac{17\pi}{6} = -\frac{\sqrt{3}}{3}$

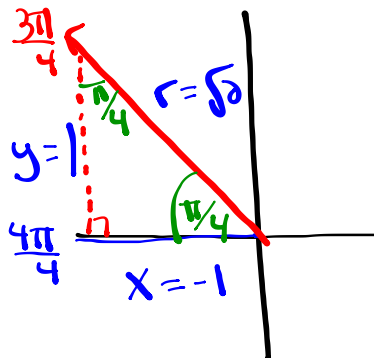
② $\frac{14\pi}{4}$ $\frac{15\pi}{4}$ $\frac{16\pi}{4}$
 4π
 (Even)



$$\sin \frac{15\pi}{4} = \frac{y}{r} = \frac{-1}{\sqrt{2}}$$

$\sin \frac{15\pi}{4} = -\frac{\sqrt{2}}{2}$

③ $-\frac{21\pi}{4} + \frac{6\pi}{1}$
 $-\frac{21\pi}{4} + \frac{24\pi}{4}$
 $\frac{3\pi}{4}$



$$\cos \left(-\frac{21\pi}{4} \right) = \cos \frac{3\pi}{4} = \frac{a}{h} = \frac{x}{r}$$

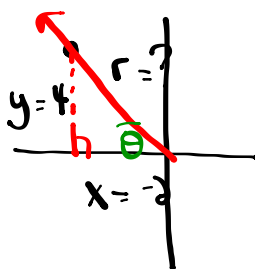
$$= -\frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$\frac{2\pi}{4}$ $\frac{3\pi}{4}$ $\frac{4\pi}{4}$
 π
 (odd)

$\cos \left(-\frac{21\pi}{4} \right) = -\frac{\sqrt{2}}{2}$

Review

ⓐ $x = -2$
 $y = 4$



ⓑ Find r

Attachments

Worksheet - Sketching Angles in Radians.doc