Questions from homework

③ a) $f(x) = 2x^3 - 3x^2 \qquad -2 \le x \le 2$

$f'(x) = 6x^2 - 6x$

$f'(x) = 6x(x-1)$

CV: $x = 0, 1$

$f(0) = 0 \qquad (0,0)$

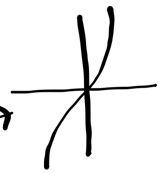
$f(1) = -1 \qquad (1,-1)$

$f(-2) = -16 - 12 = -28 \qquad (-2,-28)$ abs min

$f(2) = 16 - 12 = 4 \qquad (2,4)$ abs max

The First Derivative Test

If f has a local maximum or minimum at c, then c must be a critical value of f (Fermat's Theorem), but not all critical numbers give rise to a maximum or minimum. For instance, recall that 0 is a critical number of the function $y = x^3$ but this function has no maximum or minimum at a critical number.



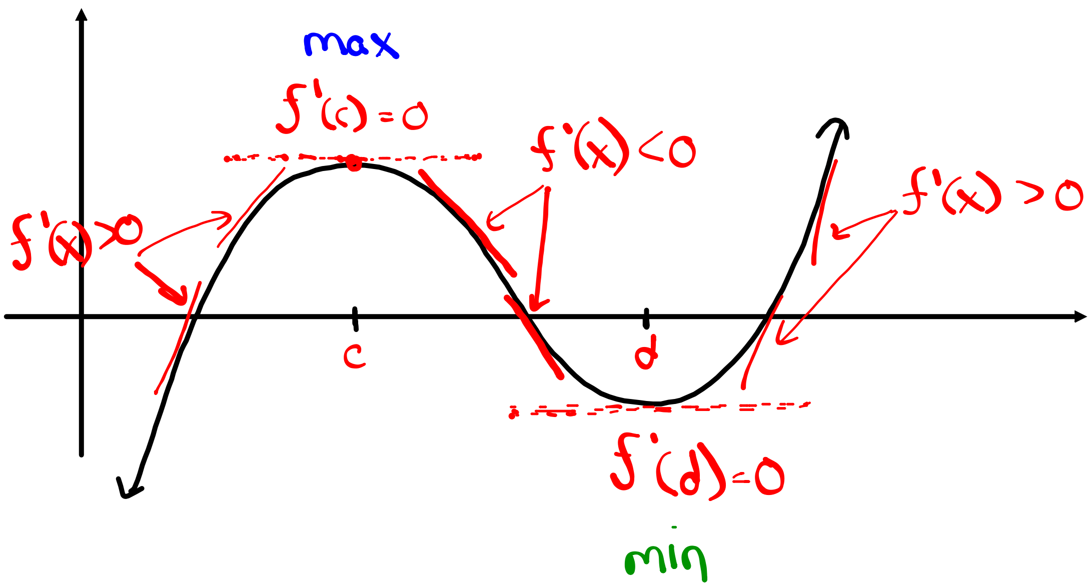
$y = x^3$

$y' = 3x^2$

CV:
$0 = 3x^2$
$0 = x^2$
$0 = x$

$+ \quad +$
$(-1) \quad 0 \quad (1)$

One way of solving this is suggested by the figure below.
If f is increasing to the left of a critical number c and decreasing to the right of c, then f has a local max at c.
If f is decreasing to the left of a critical number c and increasing to the right of c, then f has a local min at c.



The First Derivative Test

Let c be a critical number of a continuous function f.

1. If $f'(x)$ changes from positive to negative at c, then f has a local max at c.

2. If $f'(x)$ changes from negative to positive at c, then f has a local min at c.

3 If $f'(x)$ does not change signs at c, then f has no max or min at c.

Example 1

Find the local maximum and minimum values of
$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(x) = 3(x^2 - 1)$

$f'(x) = 3(x-1)(x+1)$

$0 = 3(x-1)(x+1)$

CV: $x - 1 = 0$ | $x + 1 = 0$

$x = 1$ | $x = -1$

max min

$+ \downarrow - \downarrow +$

(-2) -1 (0) 1 (2)

Increasing on $(-\infty, -1)$ + $(1, \infty)$

Decreasing on $(-1, 1)$

max @ $x = -1$

$f(x) = x^3 - 3x + 1$

$f(-1) = (-1)^3 - 3(-1) + 1$

$f(-1) = -1 + 3 + 1$

$f(-1) = 3$

$\boxed{(-1, 3)}$

min @ $x = 1$

$f(x) = x^3 - 3x + 1$

$f(1) = (1)^3 - 3(1) + 1$

$f(1) = 1 - 3 + 1$

$f(1) = -1$

$\boxed{(1, -1)}$

Example 2

Find the local maximum and minimum values of
g(x) = x⁴ - 4x³ - 8x² - 1. Use this information to sketch the graph of g.

$g'(x) = 4x^3 - 12x^2 - 16x$

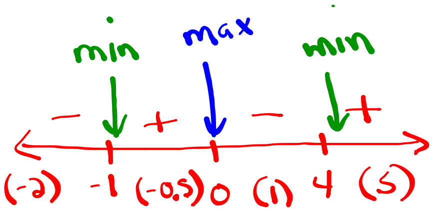
$g'(x) = 4x(x^2 - 3x - 4)$

$g'(x) = 4x(x-4)(x+1)$

$0 = 4x(x-4)(x+1)$

CV: $4x = 0$, $x = 0$ | $x - 4 = 0$, $x = 4$ | $x + 1 = 0$, $x = -1$

$\boxed{x = -1, 0, 4}$



Increasing on $(-1, 0)$ and $(4, \infty)$

Decreasing on $(-\infty, -1)$ and $(0, 4)$

min @ $x = -1$

$g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g(-1) = (-1)^4 - 4(-1)^3 - 8(-1)^2 - 1$

$g(-1) = 1 + 4 - 8 - 1$

$g(-1) = -4$

$\boxed{(-1, -4)}$

min @ $x = 4$

$g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g(4) = (4)^4 - 4(4)^3 - 8(4)^2 - 1$

$g(4) = 256 - 256 - 128 - 1$

$g(4) = -129$

$\boxed{(4, -129)}$

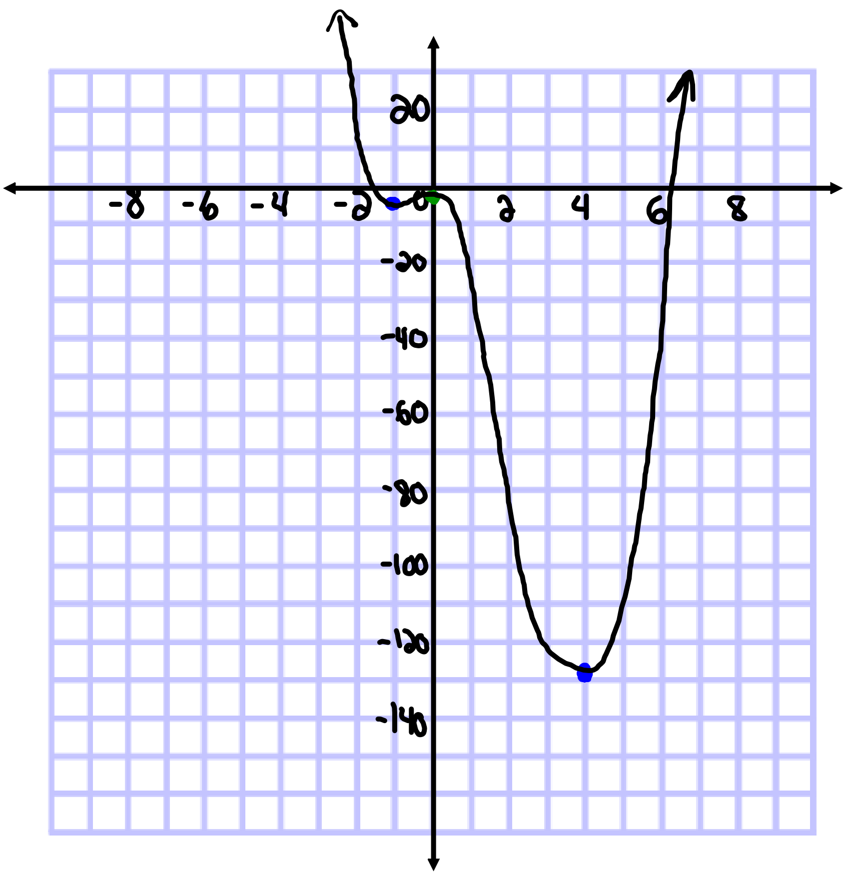
max @ $x = 0$

$g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g(0) = (0)^4 - 4(0)^3 - 8(0)^2 - 1$

$g(0) = -1$

$\boxed{(0, -1)}$


20
-8
-6
-4
-2
0
2
4
6
8
-20
-40
-60
-80
-100
-120
-140

The First Derivative Test

(for absolute extreme values)

Let c be a critical number of a continuous function f.

1. If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum value.

2. If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum value.

Homework

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