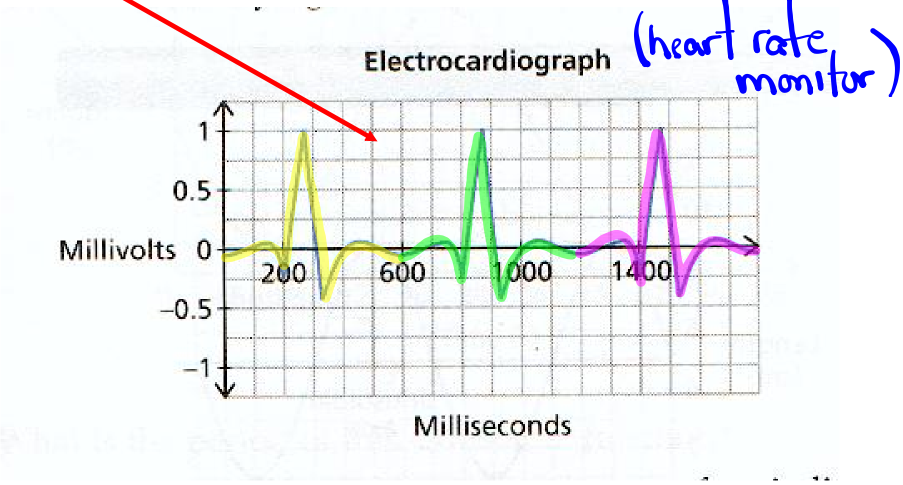
Sinusoidal Relations (Trig Graphs)

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(a function that repeats)

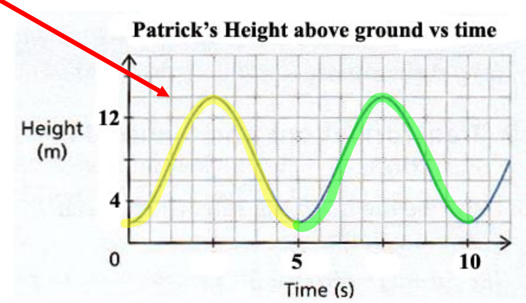
Example of periodic behavior



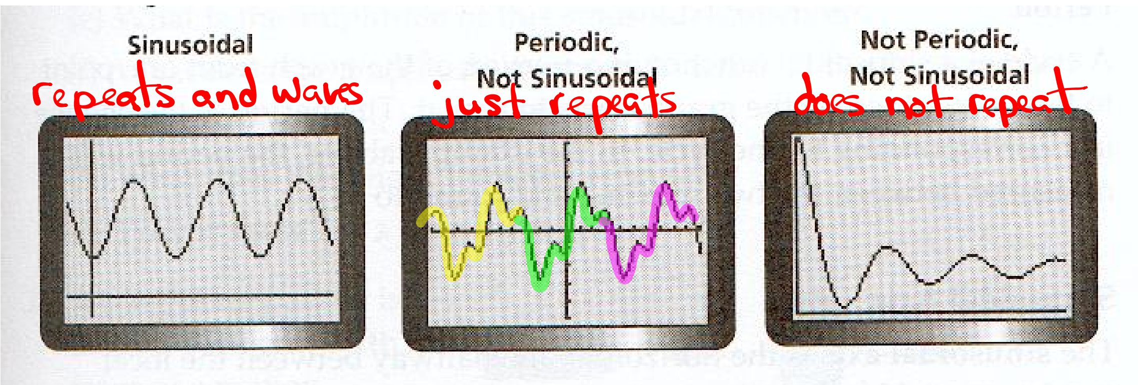
Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

(Repeats and looks like a smooth wave).

Example of sinusoidal behavior



These illustrations should summarize periodic and sinusoidal...

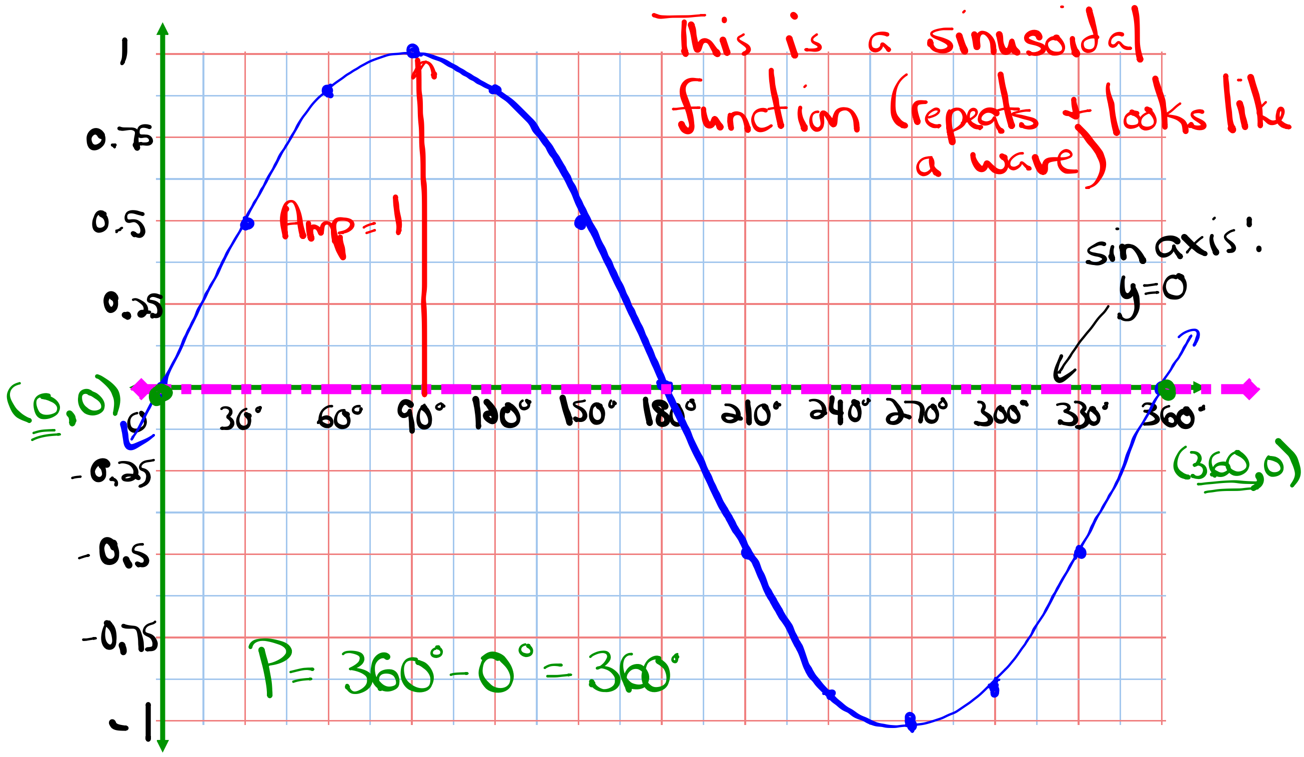


Let's examine the graph of $y = \sin\theta$

$y = \sin x$

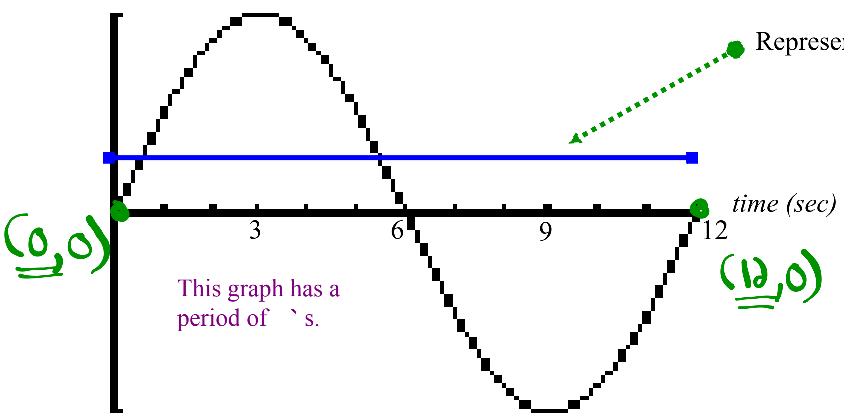
θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Now plot the above points...



Vocabulary of Sinusoidal Functions

I. Period:
The change in x corresponding to one cycle. (one repitition)



$P = 12 - 0 = 12$

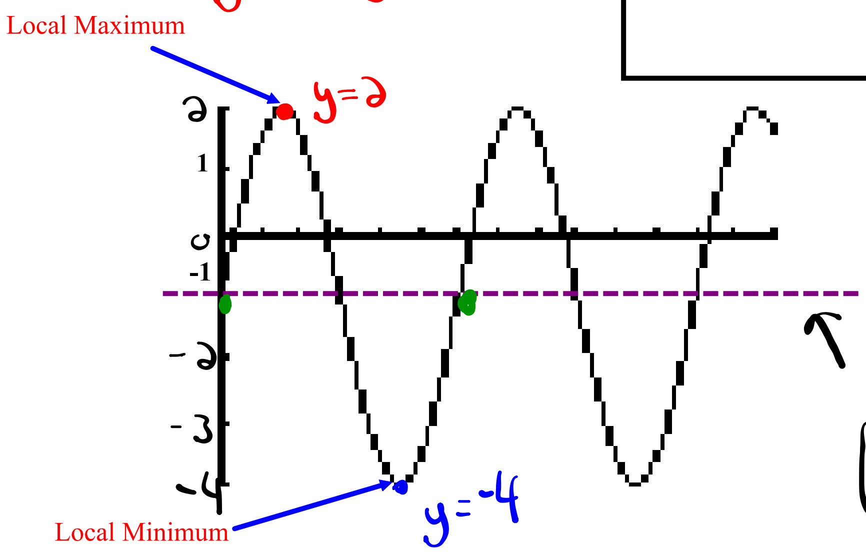
II. Sinusoidal Axis:
The horizontal line halfway between the local maximum and local minimum.

highest (y) lowest (y)

$$\boxed{\text{Sinusoidal Axis} = \frac{\text{Max} + \text{Min}}{2}}$$

$$= \frac{2 + (-4)}{2}$$

$$= \frac{-2}{2} = -1$$



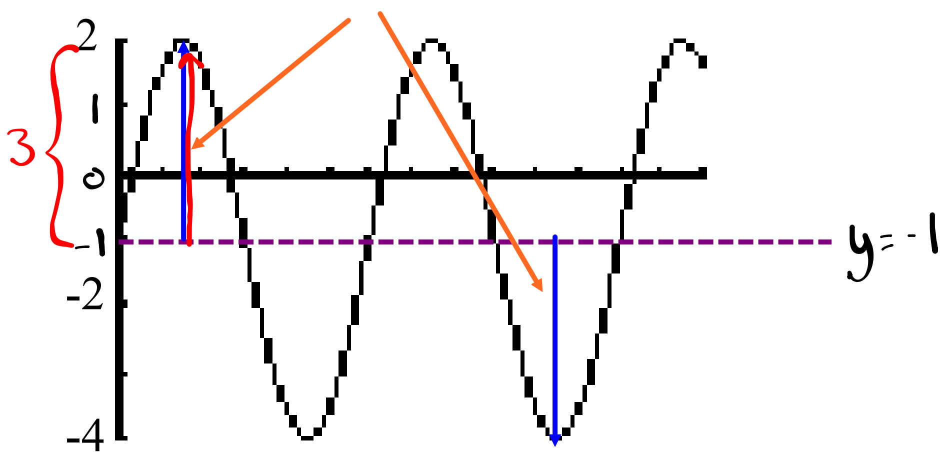
Equation of sinusoidal axis is $y = -1$

$\boxed{k = -1}$

III. Amplitude: The vertical distance from the sinusoidal axis to a local maximum or local minimum. Amplitude = $|a|$

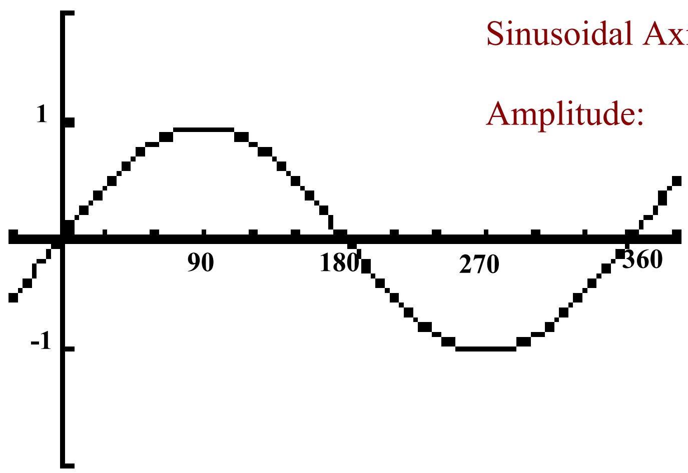
Amp is always positive

Amplitude would equal 3



Summarize...

Here is the graph of $y = \sin\theta$



Period :

Sinusoidal Axis:

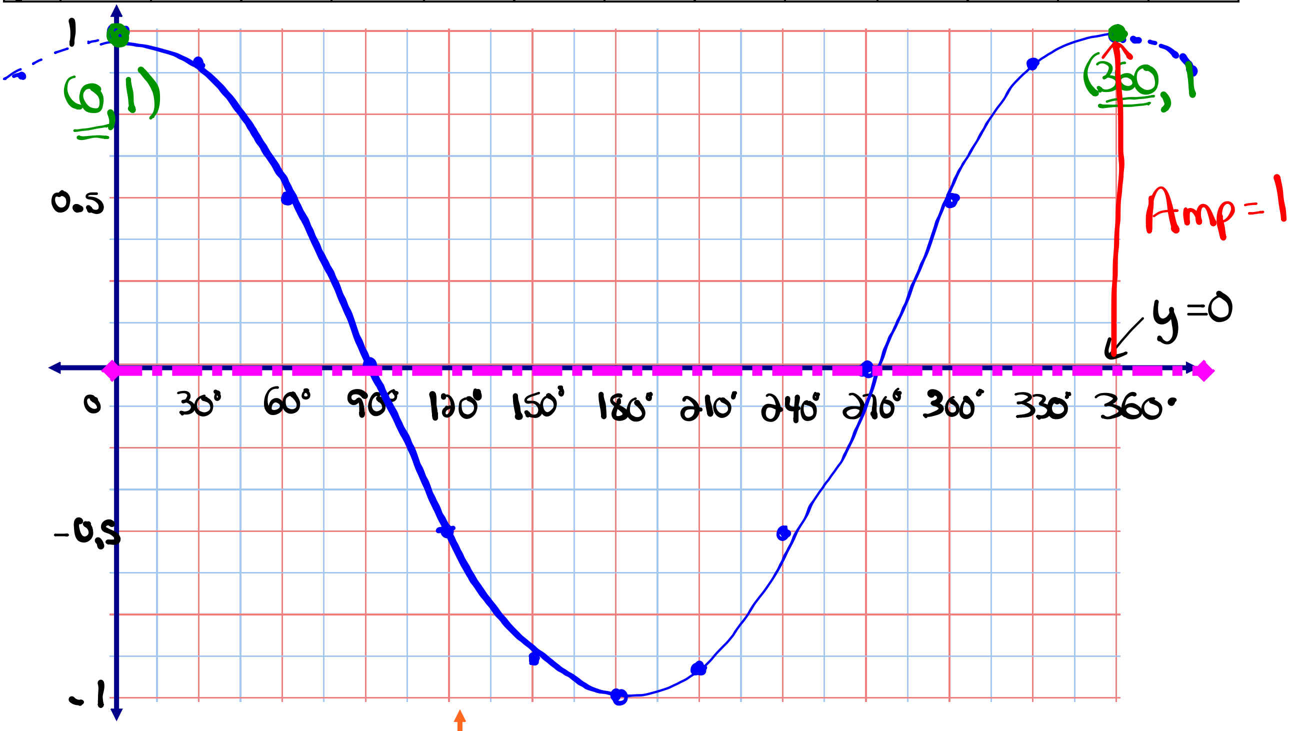
Amplitude:

What about $y = \cos\theta$?

$y = \cos x$

Complete the table of values and sketch below

θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Is this a sinusoidal function? Yes (Repeats + looks like waves)

What about the period, sinusoidal axis, and amplitude?

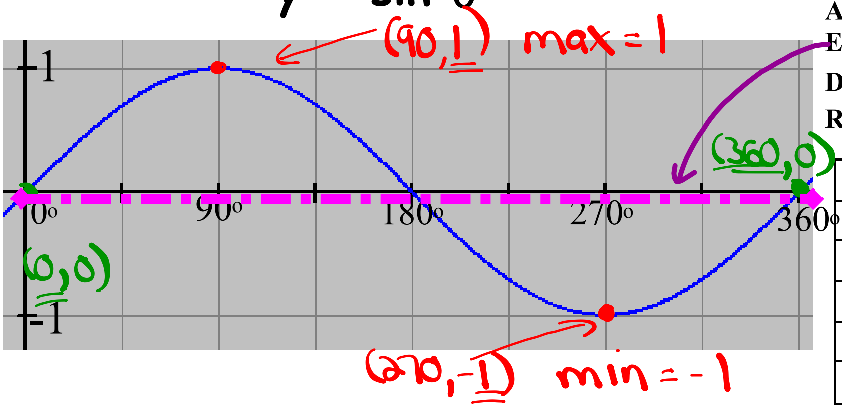
Period $= 360° - 0° = 360°$

sinusoidal axis $= \dfrac{\text{max} + \text{min}}{2} = \dfrac{1 + (-1)}{2} = \dfrac{0}{2} = 0$ $(y = 0)$

Amplitude $= 1$

Basic Trig Graphs (Base Functions)

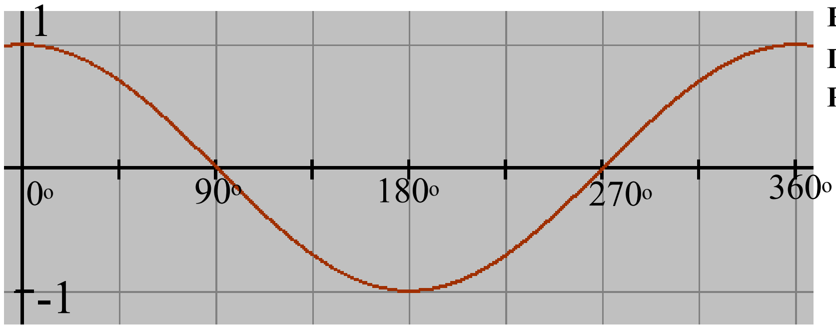
y = sin θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R}
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	0
90°	1
180°	0
270°	–1
360°	0

y = cos θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R}
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	1
90°	0
180°	–1
270°	0
360°	1

Homework

Page 233 #1-9

ex: $\frac{y}{2} - 5 = -4\cos[3x - 90°] - 7$

$2 \cdot \frac{y}{2} = 2 \cdot -4\cos[3x - 90°] - 2 \cdot 2$

$y = -8\cos[3x - 90°] - 4$

$y = -8\cos[3(x - 30°)] - 4$

$y = -\underline{8}\cos[\underline{3}(x - \underline{30°})] - \underline{4}$

a = -8 → vertically stretched by a factor of 8.
vertically reflected in the x-axis.

b = 3 → horizontally stretched by a factor of $\frac{1}{3}$.

h = 30° → horizontally translated 30° right.

k = -4 → vertically translated 4 units down.

Questions from Homework

6. Match each function with its graph.

~~a)~~ $y = 3 \cos x$ $a = 3$

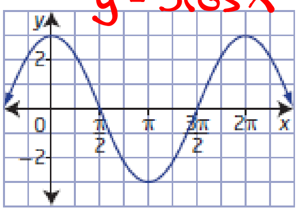
~~b)~~ $y = \cos 3x$ $b = 3$ $\rightarrow b = \frac{2\pi}{3}$

c) $y = -\sin x$ $a = -1$

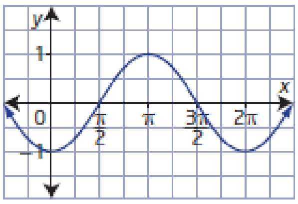
d) $y = -\cos x$ $a = -1$

$y = 3\cos x$

A



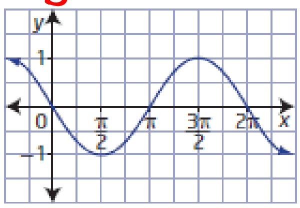
B



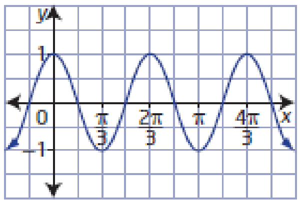
$y = -\cos x$

$y = -\sin x$

C



D



$y = \cos 3x$

From Sheet:

h) $2°$ $\frac{1}{\cancel{2}}(y+2) \overset{2\cdot}{=} 3\cos(x-90°)$

$y+2 \overset{-2}{=} 6\cos(x-90°)^{-2}$

$y = 6\cos(x-90°) - 2$

$a = 6$ $h = 90°$ equation of sin axis: $y = -2$

$b = 1$ $k = -2$ $P = \frac{360°}{b} = \frac{360°}{1} = 360°$

Worksheet

① d) $y - 5 = 6\cos\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right] - 2$

$y = 6\cos\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right] + 3$

$a = 6$ $h = \frac{\pi}{2}$ equation of sin. axis: $y = 3$

$b = \frac{1}{3}$ $k = 3$ $P = \frac{2\pi}{b} = 2\pi \div \frac{1}{3} = 2\pi \cdot \frac{3}{1} = 6\pi$

g) $y + 5 = -2\sin\left(4x + \frac{\pi}{3}\right)$

$y = -2\sin\left(4x + \frac{\pi}{3}\right) - 5$ (Factor out a 4)

$y = -2\sin\left[4\left(x + \frac{\pi}{12}\right)\right] - 5$

$\frac{\pi}{3} \div 4$

$\frac{\pi}{3} \times \frac{1}{4} = \frac{\pi}{12}$

$a = -2$ $h = -\frac{\pi}{12}$ equation of sin. axis: $y = -5$

$b = 4$ $k = -5$ $P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

Worksheet

h) $2 \cdot \frac{1}{2}(y+2) \stackrel{2\cdot}{=} 3\cos(x-90^\circ)$

$1(y+2) = 6\cos(x-90^\circ)$

$y+2 = 6\cos(x-90^\circ)$

$y = 6\cos(x-90^\circ)-2$

$a = 6$ $b = 1$ $h = 90^\circ$ $k = -2$

Amp $= 6$ $P = \frac{360^\circ}{1} = 360^\circ$ sin axis: $y = -2$

Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $y = a\sin[b(x-h)]+k$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.
b<0 " " " " " " " y-axis
2. Amplitude: The amplitude of the graph will be equal to | a | .
3. Period: The period of the graph will be equal to $\frac{360°}{b}$ or $\frac{2\pi}{b}$
4. Horizontal Phase Shift: (Translation) The graph will shift h units to the right/left
5. Vertical Translation: The graph will shift k units up./down

Mapping Notation: $(x, y) \to \left(\frac{1}{b}x+h, ay+k\right)$

Transformations of Sinusoidal Functions

Example: $f(\theta) = -2\sin[3(\theta+30°)]-2$

a = -2 b = 3 h = -30° k = -2

Range: max = k + Amp = -2 + 2 = 0
min = k - Amp = -2 - 2 = -4

Domain	{θ|θ∈R} or (-∞, ∞)
Range	{y|-4 ≤ y ≤ 0, y∈R} or [-4, 0]
Reflection	in the x-axis (a<0)
Amplitude	Amp = |a| = |-2| = 2
Horizontal Phase Shift	30° left (h = -30°)
Vertical Translation	2 down (k = -2)
Period	$P = \frac{360°}{b} = \frac{360°}{3} = 120°$

sinusoidal axis y = -2

EXAMPLE #1

Now let's sketch a graph of $y = 3\cos[2(\theta - 135°)] + 2$

$a = 3$ $b = 2$ $h = 135°$ $k = 2$

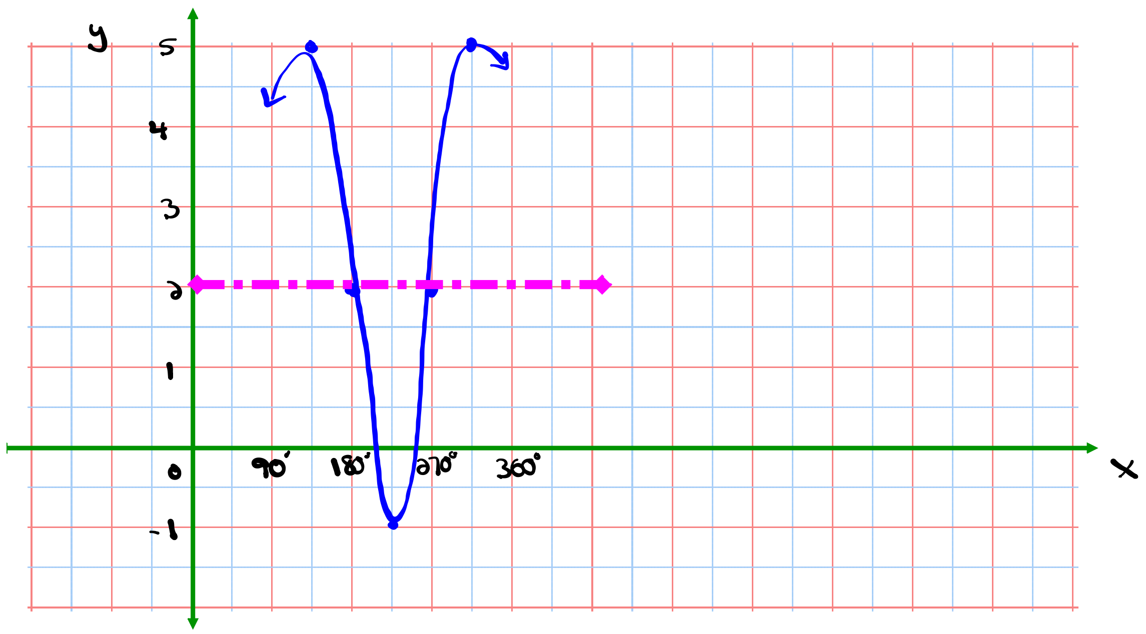
$(\theta, y) \rightarrow \left[\frac{1}{2}\theta + 135°, 3y + 2\right]$

$y = \cos\theta$

θ	y
0°	1
90°	0
180°	-1
270	0
360	1

New points after mapping

θ	y
135°	5
180°	2
225°	-1
270°	2
315°	5



DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -1 \leq y \leq 5, y \in R\}$
AMPLITUDE	$Amp = \lvert a \rvert = \lvert 3 \rvert = 3$
PERIOD	$P = \frac{360°}{b} = \frac{360°}{2} = 180°$
PHASE SHIFT	135° right ($h = 135°$)
VERTICAL TRANSLATION	2 up ($k = 2$)
EQUATION OF SINUSOIDAL AXIS	$y = 2$

Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6}{3}\cos(3x - \pi) - \frac{9}{3}$$

$a = -2$

$b = 3$

$h = \frac{\pi}{3}$

$k = -3$

$y = -2\cos(3x - \pi) - 3$

$y = -2\cos[3(x - \frac{\pi}{3})] - 3$

$y = -2\cos[3(x - \frac{\pi}{3})] - 3$

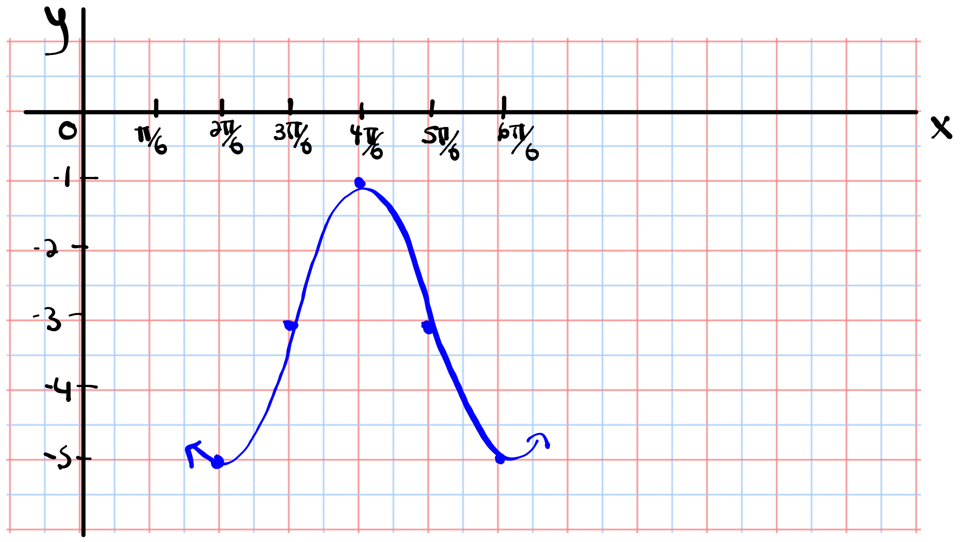
$(x, y) \rightarrow [\frac{1}{3}x + \frac{\pi}{3}, -2y - 3]$

$y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

New points after mapping

x	y
$\frac{2\pi}{6} = \pi/3$	-5
$\frac{3\pi}{6} = \pi/2$	-3
$\frac{4\pi}{6} = 2\pi/3$	-1
$\frac{5\pi}{6} = 5\pi/6$	-3
$\frac{6\pi}{6} = \pi$	-5



Homework

DOMAIN	$\{x \mid x \in R\}$
RANGE	$\{y \mid -5 \leq y \leq -1, y \in R\}$
AMPLITUDE	$Amp = \lvert -2 \rvert = 2$
PERIOD	$P = \frac{2\pi}{3}$
PHASE SHIFT	$\frac{\pi}{3}$ right $(h = \frac{\pi}{3})$
VERTICAL TRANSLATION	3 down $(k = -3)$
EQUATION OF SINUSOIDAL AXIS	$y = -3$

$\frac{1}{3}x+\frac{\pi}{3}$

(i) $\frac{1}{3}(0)+\frac{\pi}{3}$

$0+\frac{\pi}{3}$

$\frac{\pi}{3}$

$\frac{2\pi}{6}$

(ii) $\frac{1}{3}\left(\frac{\pi}{2}\right)+\frac{\pi}{3}$

$\frac{\pi}{6}+\frac{\pi}{3}$

$\frac{\pi}{6}+\frac{2\pi}{6}$

$\frac{3\pi}{6}$

$\frac{\pi}{2}$

$\frac{3\pi}{6}$

(iii) $\frac{1}{3}(\pi)+\frac{\pi}{3}$

$\frac{\pi}{3}+\frac{\pi}{3}$

$\frac{2\pi}{3}$

$\frac{4\pi}{6}$

(iv) $\frac{1}{3}\left(\frac{3\pi}{2}\right)+\frac{\pi}{3}$

$\frac{3\pi}{6}+\frac{\pi}{3}$

$\frac{3\pi}{6}+\frac{2\pi}{6}$

$\frac{5\pi}{6}$

$\frac{5\pi}{6}$

(v) $\frac{1}{3}(2\pi)+\frac{\pi}{3}$

$\frac{2\pi}{3}+\frac{\pi}{3}$

$\frac{3\pi}{3}$

π

$\frac{6\pi}{6}$

Homework

State ***a, b, h, k, and P*** from the following sinusoidal equations:

$$2y+6=4\sin(4x+\frac{\pi}{2})-2$$

$$\frac{2y}{2}=\frac{4\sin}{2}(4x+\frac{\pi}{2})-\frac{8}{2}$$

$$y=2\sin(4x+\frac{\pi}{2})-4$$

$$y=2\sin[4(x+\frac{\pi}{8})]-4$$

$$\frac{\pi}{2}\div 4=\frac{\pi}{2}\times\frac{1}{4}=\frac{\pi}{8}$$

$$y=\underline{2}\sin[\underline{4}(x+\frac{\pi}{\underline{8}})]-\underline{4}$$

$a=2$ $b=4$ $h=-\frac{\pi}{8}$ $k=-4$

$P=\frac{2\pi}{4}=\frac{\pi}{2}$ sin axis: $y=-4$

Example...

Graph the equation $y = -3\sin(2\theta + \pi) + 1$ using mapping notation.

$y = -3\sin\left[2\left(\theta + \frac{\pi}{2}\right)\right] + 1$

$a = -3$ $b = 2$ $h = -\frac{\pi}{2}$ $k = 1$

$(\theta, y) \rightarrow \left[\frac{1}{2}\theta - \frac{\pi}{2}, -3y + 1\right]$

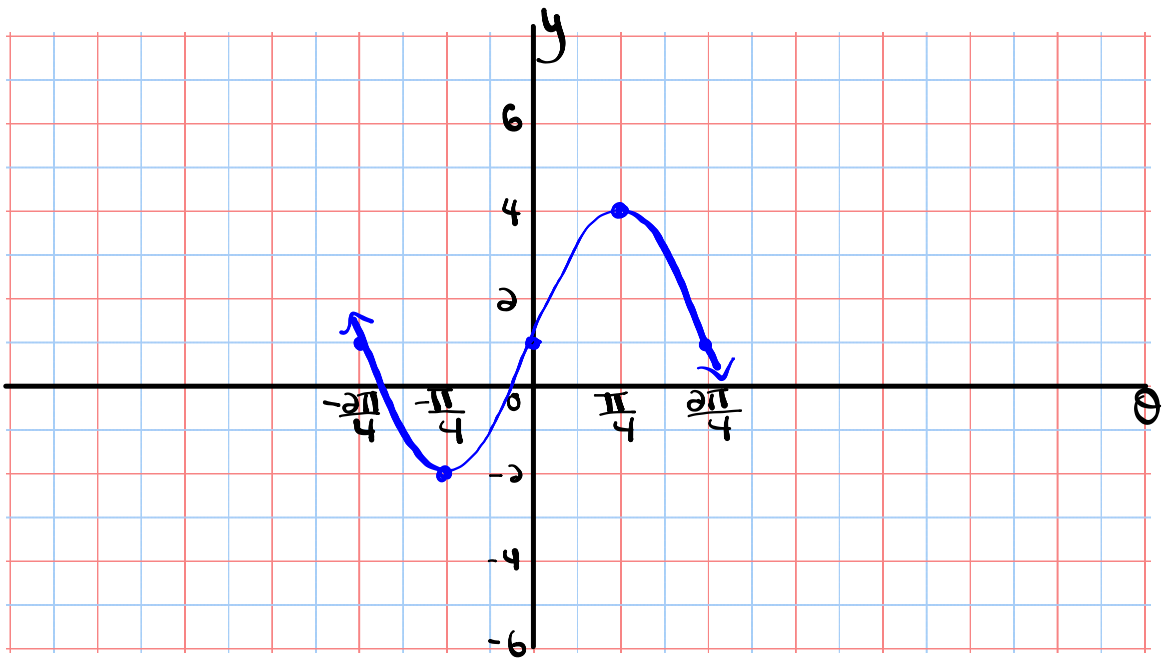
AMPLITUDE	$Amp = \lvert -3 \rvert = 3$
PERIOD	$P = \frac{2\pi}{2} = \pi$
PHASE SHIFT	$\pi/2$ left
VERTICAL TRANSLATION	1 up
EQUATION OF SINUSOIDAL AXIS	$y = 1$

$y = \sin\theta$

θ	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

New points after mapping

θ	y
$-\frac{2\pi}{4} = -\pi/2$	1
$-\pi/4$	-2
0	1
$\pi/4$	4
$\frac{2\pi}{4} = \pi/2$	1



Domain: $\{\theta \mid \theta \in R\}$

Range: $\{y \mid -2 \le y \le 4, y \in R\}$

Hopefully you are not too puzzled for this one...

2. $\frac{1}{2}(y+1)^{2\cdot} = 3\cos\left(\frac{1}{2}\theta - 90°\right)+2^{2\cdot}$ (a + k)

$y+1^{-1} = 6\cos\left[\frac{1}{2}\theta - 90°\right]+4^{-1}$

$y = 6\cos\left[\frac{1}{2}\theta - 90°\right]+3$ (factor out ½)

$y = \underline{6}\cos\left[\frac{1}{2}(\theta - \underline{180°})\right]+3$

$90 \div \frac{1}{2}$

$90 \times 2 = 180°$

$a = 6$ $b = \frac{1}{2}$ $h = 180°$ $k = 3$

Mapping:

$(x, y) \to [2x + 180°, 6y + 3]$

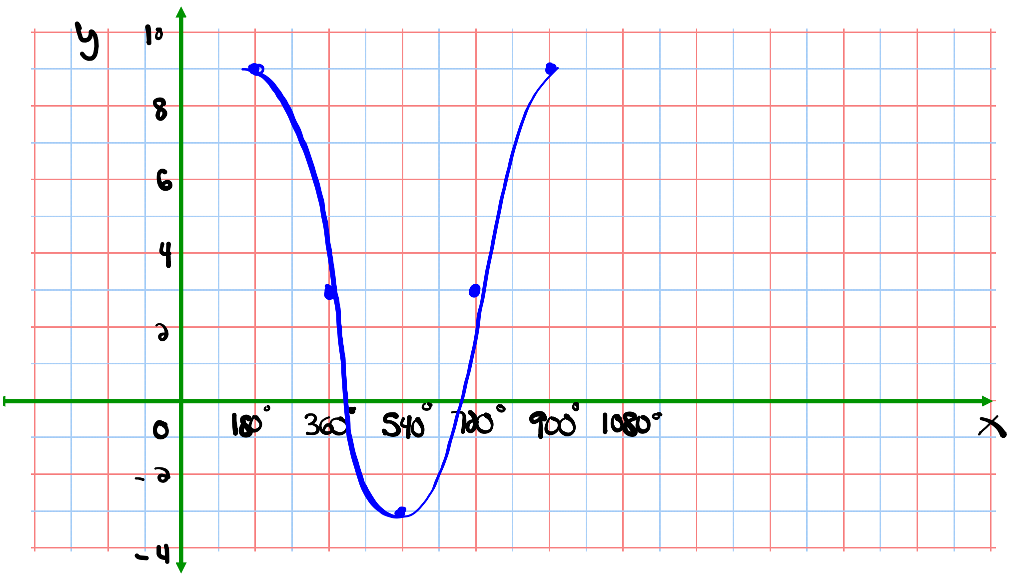
$y = \cos x$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
180°	9
360°	3
540°	-3
720°	3
900°	9

DOMAIN	$\{x \mid x \in R\}$
RANGE	$\{y \mid -3 \le y \le 9, y \in R\}$
AMPLITUDE	6
PERIOD	$P = \frac{360°}{\frac{1}{2}} = 720°$
PHASE SHIFT h	180° right
VERTICAL TRANSLATION	3 up
EQUATION OF SINUSOIDAL AXIS	$y = 3$



Solutions to the homework

① $y = 3\cos(x) - 2$

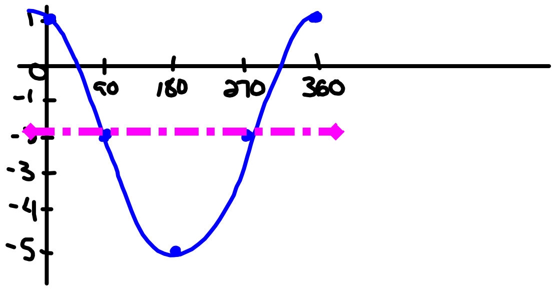
$a = 3$ $b = 1$ $h = 0$ $k = -2$ $(x, y) \rightarrow [x, 3y - 2]$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\rightarrow$

x	y
0	1
90	-2
180	-5
270	-2
360	1



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② $y = -\sin\left(2x - \frac{\pi}{6}\right)$

$y = -\sin\left[2\left(x - \frac{\pi}{12}\right)\right]$

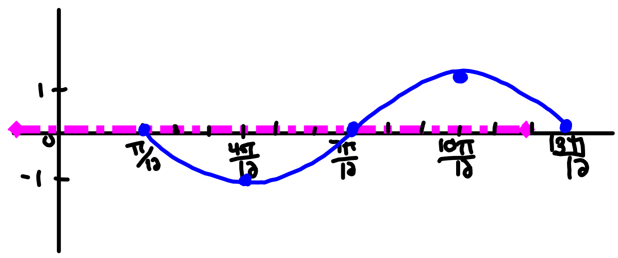
$a = -1$ $b = 2$ $h = \frac{\pi}{12}$ $k = 0$ $(x, y) \rightarrow \left[\frac{1}{2}x + \frac{\pi}{12}, -y\right]$

$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\rightarrow$

x	y
$\pi/12$	0
$\pi/3$	-1
$\frac{7\pi}{12}$	0
$5\pi/6$	1
$13\pi/12$	0



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③ $y = 4\sin(3x - 180°) + 2$

$y = 4\sin[3(x - 60°)] + 2$

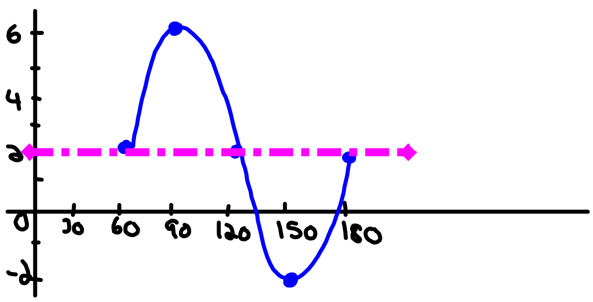
$a = 4$ $b = 3$ $h = 60°$ $k = 2$ $(x, y) \rightarrow \left[\frac{1}{3}x + 60°, 4y + 2\right]$

$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

$\rightarrow$

x	y
60	2
90	6
120	2
150	-2
180	2



⑤ $2y+3 = -4\sin\left(4x-\frac{\pi}{3}\right)-3$

$2y = -4\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-6$

$y = -2\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-3$

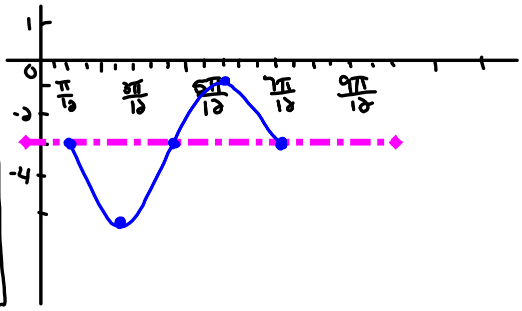
$a=-2$ $b=4$ $h=\frac{\pi}{12}$ $k=-3$ $(x,y)\rightarrow\left[\frac{1}{4}x+\frac{\pi}{12}, -2y-3\right]$

$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\rightarrow$

x	y
$\pi/12$	-3
$\frac{5\pi}{24}$	-5
$\pi/3$	-3
$11\pi/24$	-1
$7\pi/12$	-3



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⑥ $2 \cdot \frac{y-1}{\cancel{2}} = 2 \cdot 2\cos(\theta+45°)+0 \cdot 2$

$y-1 = 4\cos(\theta+45°)+0 +1$

$\boxed{y = 4\cos(\theta+45°)+1}$

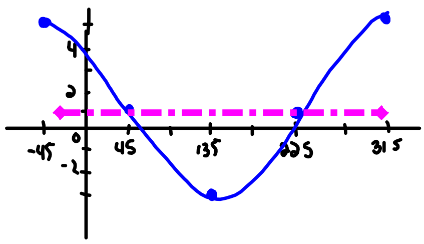
$a=4$ $b=1$ $h=-45°$ $k=1$ $(x,y)\rightarrow[x-45°, 4y+1]$

$y=\cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\rightarrow$

x	y
-45	5
45	1
135	-3
225	1
315	5



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① $\frac{1}{2}y-1 = \sin\left[2\left(x+\frac{\pi}{4}\right)\right]$

$2 \cdot \frac{1}{2}y = 2 \cdot \sin\left[2\left(x+\frac{\pi}{4}\right)\right]+1 \cdot 2$

$y = 2\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+2$

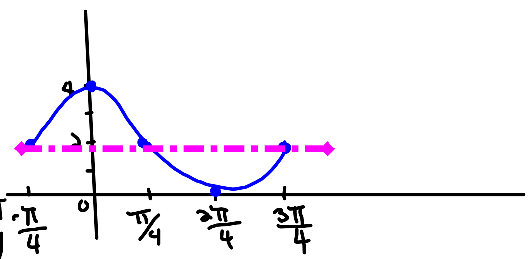
$a=2$ $b=2$ $h=-\frac{\pi}{4}$ $k=2$ $(x,y)\rightarrow\left[\frac{1}{2}x-\frac{\pi}{4}, 2y+2\right]$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\rightarrow$

x	y
$-\pi/4$	2
0	4
$\pi/4$	2
$\pi/2$	0
$\frac{3\pi}{4}$	2



⑧ $y = -4\cos(3x+90^\circ)-2$

$y = -4\cos[3(x+30)]-2$

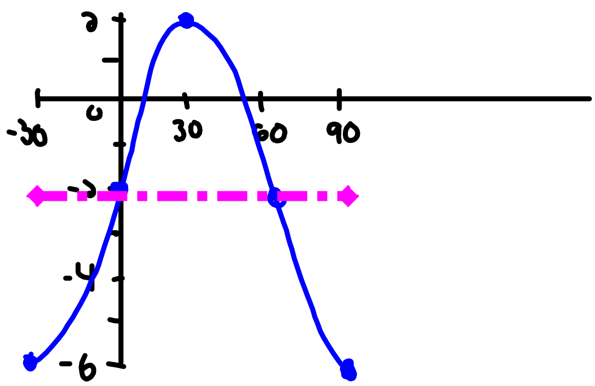
$a=-4$ $b=3$ $h=-30^\circ$ $k=-2$ $(x,y) \rightarrow \left[\frac{1}{3}x+30^\circ, -4y-2\right]$

$y=\cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\longrightarrow$

x	y
-30	-6
0	-2
30	2
60	-2
90	-6



⑦ $\frac{1}{2}y - 1 = \sin\left[2\left(\theta + \frac{\pi}{4}\right)\right]$

$$\cancel{2}\cdot \frac{1}{\cancel{2}}y \overset{2}{=} \sin\left[2\left(\theta + \frac{\pi}{4}\right)\right] + 1 \cdot 2$$

$$y = 2\sin\left[2\left(\theta + \frac{\pi}{4}\right)\right] + 2$$

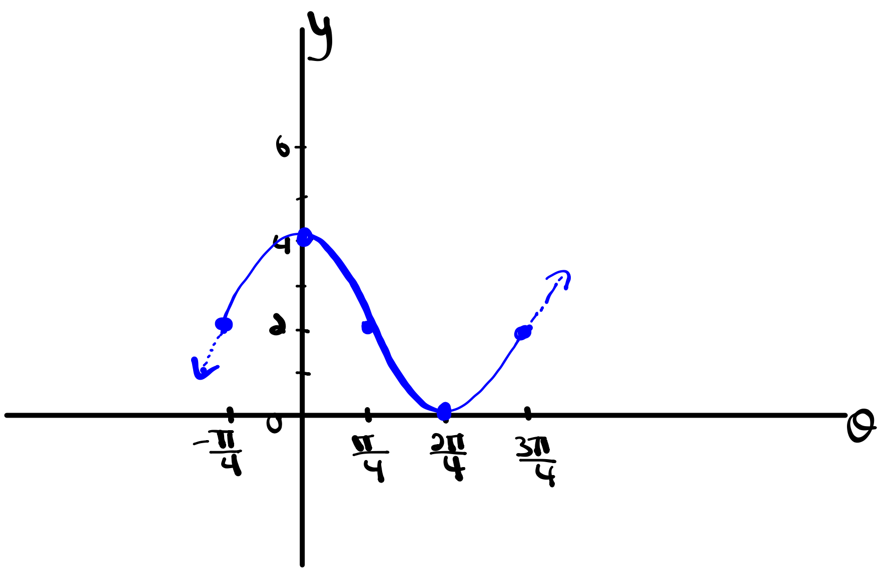
$a = 2$ $b = 2$ $h = -\frac{\pi}{4}$ $k = 2$

$$(\theta, y) \rightarrow \left[\frac{1}{2}\theta - \frac{\pi}{4},\ 2y + 2\right]$$

$y = \sin\theta$

θ	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

θ	y
$-\pi/4$	2
0	4
$\pi/4$	2
$2\pi/4$	0
$3\pi/4$	2



Extra Practice...

Worksheet # 1 - 8

Worksheet - Sketching Sinusoidal Relations

Attachments

- worksheet-sketching in radian measure.doc
- Worksheet - Finding the Equation.doc
- Worksheet - Sketching Trigonometric Functions.doc
- Worksheet Solns - Sketching Sinusoidal Relations.doc
- Worksheet - Sketching Sinusoidal relations (sept06).pdf
- Bonus Soln - Fox Population.doc
- Worksheet Solns - Applications of Sinusoidal Relations.doc
- Review - Practice Test for Sinusoidal Functions.doc
- Review - Trigonometric Functions(3)(4).doc