

② b) $xy^2 = 45$ if $y = 3$ and $\frac{dy}{dt} = 6$ Find $\frac{dx}{dt}$

(i) $xy^2 = 45$
 $x(3)^2 = 45$
 $9x = 45$
 $x = 5$

(ii) $(xy^2) = 45$
 $\frac{dx}{dt} y^2 + x 2y \frac{dy}{dt} = 0$

(iii) $\frac{dx}{dt} (3)^2 + 2(5)(3)(6) = 0$
 $9 \frac{dx}{dt} + 180 = 0$
 $9 \frac{dx}{dt} = -180$

$$\frac{dx}{dt} = -20$$

Review Sheet:

① Find $\frac{dy}{dx}$ for $x^2 + 2xy + 2y^2 = 4$

$$2x + 2y + 2x \frac{dy}{dx} + 4y \frac{dy}{dx} = 0$$

$$2x \frac{dy}{dx} + 4y \frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx} (2x + 4y) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 4y} = \frac{-2(x+y)}{2(x+2y)} = \boxed{-\frac{x+y}{x+2y}}$$

Review Sheet

$$\textcircled{2} \quad y = \sqrt{x^2+2x} = (x^2+2x)^{1/2}$$

$$y' = \frac{1}{2}(x^2+2x)^{-1/2} (2x+2)$$

$$y' = \frac{2x+2}{2(x^2+2x)^{1/2}} = \boxed{\frac{x+1}{(x^2+2x)^{1/2}}}$$

$$y'' = \frac{1(x^2+2x)^{-1/2} - (x+1)\left(\frac{1}{2}\right)(x^2+2x)^{-3/2} (2x+2)}{x^2+2x}$$

$$y'' = \frac{(x^2+2x)^{-1/2} - (x+1)^2(x^2+2x)^{-3/2}}{x^2+2x}$$

$$y'' = \frac{(x^2+2x)^{-1/2} [x^2+2x - (x+1)^2]}{x^2+2x}$$

$$y'' = \frac{x^2+2x - x^2 - 2x - 1}{(x^2+2x)(x^2+2x)^{1/2}} = \boxed{\frac{-1}{(x^2+2x)^{3/2}}}$$

Review Sheet

③ $xy^2 = 12$, $\frac{dy}{dt} = 6$, $y = 2$, find $\frac{dx}{dt}$

(i) $xy^2 = 12$
 $x(2)^2 = 12$
 $4x = 12$

$x = 3$

(ii) $xy^2 = 12$

$\frac{dx}{dt}y^2 + 2xy\frac{dy}{dt} = 0$

(iii) $\frac{dx}{dt}(2)^2 + 2(3)(2)(6) = 0$

$4\frac{dx}{dt} + 72 = 0$

$\frac{dx}{dt} = -18$

$\frac{dx}{dt} = -18$

④

$$-3xy^2 - x^3 = 3y$$

$$-3y^2 - 3x \cdot 2y \frac{dy}{dx} - 3x^2 = \frac{3dy}{dx}$$

$$-3y^2 - 6xy \frac{dy}{dx} - 3x^2 = \frac{3dy}{dx}$$

$$-3y^2 - 3x^2 = \frac{3dy}{dx} + 6xy \frac{dy}{dx}$$

$$-3y^2 - 3x^2 = \frac{dy}{dx} (3 + 6xy)$$

$$\frac{-3y^2 - 3x^2}{3 + 6xy} = \frac{dy}{dx}$$

$$\frac{-y^2 - x^2}{1 + 2xy} = \frac{dy}{dx}$$

$$-\frac{(y^2 + x^2)}{1 + 2xy} = \frac{dy}{dx}$$

$$-\frac{(x^2 + y^2)}{2xy + 1} = \frac{dy}{dx}$$

Review Sheet

$$\textcircled{5} \quad s = -2t^3 + t^2 + 4t + 1$$

$$s' = -6t^2 + 2t + 4 \quad (\text{velocity})$$

$$s'' = -12t + 2 \quad (\text{acceleration})$$

$$\begin{aligned} \text{a) } s'(3) &= -6(3)^2 + 2(3) + 4 \\ &= -54 + 6 + 4 \\ &= -44 \text{ m/s} \end{aligned}$$

$$\begin{aligned} s''(3) &= -12(3) + 2 \\ &= -36 + 2 \\ &= -34 \text{ m/s}^2 \end{aligned}$$

$$\text{b) } a = -10 \text{ m/s}^2$$

$$s'' = -12t + 2$$

$$-10 = -12t + 2$$

$$12t = 12$$

$$\boxed{t = 1 \text{ sec}}$$

$$\begin{aligned} \text{c) } s(1) &= -2(1)^3 + (1)^2 + 4(1) + 1 \\ &= -2 + 1 + 4 + 1 \\ &= \boxed{4 \text{ m}} \end{aligned}$$

$$\begin{aligned} s'(1) &= -6(1)^2 + 2(1) + 4 \\ &= -6 + 2 + 4 \\ &= \boxed{0 \text{ m/s}} \end{aligned}$$

Review Sheet.

$$\textcircled{6} \quad h = 100 + 15t - 5t^2$$

$$h' = 15 - 10t \quad (\text{velocity})$$

$$h'' = -10 \quad (\text{acceleration})$$

a) max height ($h' = 0$)

$$0 = 15 - 10t$$

$$10t = 15$$

$$\boxed{t = 1.5 \text{ sec}}$$

b) hits ground ($h = 0$)

$$0 = 100 + 15t - 5t^2$$

$$0 = 5(20 + 3t - t^2)$$

$$0 = 5(-t^2 + 3t + 20)$$

$$a = -1 \quad b = 3 \quad c = 20$$

c) s' @ 6.2 sec

$$s'(6.2) = 15 - 10(6.2)$$

$$s'(6.2) = 15 - 62 = \boxed{-47 \text{ m/s}}$$

d) constant acceleration
of -10 m/s^2

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-3 \pm \sqrt{(3)^2 - 4(-1)(20)}}{2(-1)}$$

$$t = \frac{-3 \pm \sqrt{9 + 80}}{-2}$$

$$\cancel{t = \frac{-3 + \sqrt{89}}{-2}} \quad \left| \quad t = \frac{-3 - \sqrt{89}}{-2}\right.$$

$$\cancel{t = -3} \quad \text{sec} \quad \boxed{t = 6.2 \text{ sec}}$$

Review Sheet

$$\textcircled{7} \quad s = 2t^3 - 6t^2 + 9t$$

$$s' = 6t^2 - 12t + 9$$

$$s'' = 12t - 12$$

$$a) \quad 12t - 12 = 0$$

$$12t = 12$$

$$\boxed{t = 1 \text{ sec}}$$

$$b) \quad 12t - 12 > 0$$

$$12t > 12$$

$$\boxed{t > 1}$$