Questions From Homework

② d) $\int \frac{x+1}{x^2+2x-6}\,dx$

$u = x^2+2x-6$

$du = 2x+2\,dx$

$\frac{1}{2}du = x+1\,dx$

$= \int \frac{1}{u} \cdot \left(\frac{1}{2}du\right)$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

Questions From Homework

(4) c) $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin^3\theta}\,d\theta = \int_{1/2}^{1} \frac{1}{u^3}\cdot du$

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta\, d\theta$

θ	u
$\pi/2$	1
$\pi/6$	$1/2$

$= \displaystyle\int_{1/2}^{1} u^{-3}\,du$

$= \dfrac{u^{-2}}{-2}\Big|_{1/2}^{1}$

$= -\dfrac{1}{2u^2}\Big|_{1/2}^{1}$

$= \dfrac{-1}{2(1)^2} - \dfrac{-1}{2(1/2)^2}$

$= \dfrac{-1}{2} - -2$

$= -\dfrac{1}{2} + \dfrac{4}{2} = \dfrac{3}{2}$

Warm Up

Find: $\int 5x^2 \sin(4x^3+1)dx$

$$= -\frac{5}{12}\cos(4x^3-1)+C$$

$\int x\sqrt{2x^2-5}\,dx$

$$= \frac{(2x^2-5)^{3/2}}{6}+C$$

$\int \cot x\,dx$

$$= \ln|\sin x|+C$$

Differential and Integral Calculus 120

$\int$ Integration by Parts $\int$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

As we have discussed before, every differentiation rule has a corresponding integration rule.

The rule that corresponds to the Product Rule for differentiation is called the rule for integration by parts.

The product rule stated that if f and g are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

In the notation for indefinite integrals this equation becomes... $\int[f(x)g'(x)dx + g(x)f'(x)dx] = f(x)g(x)$

or $\int f(x)g'(x)dx + \int g(x)f'(x)dx = f(x)g(x)$

which can be rearranged as:

$$\boxed{\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx}$$

this formulas above is called the **formula for integration by parts**

It is perhaps easier to remember in the following notation..... Let $u = f(x)$ and $v = g(x)$

then the differentials are: $du = f'(x)dx$ $dv = g'(x)dx$

And by the Substitution Rule, the formulas becomes...

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

Let's do an example.... Find: $\int \underbrace{x}_{u}\underbrace{\sin x dx}_{dv}$

It helps when you stick to this pattern:

we need to make an appropriate choice for u and dv

$u = x$ $dv = \sin x\, dx$

$du = 1dx$ $v = -\cos x$

Again, the goal in using integration by parts is to obtain a simpler integral than the one we started with... so we must decide on what u and dv are very carefully!

In general, when deciding on a choice for u and dv, we usually try to choose u = f(x) to be a function that becomes simpler when differentiated... (or at least NOT more complicated) as long as dv = g'(x)dx can be readily integrated to give v.

$$\int \underbrace{x}_{u}\underbrace{\sin x dx}_{dv} = x(-\cos x) - \int -\cos x\, dx$$

$$= -x\cos x + \int \cos x dx$$

$$\boxed{= -x\cos x + \sin x + C}$$

Find: $\int \underbrace{x}_{u}\underbrace{e^{x}dx}_{dv}$ $= xe^{x} - \int e^{x}dx$

$$\boxed{= xe^{x} - e^{x} + C}$$

It helps when you stick to this pattern:

$u = $ <u>x</u> $dv = $ <u>$e^{x}dx$</u>

$du = $ <u>$1\,dx$</u> $v = $ <u>e^{x}</u>

Find: $\int \underbrace{x}_{u} \underbrace{\cos(3x)dx}_{dv} = x\left(\frac{1}{3}\sin(3x)\right) - \int \frac{1}{3}\sin 3x\, dx$

It helps when you stick to this pattern:

$u =$ x $dv =$ $\cos(3x)dx$

$du =$ $1dx$ $v =$ $\frac{1}{3}\sin(3x)$

$$= \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\, dx$$

$$= \frac{1}{3}x\sin 3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x\right)$$

$$\boxed{= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C}$$

Find: $\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv}$

It helps when you stick to this pattern:

$u = \ln x$ $\quad dv = 1\,dx$

$du = \frac{1}{x}dx$ $\quad v = 1x$

$$= x\ln x - \int \cancel{x}\frac{1}{\cancel{x}}\,dx$$

$$= x\ln x - \int 1\,dx$$

$$= x\ln x - 1x + C$$

$$\boxed{= x\ln x - x + C}$$

Find: $\int \underbrace{x^2}_{u} \underbrace{\sin(3x)dx}_{dv} = x^2\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x \, 2x\,dx$

$$= -\frac{1}{3}x^2\cos 3x - \int \left(-\frac{2}{3}\right) x\cos 3x\,dx$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\int \underbrace{x}_{u}\,\underbrace{\cos 3x\,dx}_{dv}$$

It helps when you stick to this pattern:

(I) $u = x^2$ $\quad dv = \sin 3x\,dx$

$du = 2x\,dx$ $\quad v = -\frac{1}{3}\cos 3x$

(II) $u = x$ $\quad dv = \cos 3x\,dx$

$du = 1\,dx$ $\quad v = \frac{1}{3}\sin 3x$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[x\left(\frac{1}{3}\sin 3x\right) - \int \frac{1}{3}\sin 3x\,dx\right]$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x - \frac{1}{3}\int \underbrace{\sin 3x\,dx}_{-\frac{1}{3}\cos 3x}\right]$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]$$

$$\boxed{= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C}$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$$

Find: $\int \underbrace{x^2}_{u} \underbrace{e^x dx}_{dv}$

It helps when you stick to this pattern:

(i) $u = x^2$ $dv = e^x dx$

$du = 2x\,dx$ $v = e^x$

(ii) $u = x$ $dv = e^x dx$

$du = 1dx$ $v = e^x$

$$= x^2 e^x - \int e^x \, 2x \, dx$$

$$= x^2 e^x - 2\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv}$$

$$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$$

$$= x^2 e^x - 2\left[x e^x - e^x\right]$$

$$\boxed{= x^2 e^x - 2xe^x + 2e^x + C}$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

Find: $\int x^2 \ln x\,dx$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\,dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{1}{3}x^3\right)$$

$$\boxed{= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

It helps when you stick to this pattern:

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{3}x^3$

Find:

$$\int \underbrace{e^x}_{u} \underbrace{\sin x\,dx}_{dv} = e^x(-\cos x) - \int(-\cos x)e^x\,dx$$

It helps when you stick to this pattern:

$u = e^x \quad dv = \sin x\,dx$

$du = e^x\,dx \quad v = -\cos x$

$$\int e^x \sin x\,dx = -e^x\cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x\,dx}_{dv}$$

$u = e^x \quad dv = \cos x$

$du = e^x\,dx \quad v = \sin x$

$$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int \sin x\, e^x\,dx$$

$$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$$

$$\frac{2\int e^x \sin x\,dx}{2} = \frac{e^x\sin x - e^x\cos x}{2}$$

$$\int e^x \sin x\,d = \frac{1}{2}\left(e^x\sin x - e^x\cos x\right) + C$$

$$\boxed{= \frac{1}{2}e^x(\sin x - \cos x) + C}$$

this one as a little twist,
because you cannot get to a simpler integral
- rearrange for double the initial integral and
divide by two!

Find:

$$\int e^x \cos x dx$$

It helps when you
stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

this one as a little twist,
because you cannot get to a simpler integral
- rearrange for double the initial integral and
divide by two!

$$= \frac{1}{2} e^x (\cos x + \sin x) + C$$

Find: $\int \underbrace{\sin^{-1} x}_{u}\, \underbrace{dx}_{dv}$

It helps when you stick to this pattern:

$u =$ $\sin^{-1}x$ $dv =$ 1 dx

$du = \frac{1}{\sqrt{1-x^2}}dx$ $v =$ x

may require substitution rule as well...

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= x\sin^{-1}x - \int x \cdot \frac{1}{\sqrt{1-x^2}}dx$

$= x\sin^{-1}x - \int \frac{1}{(u)^{1/2}} \cdot -\frac{1}{2}du$

$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}du$

$= x\sin^{-1}x + \frac{1}{2}(2u^{1/2}) + C$

$= x\sin^{-1}x + u^{1/2} + C$

$$\boxed{= x\sin^{-1}x + \sqrt{1-x^2} + C}$$

It helps when you stick to this pattern:

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$$\int_1^e \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv} = x\ln x \Big|_1^e - \int_1^e \cancel{x}\frac{1}{\cancel{x}}dx$$

$$= x\ln x \Big|_1^e - \int_1^e dx$$

$$= x\ln x \Big|_1^e - x \Big|_1^e$$

$$= e\ln e - 1\ln 1 - (e - 1)$$

$$= e(1) - 1(0) - e + 1$$

$$= e - e + 1$$

$$= 1$$

It helps when you
stick to this pattern:

$u = \ln(\cos x)$ $dv = \sin x\,dx$

$du = \dfrac{-\sin x}{\cos x}dx$ $v = -\cos x$

$$\int_0^{\frac{\pi}{3}} \sin x \ln(\cos x)dx = -\cos x \ln(\cos x)\Big|_0^{\pi/3} - \int_0^{\pi/3} \cancel{-\cos x}\left(\frac{-\sin x}{\cancel{\cos x}}\right)dx$$

$$= -\cos x\ln(\cos x)\Big|_0^{\pi/3} - \int_0^{\pi/3} \sin x\,dx$$

$$= -\cos x\ln(\cos x)\Big|_0^{\pi/3} + \cos x\Big|_0^{\pi/3}$$

$$= -\cos\left(\frac{\pi}{3}\right)\ln\left(\cos\frac{\pi}{3}\right) + \cos(0)\ln(\cos 0) + \left(\cos\left(\frac{\pi}{3}\right) - \cos(0)\right)$$

$$= \left(-\frac{1}{2}\right)\ln\left(\frac{1}{2}\right) + 1\ln(1) + \left(\frac{1}{2} - 1\right)$$

$$= -\frac{1}{2}\ln\left(\frac{1}{2}\right) + 1(0) + \frac{1}{2} - 1$$

$$= -\frac{1}{2}\ln\left(\frac{1}{2}\right) + \frac{1}{2} - \frac{2}{2}$$

$$= -\frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{2}$$

This is cool with me!

$$= -\frac{1}{2}\ln(2)^{-1} - \frac{1}{2}$$

$$= \frac{1}{2}\ln(2) - \frac{1}{2}$$

$$\boxed{= \frac{1}{2}(\ln(2) - 1)}$$

<u>Homework</u> - Exercise 11.4 - pp. 515 - Q. 1,2,4 (omit 1 h)

Recall #6 a) on page 512

⑥ a) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$= \int \frac{1}{\cos x} \cdot \sin x \, dx$

$= \int \frac{1}{u} \cdot -du$

$= -\int \frac{1}{u} du$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln|\cos x^{-1}| + C$

$= \ln\left|\frac{1}{\cos x}\right| + C$

$= \boxed{\ln|\sec x| + C}$