Correct Homework Sheet

② $\frac{1-\boxed{\cos 2\theta}}{\sin^2\theta} = 2$

$\frac{1-(\cos^2\theta-\sin^2\theta)}{\sin^2\theta}$

$\frac{\boxed{1-\cos^2\theta}+\sin^2\theta}{\sin^2\theta}$

$\frac{\sin^2\theta+\sin^2\theta}{\sin^2\theta}$

$\frac{2\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$

2

③ $\boxed{\sin(x+y)}\boxed{\sin(x-y)} = \cos^2 y-\cos^2 x$

$(\sin x\cos y+\cos x\sin y)(\sin x\cos y-\cos x\sin y)$

$\sin^2 x\cos^2 y-\cos^2 x\sin^2 y$

$(1-\cos^2 x)\cos^2 y-\cos^2 x(1-\cos^2 y)$

$\cos^2 y-\cancel{\cos^2 x\cos^2 y}-\cos^2 x+\cancel{\cos^2 x\cos^2 y}$

$\cos^2 y-\cos^2 x$

⑤ $\boxed{\tan^4\theta} = \sec^4\theta\,(1-2\cos^2\theta+\cos^4\theta)$

$\frac{\sin^4\theta}{\cos^4\theta}$

$\boxed{\sec^4\theta}\,(1-\cos^2\theta)(1-\cos^2\theta)$

$\frac{1}{\cos^4\theta}$

⑦ $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = 2\boxed{\sec\theta}$

$\dfrac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$ $\quad 2\left(\dfrac{1}{\cos\theta}\right)$

$\dfrac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$ $\quad \dfrac{2}{\cos\theta}$

$\dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$

$\dfrac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$

$\dfrac{2}{\cos\theta}$

⑩ $\dfrac{\tan^2\theta}{\boxed{\tan^2\theta+1}} = \sin^2\theta$

$\boxed{\tan^2\theta} \div \sec^2\theta$

$\dfrac{\sin^2\theta}{\cos^2\theta} \div \dfrac{1}{\cos^2\theta}$

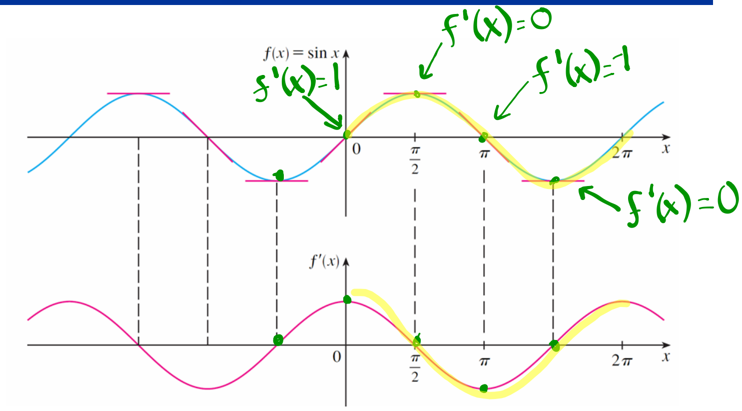
$\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta}$

$\sin^2\theta$

Derivatives of Trigonometric Functions

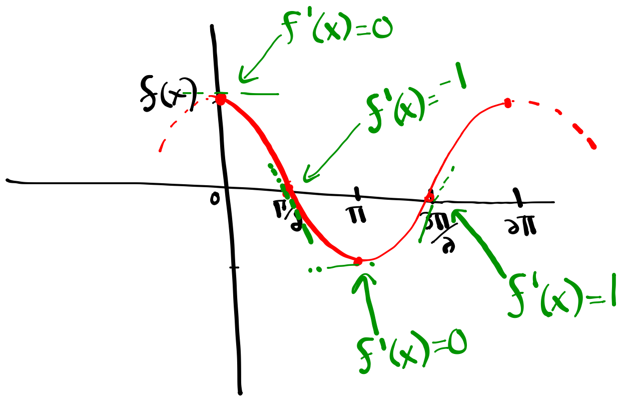
The Sine Function

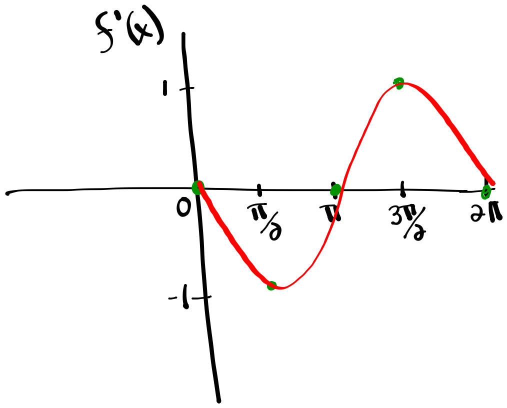
- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



if $f(x) = \sin x$

$f'(x) = \cos x$





Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin x}{h}+\frac{\cos x\sin h}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin x}{h}+\frac{\cos x\sin h}{h}\right]$$

$$= \lim_{h\to 0}\left[\sin x\left(\frac{\cos h-1}{h}\right)+\cos x\left(\frac{\sin h}{h}\right)\right]$$

$$= \lim_{h\to 0}\sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}+\lim_{h\to 0}\cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h\to 0$,

$$\lim_{h\to 0}\sin x=\sin x \quad\text{and}\quad \lim_{h\to 0}\cos x=\cos x$$

- With some work we can also show that

$$\lim_{h\to 0}\frac{\sin h}{h}=1 \quad\text{and}\quad \lim_{h\to 0}\frac{\cos h-1}{h}=0$$

$f(x) = \sin x$

$f'(x) = \cos x$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0}\sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}+\lim_{h\to 0}\cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}$$

$$= (\sin x)\cdot 0+(\cos x)\cdot 1=\cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du \qquad \frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du \qquad \frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du \qquad \frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Ex: $f(x) = \tan(5x^2)$ $\quad u = 5x^2$

$du = 10x$

$f'(x) = \sec^2(5x^2) \cdot 10x$

$f'(x) = 10x\sec^2(5x^2)$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $u = 3x$ $du = 3$

$y' = \cos(3x) \cdot 3$

$y' = 3\cos(3x)$

$y = \sin(x+2)$ $u = x+2$ $du = 1$

$y' = \cos(x+2) \cdot 1$

$y' = \cos(x+2)$

$y = \sin(kx+d)$ $u = kx+d$ $du = k$

$y' = \cos(kx+d) \cdot k$

$y' = k\cos(kx+d)$

Ex #2.

Differentiate:

$u = x^3$
$du = 3x^2$

a) $y = \sin(x^3)$

$y' = \cos(x^3) \cdot 3x^2$
$y' = 3x^2 \cos(x^3)$

$y = (\sin x)^3$ $u = x$ $du = 1$

b) $y = \sin^3 x$

$y' = 3(\sin x)^2 (\cos x) \cdot 1$
$y' = 3\sin^2 x \cos x$

$y = [\sin(x^2-1)]^3$
$u = x^2-1$ $du = 2x$

c) $y = \sin^3(x^2-1)$

$y' = 3[\sin(x^2-1)]^2 \cos(x^2-1) 2x$

$y' = 6x \sin^2(x^2-1) \cos(x^2-1)$

Ex #3.

Differentiate:

Product Rule:

$f'(x)g(x) + f(x)g'(x)$

$$y = (x^2)(\cos x)$$

$$y' = 2x\cos x + x^2(-\sin x \cdot 1)$$

$$y' = 2x\cos x - x^2\sin x$$

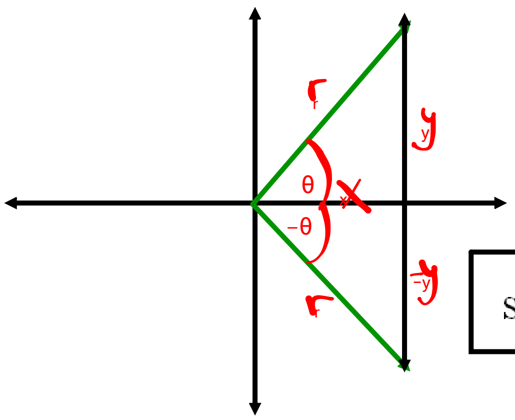
$$y' = x(2\cos x - x\sin x)$$

Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

Negative Angles



$$\sin\theta = \frac{y}{r} \qquad \cos(\theta) = \frac{x}{r}$$

$$\sin(-\theta) = -\frac{y}{r} \qquad \cos(-\theta) = \frac{x}{r}$$

$$\boxed{\sin(-\theta) = -\sin\theta} \qquad \boxed{\cos(-\theta) = \cos\theta}$$

Exercise 7.2

① a) $y = \cos(-4x)$

$u = -4x$

$du = -4$

$y' = -\sin u \cdot du$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

$y = \cos(-4x)$

$y = \cos(4x)$

$u = 4x$

$du = 4$

$y' = -\sin u \cdot du$

$y' = -\sin(4x) \cdot 4$

$y' = -4\sin(4x)$

7.2

1 c) $y = 4\sin(-2x^2-3)$

$y' = \cancel{0(\sin(-2x^2-3))} + 4(\cos(-2x^2-3))(-4x)$

$y' = -16x\cos(-2x^2-3)$

g) $y = \sin^{-2}(x^3)$

$y = [\sin(x^3)]^{-2}$

$y' = -2[\sin(x^3)]^{-3}(\cos(x^3))(3x^2)$

$y' = -6x^2\sin^{-3}(x^3)\cos(x^3)$

$y' = -\dfrac{6x^2\cos(x^3)}{\sin^3(x^3)}$

i) $y = 3\sin^4(2-x)^{-1}$

$y = 3[\sin(2-x)^{-1}]^4$

$u = (2-x)^{-1}$
$du = -(2-x)^{-2}(-1)$
$= (2-x)^{-2}$

$y' = 12[\sin(2-x)^{-1}]^3\cos(2-x)^{-1} \cdot (2-x)^{-2}$

$y' = \dfrac{12\sin^3(2-x)^{-1}\cos(2-x)^{-1}}{(2-x)^2}$

l) $y = \dfrac{\sin x}{1+\cos x}$

$\dfrac{dy}{dx} = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$\dfrac{dy}{dx} = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ ← Pythagorean Identity

$\dfrac{dy}{dx} = \dfrac{\cancel{1+\cos x}}{(1+\cos x)^{\cancel{2}}} = \dfrac{1}{1+\cos x}$

Attachments

Derivatives Worksheet.doc