# Developing Trigonometric Functions from Properties...

**Develop a trigonometric function that fits the following description...**

- **Models a sine function**
- **Period is 120°**
- **Graph is reflected in x-axis**
- **Wave has a range of -8** $\leq y \leq 2$
- **Graph has a phase shift of 60° right**
- **Graph has a vertical translation of 3 units down**

$P = 120°$

$b = \frac{360°}{120°} = 3$

reflected in x-axis

$a < 0$ (negative)

$a = -5$

max = 2

min = -8

sin axis $= k = \frac{2 + (-8)}{2} = -3$

Amp = 5 (sin axis → max)

$h = 60°$

$$\boxed{y = -5\sin[3(x - 60°)] - 3}$$

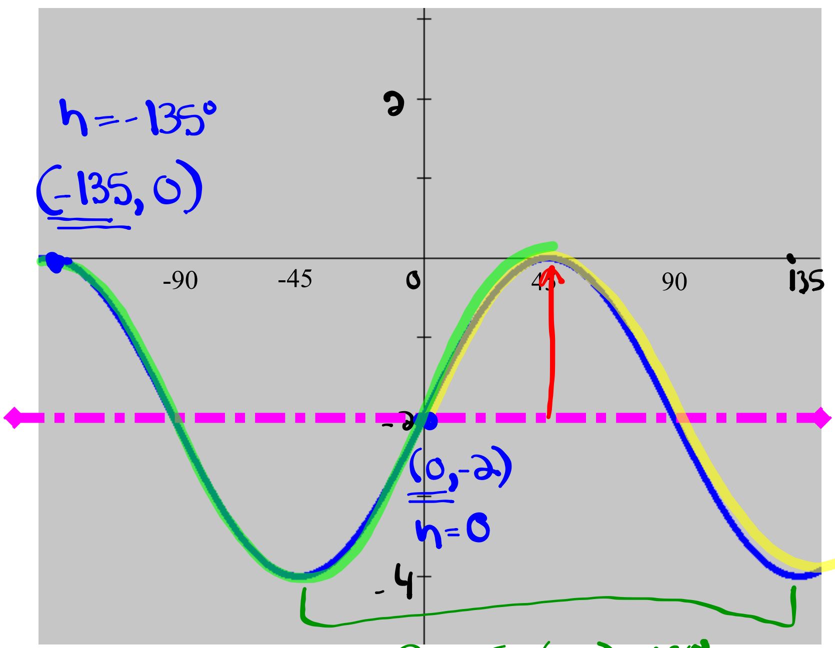
...Now we must learn how to identify all of the above information from a graph.

# Developing the Equation of a Sinusoidal Function

STEPS:

1) Identify & label the sinusoidal axis.
2) Determine the amplitude, period & vertical translation.
3) Pick a trig function & determine the corresponding phase shift.

- the choices are: positive sine, positive cosine, negative sine, negative cosine




P = 135 - (-45) = 180°

max = 0

min = -4

sin axis = $\frac{0+(-4)}{2} = -2$

Amp = 2 a = ±2

P = 180°

b = $\frac{360°}{180°} = 2$

k = -2

$y = \sin x$ (h = 0°)

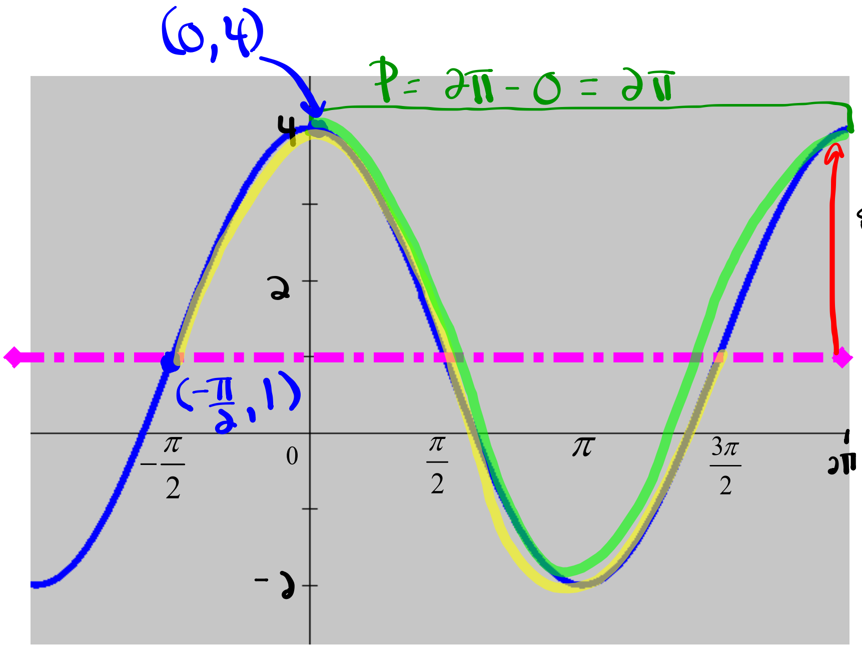
$y = 2\sin[2(x - 0°)] - 2$

$y = \cos x$ (h = -135°)

$y = 2\cos[2(x + 135°)] - 2$

# Finding an Equation from a Graph:

Determine a sine and a cosine equation for this graph




max = 4

min = -2

sin axis = $\frac{4+(-2)}{2} = 1$

Amp = 3  a = ±3

P = 2π

b = 1

k = 1

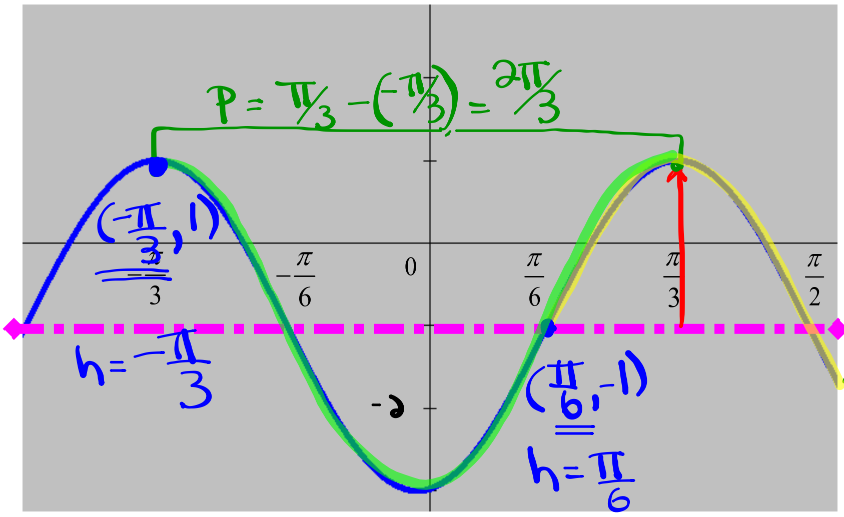
$y = \sin x \left(h = -\frac{\pi}{2}\right)$

$y = 3\sin\left[1\left(x + \frac{\pi}{2}\right)\right] + 1$

$y = \cos x \ (h = 0)$

$y = 3\cos[1(x - 0)] + 1$

Determine a sine and a cosine equation for this graph $2\pi \div \frac{2\pi}{3}$




max = 1

min = -3

sin axis: $\frac{1+(-3)}{2} = -1$

Amp = 2 $a = \pm 2$

$P = \frac{2\pi}{3}$

$b = 2\pi \times \frac{3}{2\pi} = 3$

$k = -1$

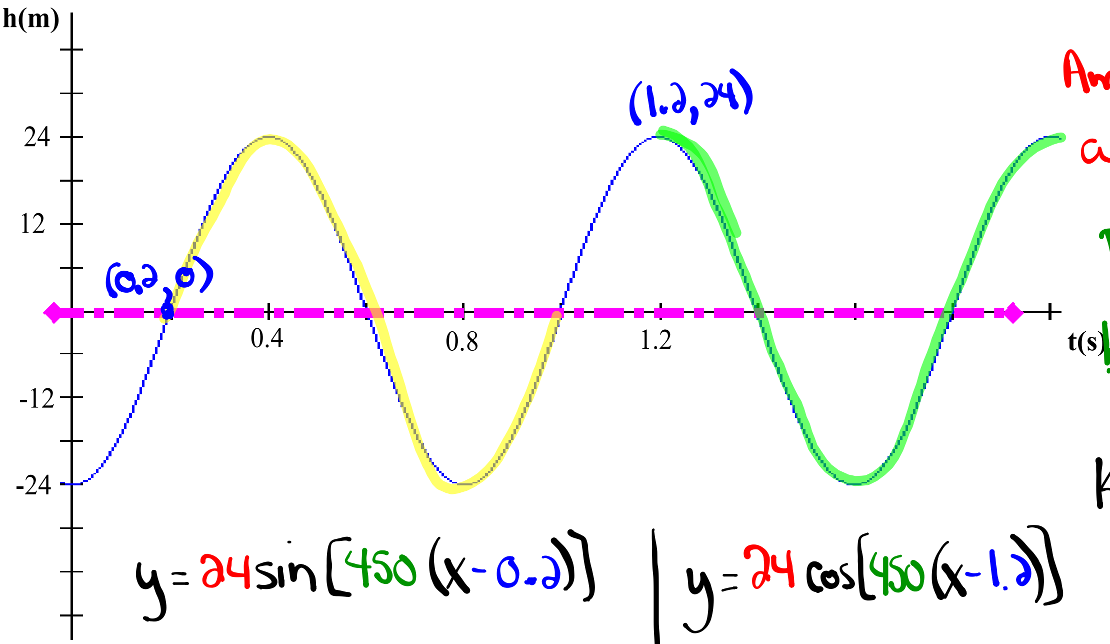
$y = \sin x \quad (h = \frac{\pi}{6})$

$y = 2\sin[3(x - \frac{\pi}{6})] - 1$

$y = \cos x \quad (h = -\frac{\pi}{3})$

$y = 2\cos[3(x + \frac{\pi}{3})] - 1$

Determine a sine and a cosine equation for this graph




Amp = 24

a = ± 24

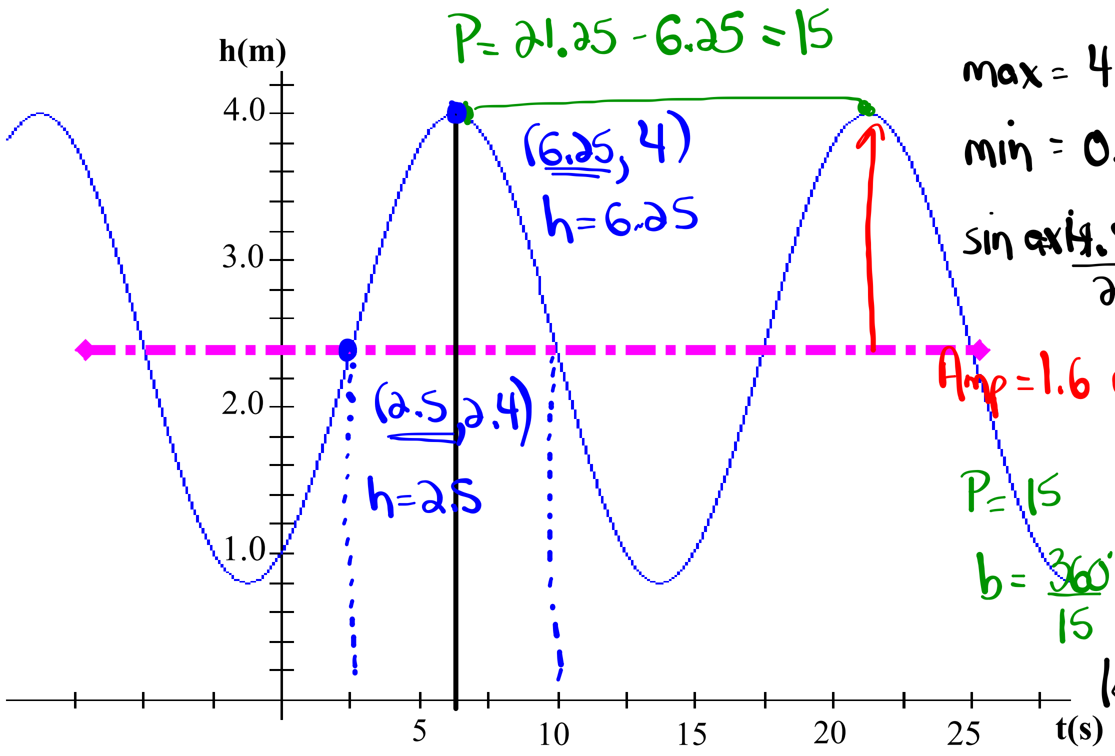
P = 0.8

$b = \frac{360}{0.8} = 450$

k = 0

$y = 24\sin[450(x - 0.2)]$ | $y = 24\cos[450(x - 1.2)]$

Find 4 equations to describe the graph.




max = 4

min = 0.8

sin axis $= \frac{4.8}{2} = 2.4$

Amp = 1.6 a = ($\pm$1.6)

P= 15

$b = \frac{360°}{15} = 24$

K = 2.4

$y = 1.6 \sin[24(x - 2.5)] + 2.4$

$y = -1.6 \sin[24(x - 10)] + 2.4$

# EXTRA PRACTICE...

Worksheet: #28 a) - f)

# Applications of Sinusoidal Relations

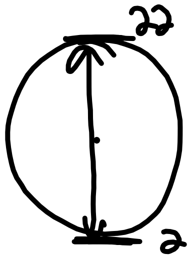
Strategy: (1) Translate ALL key pieces of information from the problem.
(2) Draw a sketch with ALL key points identified.
(3) Develop an equation that models the problem.
(4) Answer the question(s) being asked.

CHECK??? Do the numbers make sense?

* Radius = Amp.

* Count by $\frac{P}{4}$ on x-axis

* min + diameter = max
  min + radius = sin axis




$r = 10$
min = 2
max = ?

* From max to min or min to max is half the period

Ex: max @ 10s P = 10s
min @ 15s
max @ 20s

# Applications of Sinusoidal Functions

A carnival Ferris wheel with a radius of 14 m makes one complete revolution every 16 seconds. The bottom of the wheel is 1.5 m above the ground. If a person is at the top of the wheel when a stop watch is started, determine how high above the ground that person will be after 1 minute and 7 seconds? Sketch one period of this function.

Amp = 14 P = 16 min = 1.5 k = 15.5

$a = \pm 14$ $b = \frac{360}{16} = 22.5$ max = min + diameter $h = 0$

= 1.5 + 28

= 29.5 $y = \cos x$

sin axis = min + radius

= 1.5 + 14

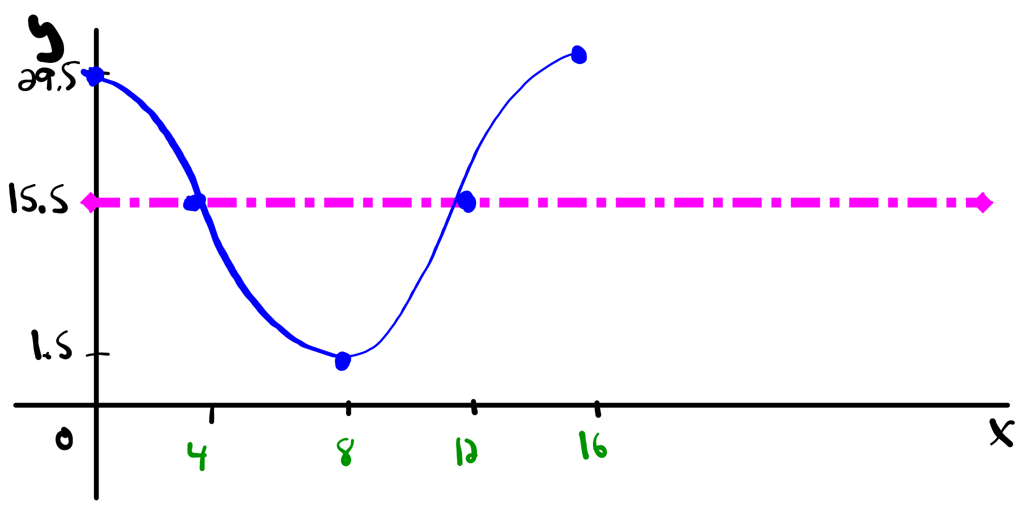
= 15.5

equation: $y = 14\cos[22.5(x)] + 15.5$

x = 1 min and 7 sec = 67 sec → Find y

$y = 14\cos[22.5(67)] + 15.5$

$y = 20.86$ m




Count by $\frac{P}{4} = \frac{16}{4} = 4$ s

# Ocean Tides

The alternating half-daily cycles of the rise and fall of the ocean are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5m above mean sea-level and to drop 6.5m below. The tide completes one cycle every 12 h. Assuming the height of water with respect to mean sea-level to be modelled by a sine function,

(a) draw the graph for a the motion of the tides for one complete day;
(b) find an equation for the graph in (a).

Amp = 6.5
$a = \pm 6.5$

max = 6.5
min = -6.5

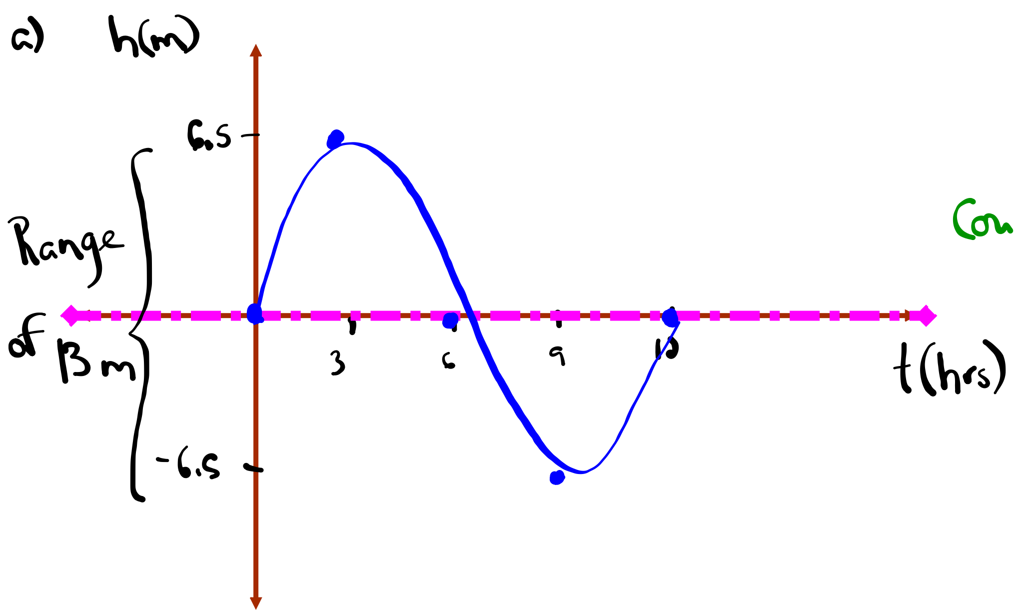
sin axis = $\frac{6.5 + (-6.5)}{2} = 0$
K = 0

P = 12 h

$b = \frac{360}{12} = 30$

h = 0

a)




Count by $\frac{12}{4} = 3$ on x axis

b) $y = 6.5\sin[30(x - 0)] + 0$

$y = 6.5\sin[30(x)]$

Homework

# Solutions to Homework

**Applications of Sinusoidal Functions**

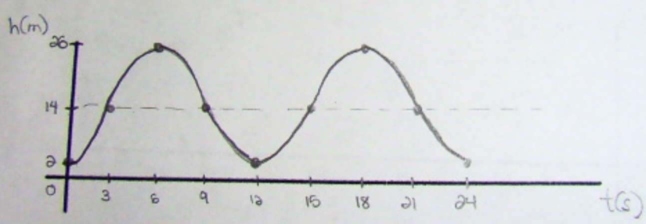
1. A Ferris wheel has a radius of 12m and makes one complete revolution every 12 seconds. The bottom of the wheel is 2m above the ground. ***If a person gets on at the bottom and goes up,*** determine the following:

a) Amplitude: = 12m b) Period: 12s c) b: 30 d) d: = 14m

e) Minimum Height: = 2m f) Maximum Height: = 26m → (2+24) min + diameter

g) Equation: $y = -12\cos(30x) + 14$

h) Sketch the graph for two revolutions (periods):



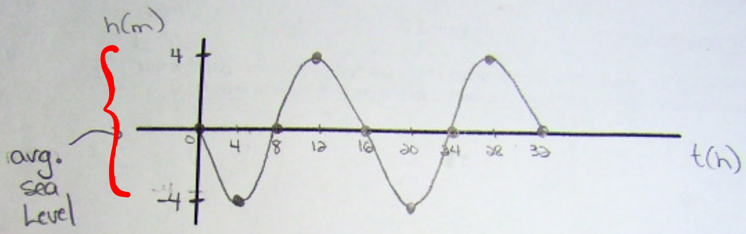

i) How high is the person at these times?

i) 12 seconds → $y = -12\cos(30(12)) + 14 = \boxed{2m}$

ii) 5 minutes → $y = -12\cos(30(300)) + 14 = \boxed{2m}$

2. The tide on the Miramichi River has a range of 8m. At time 0 the tide is midway and at 4 hours the tide is low. The tide completes one full cycle in 16 hours.

a) Draw the graph for a 32 hour period.




$a = 4$

$P = 16$

$b = \frac{360}{16} = 22.5$

$c = 0$

$d = 0$

b) Write the equation for this graph.

$y = -4\sin(22.5x)$

# Solutions to Homework

min = −3

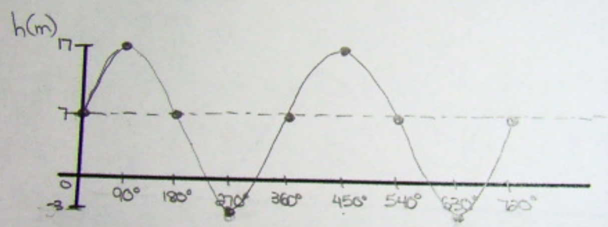
**3.** A water wheel has a radius of 10m. 3m of the wheel is submerged under water. If the wheel makes one revolution in 360 degrees and the bucket starts at the center and goes up, find:

a) Amplitude: = 10m b) Period: = 360° c) b: = 1 d) d: = 7

e) Min Height: = −3m f) Max Height: = 17m → (−3 + 20)
min + diameter

g) Equation of Graph: $y = 10\sin(x) + 7$

h) Sketch the graph for 2 revolutions:




i) How high will the bucket be after?

(i) 40° → $y = 10\sin(40°) + 7 = \boxed{13.43m}$

(ii) 110° → $y = 10\sin(110°) + 7 = \boxed{16.4m}$

(iii) 200° → $y = 10\sin(200°) + 7 = \boxed{3.58m}$

j) After how many degrees would the height be equal to 11? (Hint sub 11 in for y)

$11 = 10\sin(x) + 7$
$4 = 10\sin(x)$
$0.4 = \sin(x)$
$\boxed{x = 23.58°}$
$\sin^{-1}(0.4) = x$

**4.** A water wheel is defined by the equation $y = 7\cos[18(x)] + 4$

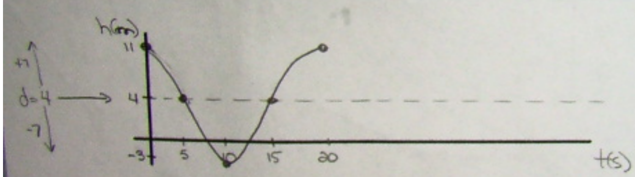
**Find:**

a = 7
b = 18
c = 0
d = 4

a) Amplitude = 7

b) Period = $\frac{360}{18}$ = 20s

c) Sketch the graph if the bucket starts at the top and goes down. Assume this function models the height of the bucket in meters over time in seconds.



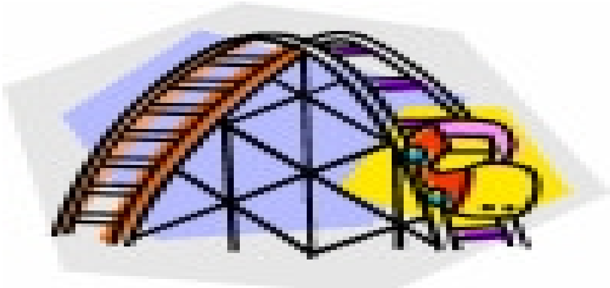

d) How much of the wheel is submerged? 3m (min = −3)

e) What is the Radius? = 7m
↑
Amp.

f) When is the bucket 5m high?

$5 = 7\cos(18x) + 4$
$1 = 7\cos(18x)$
$0.1428 = \cos(18x)$
$81.79 = 18x$
$\boxed{4.54s = x}$
$\cos^{-1}(0.1428) = 18x$

# Roller Coaster



John climbs on a roller coaster at Six Flags Amusement Park. An observer starts a stopwatch and observes that John is at a maximum height of 12 m at t = 13.2 s. At t = 14.6 s, John reaches a minimum height of 4 m.

a) Sketch a graph of the function.

b) Find an equation that expresses John's height in terms of time.

c) How high is John above the ground at t = 20.8 s? $(x = 20.8)$

Given:

min = 4m

max = 12m

sin axis = $k = \frac{12+4}{2} = 8$

$h = 13.2$

Amp = 4

$a = \pm 4$

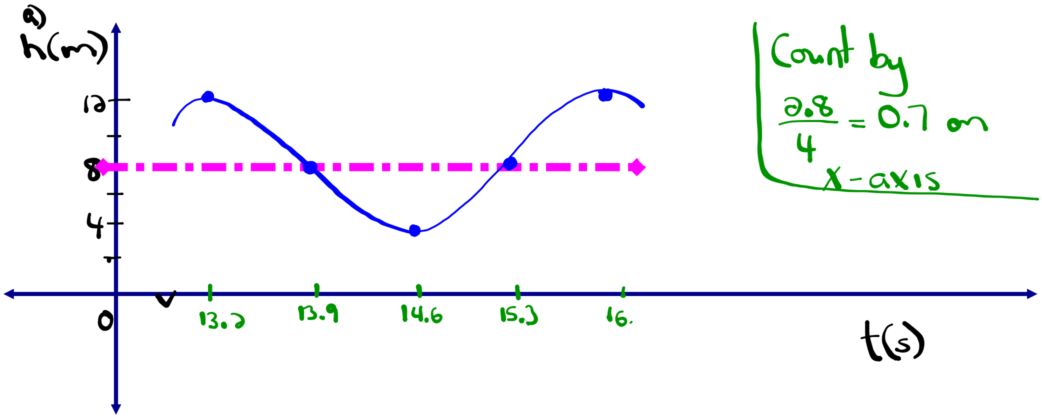
$P = 2(14.6 - 13.2)$

$P = 2(1.4) = 2.8s$

$b = \frac{360}{2.8} = 128.57$

Count by $\frac{2.8}{4} = 0.7$ on x-axis

a)




b) $y = 4\cos[128.57(x - 13.2)] + 8$

c) $y = 4\cos[128.57(20.8 - 13.2)] + 8$

$y = 4\cos[128.57(7.6)] + 8$

$\boxed{y = 7.1m}$

# Spring Problem

A weight attached to a long spring is being bounced up and down by an electric motor. As it bounces, its distance from the floor varies periodically with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight reaches its first low point 40 cm above the ground. The next high point, 60 cm above the ground, occurs at 1.9 seconds.

a) Sketch a graph of the function.

b) Write an equation expressing the distance above the ground in terms of the numbers of seconds the stopwatch reads.

c) When is the mass 43.75m high? $(y = 43.75)$

Given:

min = 40cm

max = 60cm

sin axis $= k = \frac{60+40}{2} = 50$

$h = 0.3$

Amp = 10

$a = \pm 10$

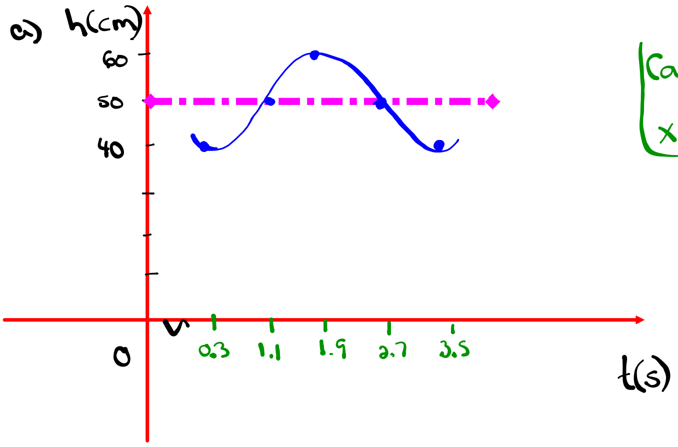
$P = 2(1.9 - 0.3)$

$P = 2(1.6) = 3.2s$

$b = \frac{360}{3.2} = 112.5$

Count by $\frac{3.2}{4} = 0.8$ x-axis.

a)




b) $y = -10\cos[112.5(x - 0.3)] + 50$

c) $y = -10\cos[112.5(x-0.3)] + 50$

$$43.75 = -10\cos[112.5(x-0.3)] + 50$$

$$\frac{-6.25}{-10} = \frac{-\cancel{10}\cos[112.5(x-0.3)]}{\cancel{-10}}$$

$$0.625 = \cos[112.5(x-0.3)]$$

$$\cos^{-1}(0.625) = 112.5(x-0.3)$$

$$\frac{51.32}{112.5} = \frac{\cancel{112.5}(x-0.3)}{\cancel{112.5}}$$

$$0.46 = x - 0.3$$

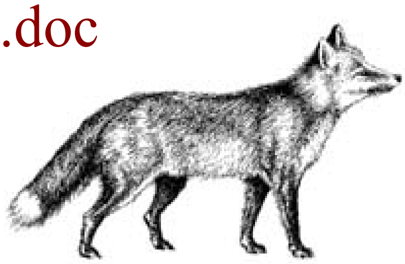
$$\boxed{0.76s = x}$$

# Biology!

Naturalists find that the populations of some animals varies periodically with time. Records started being taken at $t = 0$ years. A minimum number, 200 foxes, occurred when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t = 5.1$ years.

**Give two different times at which the fox population is 625.**

Bonus Soln - Fox Population.doc



Chapter 5 Test

10 Multiple Choice

① Sketch a sinusoidal function (mapping rule) Review #4

② Find an equation from the graph ($\sin\theta$ + $\cos\theta$) Review #5

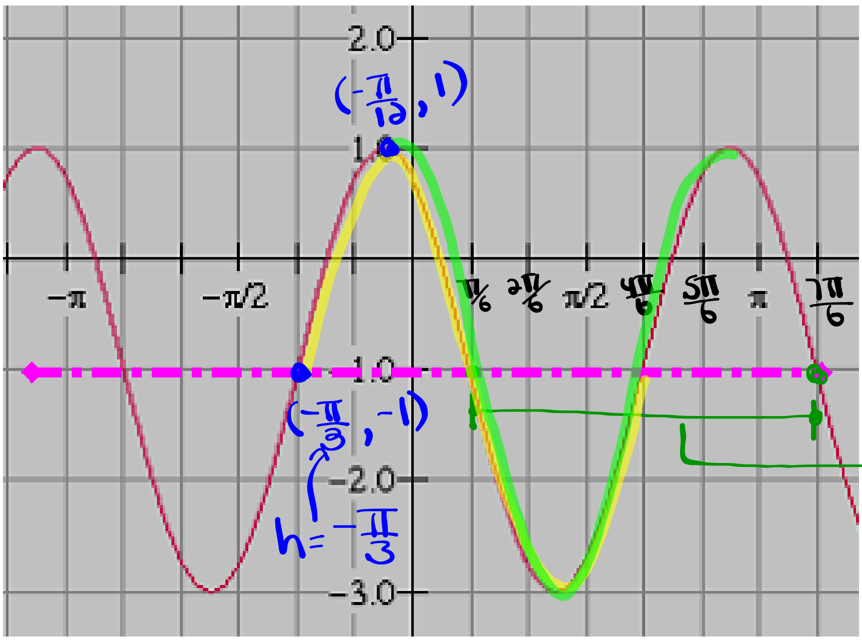
③ Word Problem ($h=0$) Review #1

④ Word Problem ($h=\#$) Review #2

## REVIEW - Sketching Trigonometric Functions

- sinusoidal functions
  - properties: domain/range, amplitude, period, phase shift, vertical translation, eq'n of sinusoidal axis, mapping notation.
  - sketching equation in standard form.
- finding the function (both a sine/cosine) given a graph
- solving trigonometric equations where period is not 360
- applications of sinusoidal functions.
  - sketch
  - develop a function
  - use function to answer question
- sketches of all SIX trigonometric ratios

Write both a cosine and sine function to describe the graph shown




(1) max = 1
min = -3
sin axis = $\frac{1-3}{2} = -1$

(1) $k = -1$

Amp = 2
$a = \pm 2$

$P = \frac{7\pi}{6} - \frac{\pi}{6} = \frac{6\pi}{6} = \pi$

$b = \frac{2\pi}{\pi} = 2$

| | |
|---|---|
| $y = \sin\theta$ $\left(h = -\frac{\pi}{3}\right)$ | $y = \cos\theta$ $\left(h = -\frac{\pi}{12}\right)$ |
| $y = 2\sin\left[2\left(\theta + \frac{\pi}{3}\right)\right] - 1$ | $y = 2\cos\left[2\left(\theta + \frac{\pi}{12}\right)\right] - 1$ |

Complete the chart shown below and sketch one full cycle of this function

$$-2\cdot \quad -\frac{1}{2}(y+2) = -2\cdot\sin\left(3\theta+\frac{\pi}{8}\right)-2 \cdot -2$$

| | |
|---|---|
| DOMAIN | $\{\theta \mid \theta \in R\}$ |
| RANGE | $\{y \mid 0<y<4, y \in R\}$ |
| AMPLITUDE | 2 |
| PERIOD | $\frac{2\pi}{3}$ |
| PHASE SHIFT | $\pi/24$ left |
| VERTICAL TRANSLATION | 2 up |
| EQUATION OF SINUSOIDAL AXIS | $y=2$ |

$$y+2 = -2\sin\left(3\theta+\frac{\pi}{8}\right)+4$$

$$y = -2\sin\left(3\theta+\frac{\pi}{8}\right)+2$$

$$y = -2\sin\left[3\left(\theta+\frac{\pi}{24}\right)\right]+2$$

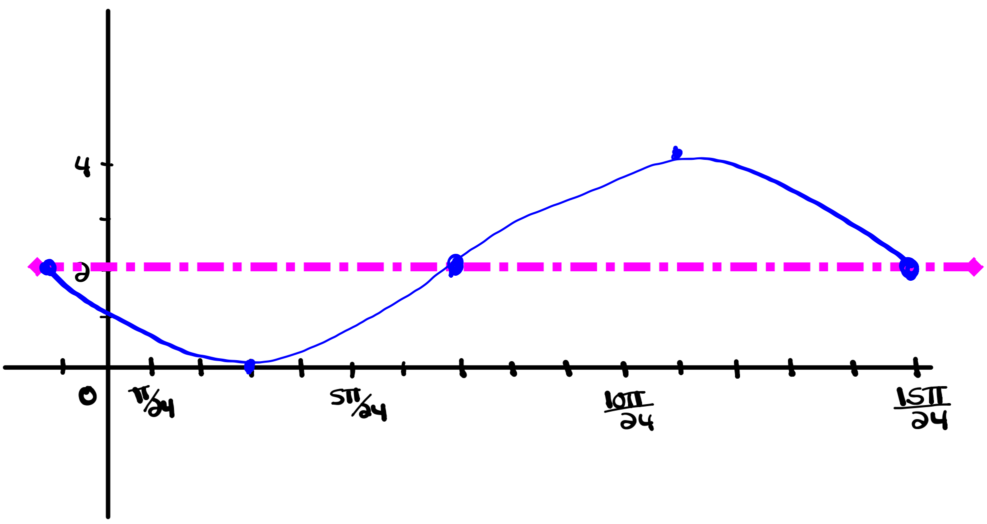
$$y = -2\sin\left[3\left(\theta+\frac{\pi}{24}\right)\right]+2$$

$a=-2$ $\quad b=3$ $\quad h=-\frac{\pi}{24}$ $\quad k=2$ $\quad (\theta, y) \rightarrow \left[\frac{1}{3}\theta-\frac{\pi}{24}, -2y+2\right]$

$y=\sin\theta$ $\longrightarrow$

| $\theta$ | $y$ |
|---|---|
| 0 | 0 |
| $\pi/2$ | 1 |
| $\pi$ | 0 |
| $3\pi/2$ | -1 |
| $2\pi$ | 0 |

| $\theta$ | $y$ |
|---|---|
| $-\pi/24$ | 2 |
| $3\pi/24$ | 0 |
| $7\pi/24$ | 2 |
| $11\pi/24$ | 4 |
| $15\pi/24$ | 2 |



The Canadian National Historic Windpower Centre, at Etzikom, Alberta, has various styles of windmills on display. The tip of the blade of one windmill reaches its minimum height of 8 m above the ground at a time of 2 s. Its maximum height is 22 m above the ground. The tip of the blade rotates 12 times per minute.

**a)** Write a sine or a cosine function to model the rotation of the tip of the blade.

**b)** What is the height of the tip of the blade after 4 s? $x = 4$

**c)** For how long is the tip of the blade above a height of 17 m in the first 10 s?

min = 8m

max = 22m

sin axis = $k = \frac{22+8}{2} = 15$m

$P = \frac{60\text{s}}{12} = 5$s

Amp = 7

$a = \pm 7$

$b = \frac{360°}{5} = 72$ | $b = \frac{2\pi}{5}$

h = 2 s (at its min)

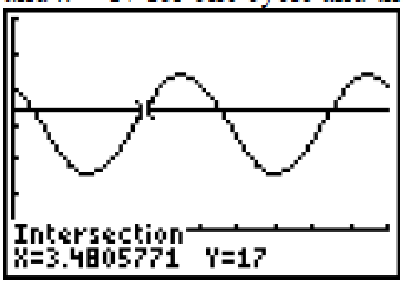
a) $y = -7\cos[72(x-2)] + 15$

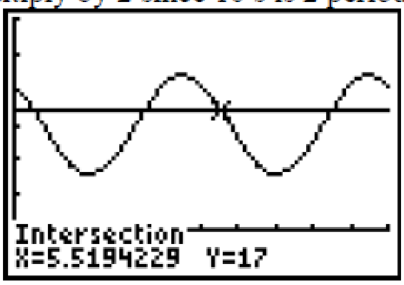
or $y = -7\cos\left[\frac{2\pi}{5}(x-2)\right] + 15$

b) $y = -7\cos[72(4-2)] + 15$

$\boxed{y = 20.7\text{m}}$

**c)** Determine the points of intersection of the graphs of $h = 7\cos\frac{2\pi}{5}(t + 0.5) + 15$ and $h = 17$ for one cycle and then multiply by 2 since 10 s is 2 periods.




$2(5.5194 - 3.4806) = 4.0776$

The tip of the blade above a height of 17 m in the first 10 s is approximately 4.08 s.

# PRACTICE TIME...

Review - Practice Test for Sinusoidal Functions.doc

# Practice Test Solutions

**Part A: Multiple Choice**

1. A
2. D
3. A
4. C
5. B
6. D
7. A
8. D
9. B
10. A
11. A (second hand)
12. C
13. A
14. C
15. D
16. D
17. B
18. D
19. A
20. A

**Part B: Open Response**

1. $-\frac{5}{4}$

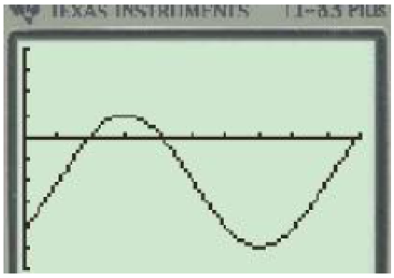
2. (i)

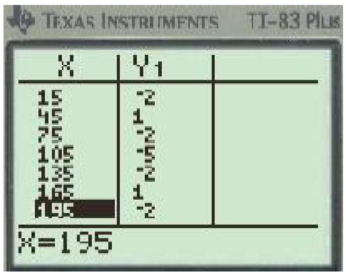
$$y = 3\sin\frac{3}{2}(x - 160°) - 6$$

$$y = 3\cos\frac{3}{2}(x + 20°) - 6$$

(ii) $(x, y) \rightarrow \left(\frac{2}{3}x + 160°, 3y - 6\right)$

3.






4. 10.28 m

# MORE PRACTICE???

Review - Trigonometric Functions.doc

## SOLUTIONS

1. (a) 39° (b) 53°

2. (a) −2 (b) $\frac{7-2\sqrt{3}}{4}$

3. (a) II (b) II

4. (a) −1.2799 (b) −1.0864
   c) 1.2690 (d) 39°
   (e) −5 (f) 25°

5. $\sin\theta = \frac{-\sqrt{5}}{5}$ $\csc\theta = -\sqrt{5}$

   $\cos\theta = \frac{-2\sqrt{5}}{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$

   $\tan\theta = \frac{1}{2}$ $\cot\theta = 2$

6. $\frac{-\sqrt{10}}{2}$

8. Amp = 3
Period = 180°
V.T. = Up 2
P.S. = none
Domain: $0° \le \theta \le 360°$

(b) Amp = 2
Period = 120°
V.T. = Down 2
P.S. = 60° left
Domain: $\theta \in R$

(c) Amp = 2
Period = 720°
V.T. = Up 5
P.S. = none
Domain: $-90 \le \theta \le 360°$
Range: $-3 \le y \le 7$

(d) Amp = 6
Period = 360°
V.T. = None
P.S. = 90° right
Domain: $\theta \in R$
Range: $-6 \le y \le 6$

10. 11.9 m

11. 46.2 cm

# Let's look at the detailed solutions...

Worksheet Solns - Applications of Sinusoidal Relations.doc

## Check-Up...

Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t, and that y is positive when Tarzan is over water and negative when he is over land. Jane finds that when t = 2.8 seconds, Tarzan is at one end of his swing, 23 feet from the riverbank, over the water. She finds when t = 6.3 seconds he reaches the other end of his swing and is situated 17 feet from the riverbank, however this time over land.

(a) Where was Tarzan when Jane started the stopwatch?

(b) Provide three instances when Tarzan was located at a position 14 feet from the riverbank, over the water.

What about graphs of other trigonometric functions ???

## Graph the Tangent Function

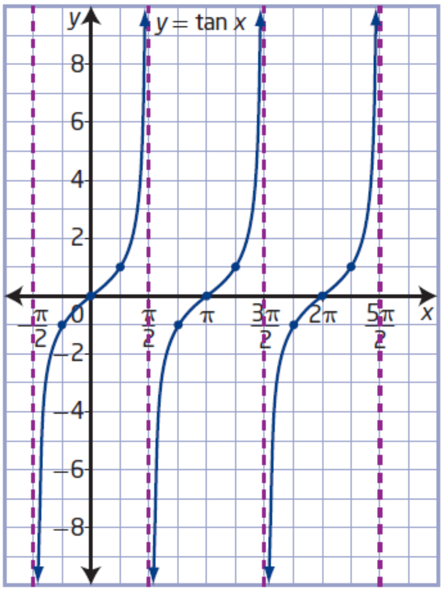
Graph the function $y = \tan \theta$ for $-2\pi \leq \theta \leq 2\pi$. Describe its characteristics.

| Angle Measure | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| *y*-coordinate on Tangent Line | | | | | | | | | |

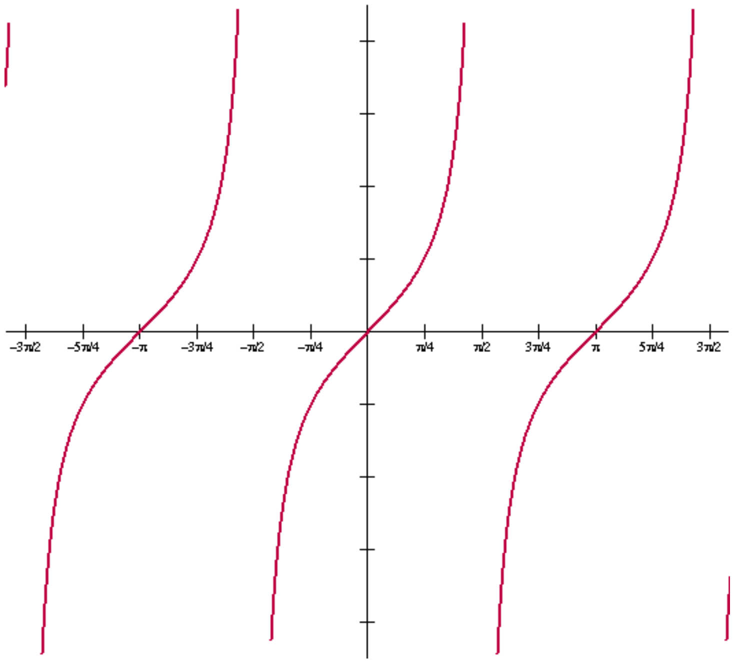
## Key Ideas

- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph $y = \tan x$, draw one asymptote; draw the points where $y = -1$, $y = 0$, and $y = 1$; and then draw another asymptote.
- The tangent function $y = \tan x$ has the following characteristics:
  - The period is $\pi$.
  - The graph has no maximum or minimum values.
  - The range is $\{y \mid y \in R\}$.
  - Vertical asymptotes occur at $x = \frac{\pi}{2} + n\pi$, $n \in I$.
  - The domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in R, n \in I\right\}$.
  - The $x$-intercepts occur at $x = n\pi$, $n \in I$.
  - The $y$-intercept is 0.

How can you determine the location of the asymptotes for the function $y = \tan x$?



# y = tan θ



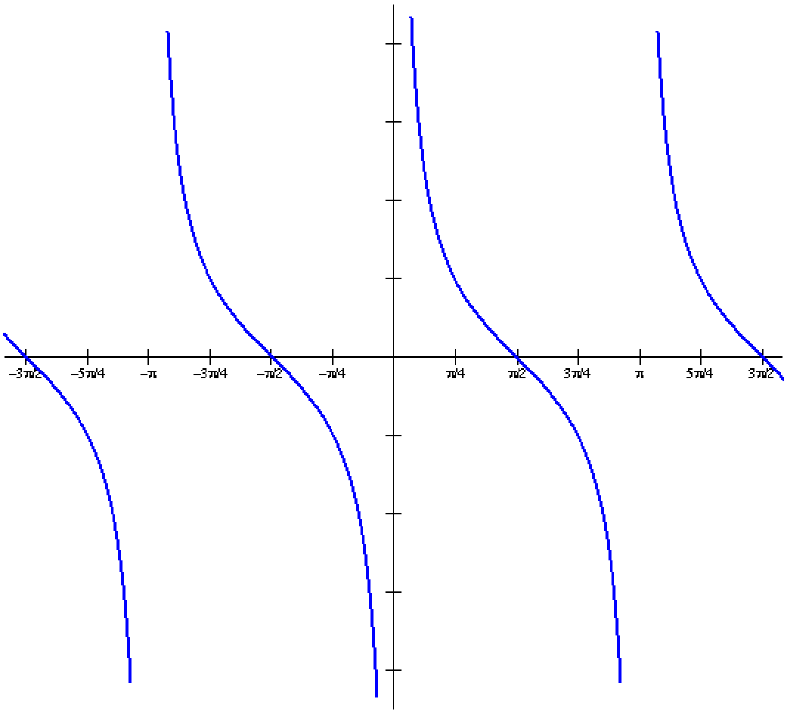

What would the graph of **cot θ** look like?

**REMEMBER:**

$$\tan x = \frac{1}{\cot x}$$

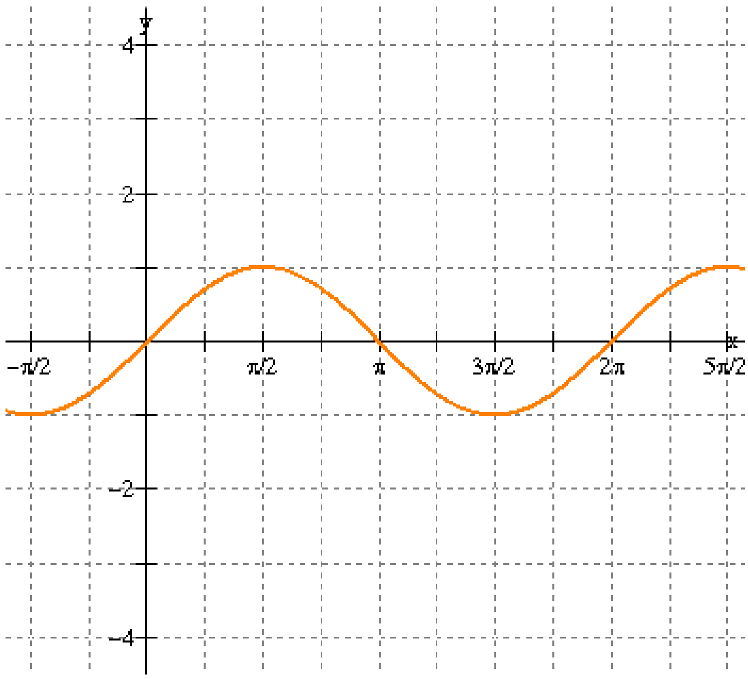
where tan x = 0,
cot x is undefined

# y = cot θ



# Graphs of Other Trigonometric Functions

y = sin θ



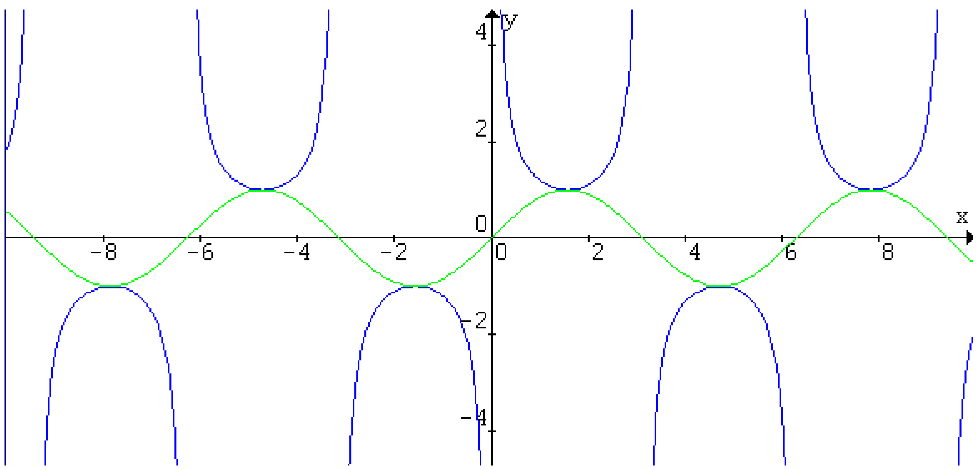

What would the graph of **csc θ** look like?

**REMEMBER:**

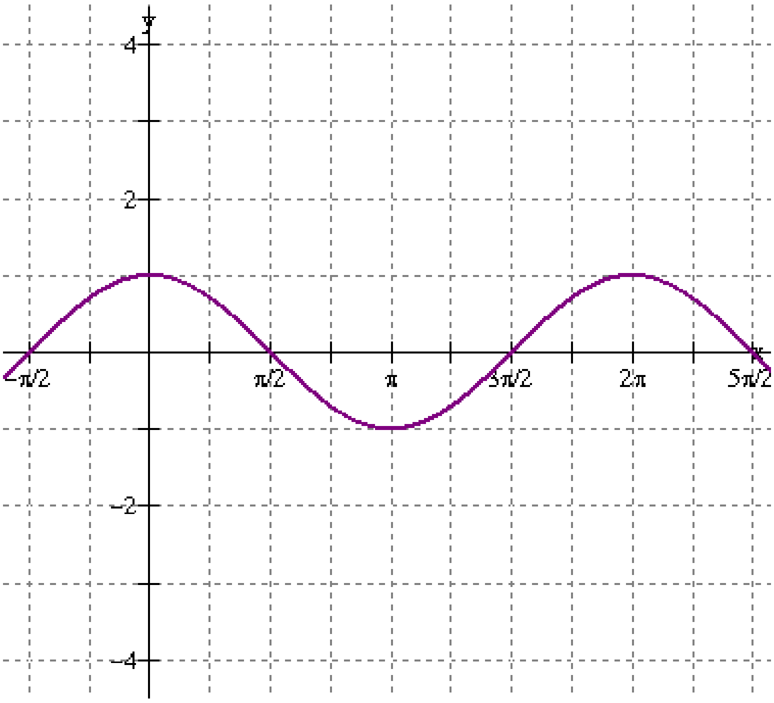
$$\csc\theta = \frac{1}{\sin\theta}$$

where sin x = 0,
csc x is undefined

y = sin x y = csc x



## $y = \cos \theta$



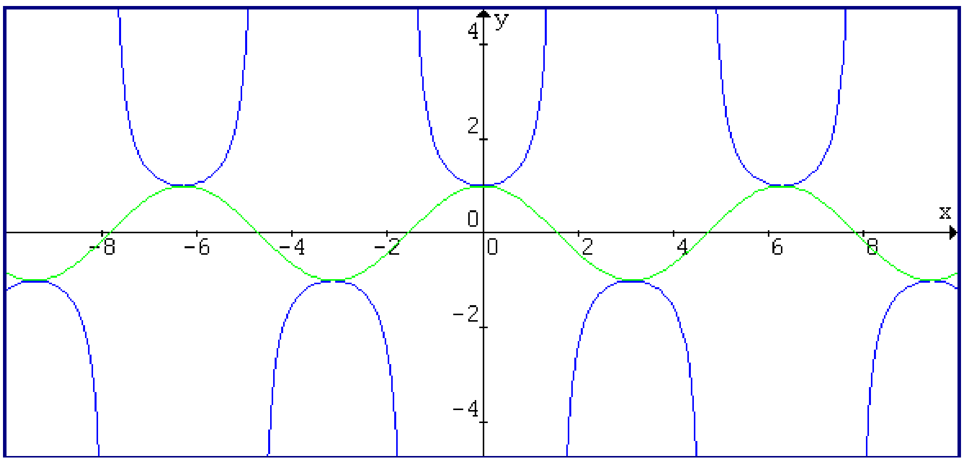

What would the graph of **sec θ** look like?

**REMEMBER:**

$$\sec\theta = \frac{1}{\cos\theta}$$

where cos x = 0,
sec x is undefined

$y = \cos x$ $y = \sec x$



Attachments

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc

Sketching Sinusoidal Functions #2.pdf

Sketching Sinusoidal Functions #2.doc

Sketching Sinusoidal Functions #3 (Solutions).doc

worksheet-sketching in radian measure.doc