Questions From Homework

Making a Complete Sketch

Example:

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to…

- Intercepts $f(x)$
- ~~Symmetry~~
- Asymptotes (No asymptotes for polynomial functions)
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values $f(x)$
- $f''(x)$ ~~Concavity~~ and ~~Points of Inflection~~. $f(x)$
- Sketch the Curve

$f(x) = 3x^5 - 5x^3$ $f'(x) = 15x^4 - 15x^2$ $f''(x) = 60x^3 - 30x$

$f(x) = x^3(3x^2 - 5)$ $f'(x) = 15x^2(x^2 - 1)$ $f''(x) = 30x(2x^2 - 1)$

$f'(x) = 15x^2(x-1)(x+1)$

① x-int ($y=0$)

$f(x) = x^3(3x^2 - 5)$

$0 = x^3(3x^2 - 5)$

$x^3 = 0$ | $3x^2 - 5 = 0$

$x = 0$ | $\frac{3x^2}{3} = \frac{5}{3}$

$(0,0)$ | $x^2 = \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}}$

$(1.29, 0)$ & $(-1.29, 0)$

② y-int ($x=0$)

$f(x) = 3x^5 - 5x^3$

$f(0) = 3(0)^5 - 5(0)^3$

$f(0) = 0$

$(0,0)$

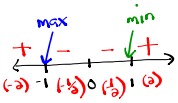
③ Intervals of Inc/Dec.

$0 = 15x^2(x-1)(x+1)$

$15x^2 = 0$ | $x - 1 = 0$ | $x + 1 = 0$

$x^2 = 0$ | $x = 1$ | $x = -1$

$x = 0$



Increasing on $(-\infty, -1) \cup (1, \infty)$

Decreasing on $(-1, 0)$ & $(0, 1)$ or $(-1, 1)$

CV: $x = -1, 0, 1$

④ max @ $x = -1$

$f(x) = 3x^5 - 5x^3$

$f(-1) = 3(-1)^5 - 5(-1)^3$

$f(-1) = -3 + 5$

$f(-1) = 2$

$(-1, 2)$

⑤ min @ $x = 1$

$f(x) = 3x^5 - 5x^3$

$f(1) = 3(1)^5 - 5(1)^3$

$f(1) = 3 - 5$

$f(1) = -2$

$(1, -2)$

⑥ Intervals of Concavity:

$f''(x) = 30x(2x^2 - 1)$

$0 = 30x(2x^2 - 1)$

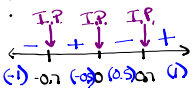
$30x = 0$ | $2x^2 - 1 = 0$

$x = 0$ | $2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

$x \approx \pm 0.707$



CV: $x = -0.7, 0, 0.7$

CD on $(-\infty, -0.7) \cup (0, 0.7)$

CU on $(-0.7, 0) \cup (0.7, \infty)$

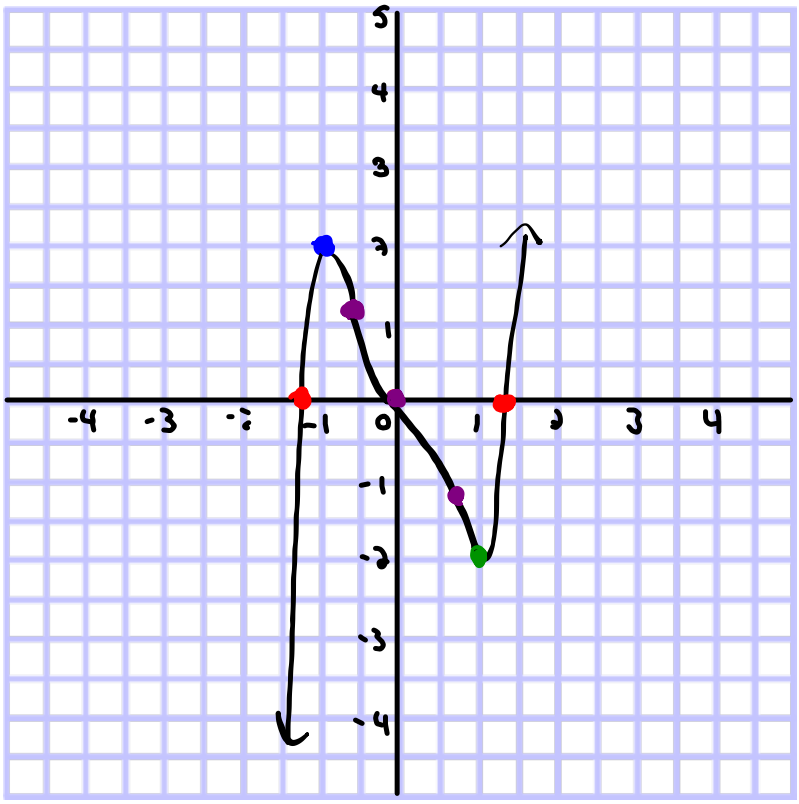
⑦ Inflection Points

$f(x) = 3x^5 - 5x^3$

$f(0) = 3(0)^5 - 5(0)^3 = 0 - 0 = 0$ $(0,0)$

$f(-0.7) = 3(-0.7)^5 - 5(-0.7)^3 = 0.504 - 1.715 = -1.2$ $(-0.7, 1.2)$

$f(0.7) = 3(0.7)^5 - 5(0.7)^3 = 0.504 - 1.715 = -1.2$ $(0.7, -1.2)$


5
4
3
2
1
-4
-3
-2
-1
0
1
2
3
4
-1
-2
-3
-4

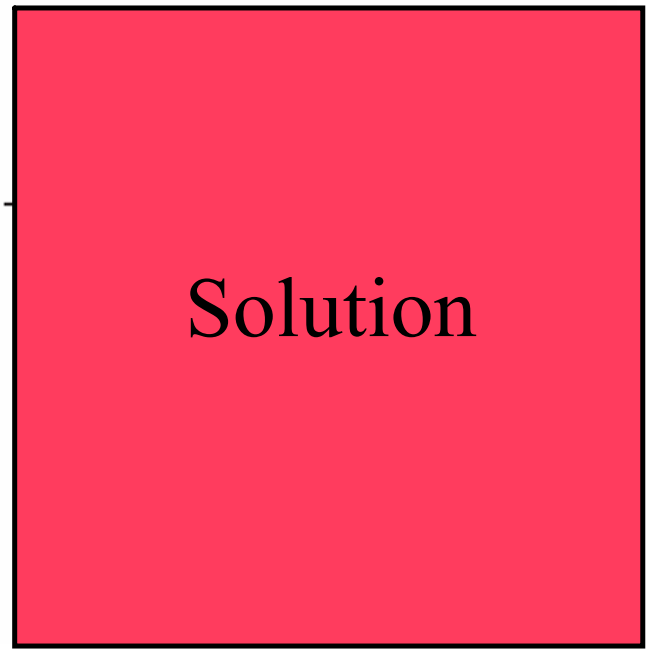
Assignment:

Sketch $f(x) = x^2 + x^3$

homework

Examine the function $f(x) = x^4 - 4x^3$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve


Solution

homework

Examine the function $f(x) = \frac{x^2}{1-x^2}$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

homework

Examine the function $f(x) = \frac{x^2}{x-7}$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve