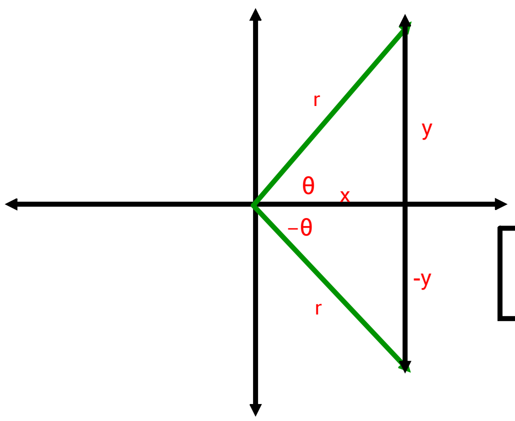
Negative Angles



$$\sin\theta = \frac{y}{r}$$

$$\cos(\theta) = \frac{x}{r}$$

$$\sin(-\theta) = -\frac{y}{r}$$

$$\cos(-\theta) = \frac{x}{r}$$

$$\boxed{\sin(-\theta) = -\sin\theta}$$

$$\boxed{\cos(-\theta) = \cos\theta}$$

Questions from Homework

① m) $y = (1 + \cos^2 x)^6$ $\rightarrow (\cos x)^2$

$\frac{dy}{dx} = 6(1 + \cos^2 x)^5 \left(2(\cos x)(-\sin x)\right)$

$\frac{dy}{dx} = -6(1 + \cos^2 x)^5 (2 \sin x \cos x)$ ← Double Angle Identity

$\frac{dy}{dx} = -6(1 + \cos^2 x)^5 (\sin 2x)$

n) $y = \sin\left(\frac{1}{x}\right) = \sin(x^{-1})$

$y' = \cos(x^{-1}) \cdot (-x^{-2})$

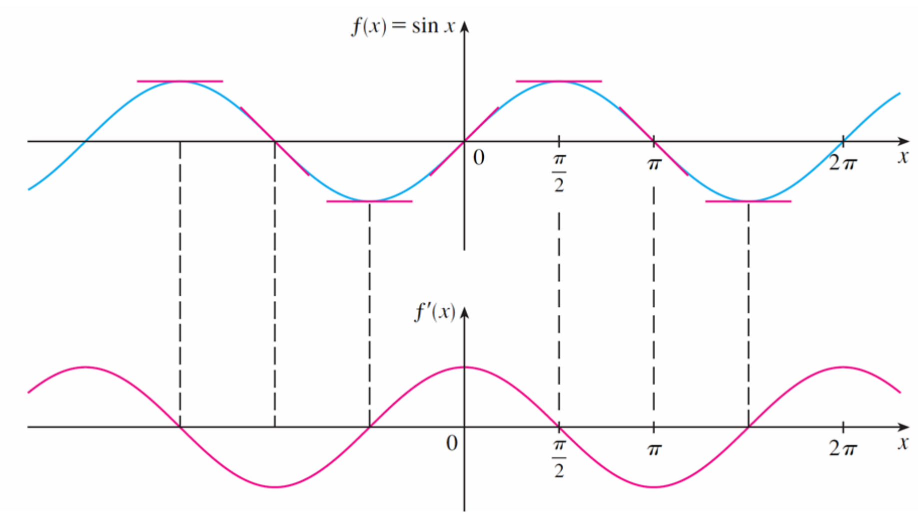
$y' = \cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

$y' = -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

    $$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

  $$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du \qquad \frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du \qquad \frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du \qquad \frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Examples:

① $y = \sin(2x)$

$u = 2x$

$d_u = 2$

$y' = \cos(2x) \cdot 2$

$y' = 2\cos(2x)$

② $f(x) = \cos(x^2)$

$u = x^2$

$d_u = 2x$

$f'(x) = -\sin(x^2) \cdot 2x$

$f'(x) = -2x\sin(x^2)$

$y = \tan(3x^2 - 5)$

$u = 3x^2 - 5$

$d_u = 6x$

$\frac{dy}{dx} = \sec^2(3x^2 - 5) \cdot 6x$

$\frac{dy}{dx} = 6x\sec^2(3x^2 - 5)$

$g(x) = \csc(6x + 5)$

$u = 6x + 5$

$d_u = 6$

$g'(x) = -\csc(6x+5)\cot(6x+5) \cdot 6$

$g'(x) = -6\csc(6x+5)\cot(6x+5)$

Let's Practice...

Differentiate the following:

$$f(x) = \frac{1}{1+\tan x} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$f'(x) = \frac{(0)\cancel{(1+\tan x)} - (1)(\sec^2 x)}{(1+\tan x)^2}$$

$$f'(x) = \frac{-\sec^2 x}{(1+\tan x)^2}$$

$$f(x) = 1(1+\tan x)^{-1}$$

$$f'(x) = -1(1+\tan x)^{-2}(\sec^2 x)$$

$$f'(x) = \frac{-\sec^2 x}{(1+\tan x)^2}$$

Ex #2.

Differentiate:

$$f(x) = 2\csc^3(3x^2) = 2[\csc(3x^2)]^3$$

$$f'(x) = 6[\csc(3x^2)]^2[-\csc(3x^2)\cot(3x^2)\cdot 6x]$$

$$f'(x) = -36x\csc^2(3x^2)\csc(3x^2)\cot(3x^2)$$

$$f'(x) = -36x\csc^3(3x^2)\cot(3x^2)$$

Homework

Worksheet on derivatives of trigonometric functions

Page 314 #3 c,e **Ex (7.2)**

Page 319 #1 Ex (7.3)

③ a) Find the equation of the tangent line to the given curve at the given point.

a) $y = 2\sin x$ at $(\pi/6, 1)$ $x_1 = \frac{\pi}{6}$ $y_1 = 1$

(i) Find the derivative:

$y = 2\sin x$

$y' = 2\cos x$

(ii) Find m:

$y'(\pi/6) = 2\cos(\pi/6)$

$y' = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$

$y' = \sqrt{3}$

$m = \sqrt{3}$

(iii) Find equation:

$y - y_1 = m(x - x_1)$

$y - 1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

$y - 1 = x\sqrt{3} - \frac{\pi\sqrt{3}}{6}$

$6y - 6 = 6x\sqrt{3} - \pi\sqrt{3}$

$\boxed{0 = 6x\sqrt{3} - 6y - \pi\sqrt{3} + 6}$

Attachments

Derivatives Worksheet.doc