① $f(x) = \begin{cases} x^2-1 & x \geq 1 \\ -1 & -1 \leq x < 1 \\ |x| & x < -1 \end{cases}$

x^2-1

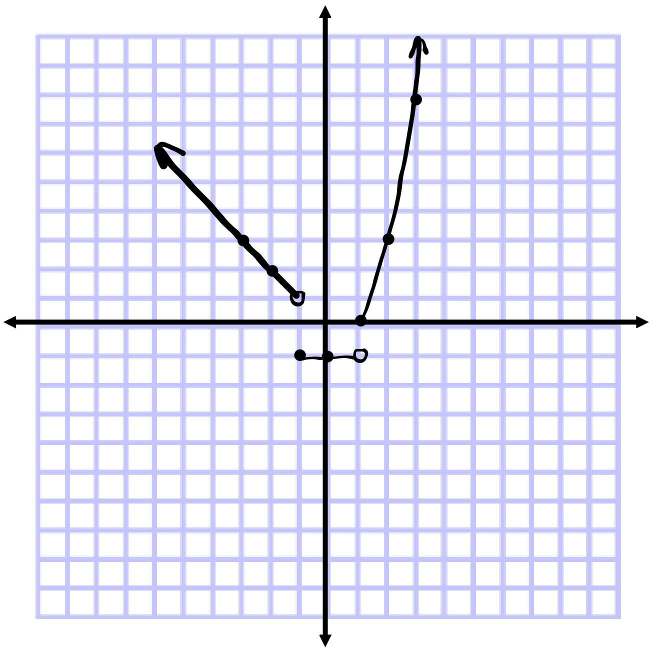
x	y
• 1	0
• 2	3
• 3	8

-1

x	y
• -1	-1
• 0	-1
o 1	-1

$|x|$

x	y
o -1	1
• -2	2
• -3	3



Discontinuous @ $x = \pm 1$

2 a) $\lim_{x\to\infty} \frac{(x^2-4)^2}{5x^4-2}$

$\lim_{x\to\infty} \frac{x^4-8x^2+16}{5x^4-2} = \frac{1}{5}$

b) $\lim_{x\to 1} \frac{x^2+3x-4}{x-1}$

$\lim_{x\to 1} \frac{2x+3}{1} = 5$

$\lim_{x\to 1} \frac{x^2+3x-4}{x-1}$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}} = 5$

c) $\lim_{x\to -2} \frac{\frac{1}{2}+\frac{1}{x}}{x^3+8}$

$\lim_{x\to -2} \frac{-\frac{1}{x^2}}{3x^2} = -\frac{1}{4}\cdot\frac{1}{12} = -\frac{1}{48}$

$\lim_{x\to -2} \frac{\cancel{2x}\,\frac{1}{\cancel{2}}+\frac{1}{\cancel{x}}\,\cancel{2x}}{(x^3+8)2x}$

$\lim_{x\to -2} \frac{\cancel{(x+2)}}{2x\underbrace{\cancel{(x+2)}(x^2-2x+4)}_{\text{sum of cubes}}}$

$= \frac{1}{-4(12)} = -\frac{1}{48}$

d) $\lim_{x\to 2} \frac{x^4-16}{x-2}$

$\lim_{x\to 2} \frac{4x^3}{1} = 32$

e) $\lim_{x\to 0} \frac{e^x-1}{\sin x}$

$\lim_{x\to 0} \frac{e^x}{\cos x} = \frac{1}{1} = 1$

f) $\lim_{x\to 1} \frac{x-1}{(\sqrt{10-x}-3)}\cdot\frac{(\sqrt{10-x}+3)}{(\sqrt{10-x}+3)}$

$\lim_{x\to 1} \frac{(x-1)(\sqrt{10-x}+3)}{10-x-9}$

$\lim_{x\to 1} \frac{\overset{-1}{\cancel{(x-1)}}(\sqrt{10-x}+3)}{\cancel{(1-x)}} = -6$

$\frac{1}{\frac{1}{2}(10-x)^{-1/2}(-1)}$

$\frac{1}{\frac{-1}{2\sqrt{10-x}}}$

$\frac{1}{\frac{-1}{6}}$

$1\cdot -6$

-6

g) $$\lim_{h\to 0} \frac{\left(\sqrt{2+h}-\sqrt{2-h}\right)\left(\sqrt{2+h}+\sqrt{2-h}\right)}{h\left(\sqrt{2+h}+\sqrt{2-h}\right)}$$

$$\lim_{h\to 0} \frac{2+h-(2-h)}{h\left(\sqrt{2+h}+\sqrt{2-h}\right)}$$

$$\lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2+h}+\sqrt{2-h}\right)} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

h) $$\lim_{x\to \infty} \frac{x-4}{(x-1)^3} = 0$$

i) $$\lim_{x\to 1} \frac{(x+3)^3-64}{x-1}$$

$$\lim_{x\to 1} \frac{3(x+3)^2(1)}{1} = 48$$

j) $$\lim_{x\to 0} \frac{\sin 2x}{2x^2+x}$$

$$\lim_{x\to 0} \frac{\sin 2x}{x(2x+1)}$$

$$\lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{1}{2x+1} \cdot 2$$

$$2\lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{1}{2x+1}$$

$$= 2(1)(1)$$

$$= 2$$

$$\lim_{x\to 0} \frac{2\cos 2x}{4x+1} = \frac{2}{1} = 2$$

k) $\lim\limits_{x\to 0} \dfrac{\tan 5x}{x}$

$\lim\limits_{x\to 0} \dfrac{5\sec^2 5x}{1} = 5(1)^2 = 5$

$\lim\limits_{x\to 0} \dfrac{\sin 5x}{\cos 5x} \cdot \dfrac{1}{x}$

$\lim\limits_{x\to 0} \dfrac{\sin 5x}{5x} \cdot \dfrac{1 \cdot 5}{\cos 5x}$

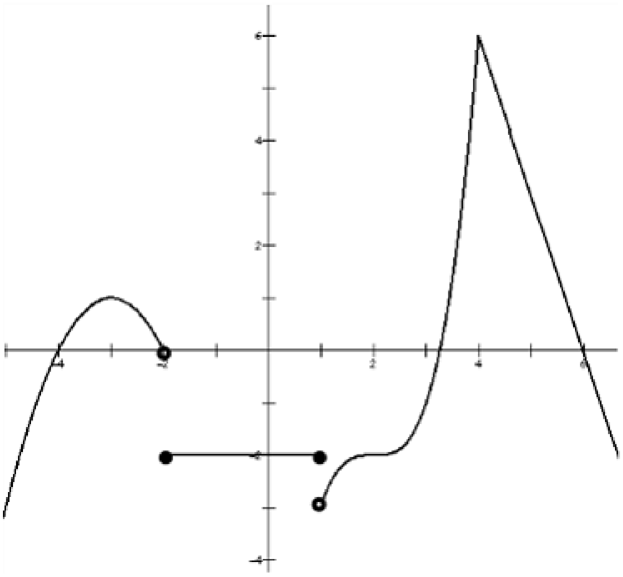
$5 \lim\limits_{x\to 0} \dfrac{\sin 5x}{5x} \cdot \dfrac{1}{\cos 5x}$

$= 5(1)(1) = 5$

l) $\lim\limits_{x\to 0} \dfrac{\sin 11x}{3x}$

$\lim\limits_{x\to 0} \dfrac{11\cos 11x}{3} = \dfrac{11(1)}{3} = \dfrac{11}{3}$

3



a) $f(1) = -2$

b) $\lim_{x \to 1^-} f(x) = -2$

c) $\lim_{x \to -2} f(x) = \text{DNE}$

d) $f(0) = -2$